



Atomic squeezing in three-level atoms with effective dipole–dipole atomic interaction

O. Civitarese*, M. Reboiro, L. Rebón, D. Tielas

Department of Physics, University of La Plata, c.c. 67 1900, La Plata, Argentina

ARTICLE INFO

Article history:

Received 5 November 2009
 Received in revised form 10 February 2010
 Accepted 8 March 2010
 Available online 11 March 2010
 Communicated by P.R. Holland

Keywords:

Squeezing
 Three-level atoms
 Laser field
 Squeezed states

ABSTRACT

The appearance of atomic squeezing in a system of three-level atoms placed in a two-mode cavity, is analyzed. The effects of effective dipole–dipole interactions between atoms are discussed. It is found that these interactions washed-out the squeezing, while the increase in the mean number of photons, of the initial coherent state, moderates this effect significantly.

© 2010 Elsevier B.V. All rights reserved.

1. Introduction

In the last years, several proposals to build quantum information devices have been made [1]. Among them, quantum-computing devices with neutral atoms [2] seems particularly attractive, because of the very large coherence times of the internal atomic states and of the well developed techniques for cooling and trapping atoms in optical lattices, far off-resonance light-traps, and magnetic microtraps [3]. In particular, dipole–dipole coupling of Rydberg states provides a strong interaction suitable for the implementation of two-qubit quantum gate for neutral atoms (with the gate operation time much faster than the time scales associated with the motion of the atoms in the trapping potential) [4]. A detailed analysis of a quantum logic device based on dipole–dipole interactions of optically trapped Rydberg atoms was presented in [5]. More recently [6], some attention was devoted to the so-called “dipole-blockade” phenomenon. When several atoms are sufficiently close, the presence of an excited atom can cause a shift in the energy of all atoms which is large enough to prevent resonant excitation of more than one atom in the sample [6]. This “dipole-blockade” phenomenon has the potential for creating strongly coupled ensembles with a moderate number of atoms [7]. Recent experiments have revealed signatures of the Rydberg interactions needed for dipole-blockade at large principal quantum number [8]. The authors of [9] have studied the Rabi oscillations between ground and Rydberg states of ^{87}Rb . They have observed

coherent population oscillations for a single atom, while the presence of two or more atoms destroys the coherence of the oscillations. The generation of an efficient multiparticle entanglement via asymmetric Rydberg blockade is discussed in [10]. Among the systems to study, ensembles of three-level atoms driven by laser fields and interacting via dipole–dipole interactions are of interest. See for instance [11,12] where the coherent manipulation of three-level atoms interacting via dipole–dipole interactions is implemented.

In this work, we are considering a system of three-level atoms in a two-mode cavity [13,14]. We have adopted a coherent state to model the initial condition of the photon field, and included an effective dipole–dipole interaction between the atoms [15,16]. The appearance of atomic squeezing [17] is investigated upon the asymmetry of the coupling constants in the photon–atom interaction sector of the proposed Hamiltonian, as well as upon the inclusion of the dipole–dipole interaction [18].

The Letter is organized as follows. In Section 2 we present the details of the formalism. The appearance of squeezing in a system of N three-level atoms and photons is discussed in Section 3, where we present the solutions of the model advanced in Section 2. In performing the calculations we have used different couplings between the atoms and the radiation field. The conclusions are drawn in Section 4.

2. Formalism

We have considered a system which consists of N identical three-level atoms in interaction with a radiation field [14,18,19].

* Corresponding author.

E-mail address: civitare@fisica.unlp.edu.ar (O. Civitarese).

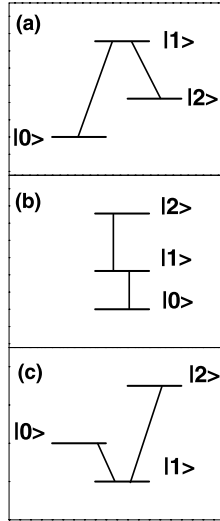


Fig. 1. Different configurations for the H_{ph-at} interaction. Insets (a), (b), and (c) correspond to the Λ -, ladder-, and V-configuration, respectively.

The atoms and the photons are placed in a cavity. The Hamiltonian proposed is of the form

$$H = H_{0,ph} + H_{0,at} + H_{dd} + H_{ph-at}, \quad (1)$$

with

$$\begin{aligned} H_{0,ph} &= \omega_a a^\dagger a + \omega_b b^\dagger b, \\ H_{0,at} &= \sum_i E_i S^{ii}. \end{aligned} \quad (2)$$

The effective dipole-dipole interaction between atoms is defined as

$$H_{dd} = g \sum_{\gamma, \gamma'} (S_+^{01}(\gamma) + S_-^{12}(\gamma)) (S_-^{01}(\gamma') + S_+^{12}(\gamma')). \quad (3)$$

In writing this interaction (3) we have neglected self-energy terms and taken averages on the position of the atoms, absorbing the interatomic distances in the effective coupling [20]. Because the configurations we are dealing with include a relatively small number of atoms, this approximation should not affect the calculations, though it should be not quite adequate for spatial arrays with large number of atoms, where attenuation effects do indeed depend upon the position of the atoms.

The atoms interact with two laser fields of frequencies ω_a and ω_b , respectively. We shall study the behavior of the system for different possible configurations. For the Λ configuration, see Fig. 1(a), H_{ph-at} reads

$$H_{ph-at} = g_1 (aS_+^{01} + a^\dagger S_-^{01}) + g_2 (bS_-^{12} + b^\dagger S_+^{12}). \quad (4)$$

If the photons and the atoms interact via a ladder configuration, see Fig. 1(b), the last term of the Hamiltonian reads

$$H_{ph-at} = g_1 (aS_+^{01} + a^\dagger S_-^{01}) + g_2 (bS_+^{12} + b^\dagger S_-^{12}), \quad (5)$$

see Fig. 1(c), and in the V-configuration it is given by

$$H_{ph,at} = g_1 (aS_-^{01} + a^\dagger S_+^{01}) + g_2 (bS_+^{12} + b^\dagger S_-^{12}). \quad (6)$$

The operators $S_+^{ij}, S_-^{ij}, S_z^{ij}$ generate the $(N+1)(N+2)/2$ -dimensional symmetric representation of the $su(3)$ algebra. This can be shown easily by writing these operators in terms of creation (annihilation), b_i^\dagger (b_i), boson operators for the i -th atomic level ($i=0, 1, 2$). Thus, the operators

$$S^{ij} = b_j^\dagger b_i, \quad i, j = 0, 1, 2, \quad (7)$$

fulfill the commutation relations

$$[S^{ij}, S^{km}] = \delta_{im} S^{kj} - \delta_{jk} S^{im}. \quad (8)$$

The atomic inversion operators

$$S_z^{ij} = \frac{1}{2}(S^{jj} - S^{ii}), \quad (9)$$

and the transition operators S_\pm^{ij}

$$S_+^{ij} = S^{ij}, \quad S_-^{ij} = (S_+^{ij})^\dagger = S^{ji}, \quad i, j = 0, 1, 2, \quad i < j, \quad (10)$$

are defined in terms of the operators (7) and obey the same algebra.

In the expression of H , of Eq. (2), a^\dagger (a) is the one photon-creation (-annihilation) operator of the photon mode of energy ω_a , while b^\dagger (b) correspond to the photon of energy ω_b . E_i is the energy of the i -th atomic level. In Eqs. (3)–(6), g_1 and g_2 are coupling constants describing the absorption (emission) of a photon in the presence of an upward (downward) atomic excitation between levels 0 and 1 (term proportional to g_1), and between levels 1 and 2 (term proportional to g_2), and g is the effective coupling constant of the dipole atom-atom interaction.

2.1. The exact solution

The exact solution of the model can be obtained straightforwardly. For details the reader is kindly referred to [18].

We consider the collective state with n_1 atoms in the first excited state and n_2 atoms in the second excited state ($n_1 + n_2 \leq N$)

$$|n_1 n_2\rangle = N(n_1, n_2) \sum_P |n_1^P(1) \cdots n_1^P(N) n_2^P(1) \cdots n_2^P(N)\rangle,$$

$$N(n_1, n_2) = \left(\binom{N}{n_1} \binom{N-n_1}{n_2} \right)^{-1/2} \quad (11)$$

with $n_1 = \sum_{j=1}^N n_1^P(j)$, $n_2 = \sum_{j=1}^N n_2^P(j)$ and $n_1^P(j) = n_2^P(j) = 0, 1$. Note that the internal degeneracy of each of the two available atomic states is included in the definition of the basis $|n_1^P(1) \cdots n_1^P(N) n_2^P(1) \cdots n_2^P(N)\rangle$.

Since the Hamiltonian of Eq. (1), contains a bosonic degree of freedom, the state which represents (n_a, n_b) photons is written as the number state

$$|n_a n_b\rangle = \frac{1}{\sqrt{n_a! n_b!}} a^{\dagger n_a} b^{\dagger n_b} |0\rangle. \quad (12)$$

We shall then express the wave function of the photons and atoms as (dressed state)

$$|n_a, n_b, n_1, n_2\rangle = |n_a, n_b\rangle \otimes |n_1, n_2\rangle. \quad (13)$$

We shall diagonalize the Hamiltonian (1) in the basis (13) by enforcing the constraints

$$n_0 + n_1 + n_2 = N,$$

$$n_a - n_b + n_1 - N = L \quad (\text{ladder-configuration}),$$

$$n_a + n_b + n_1 - N = L \quad (\Lambda\text{-configuration}),$$

$$n_a + n_b - n_1 - N = L \quad (\text{V-configuration}), \quad (14)$$

where N is the number of atoms.

In the basis of states with good L , the exact solution is written [18]

$$|\Psi_\alpha\rangle = \sum_{\eta=\{n_a, n_b, n_1, n_2\}} c_\alpha(\eta) |\eta\rangle. \quad (15)$$

2.2. Time evolution

In the basis of eigenvectors of H , constructed as discussed before, the time evolution of a given operator O is expressed as

$$O(t) = U^\dagger(t) O U(t), \quad U(t) = e^{-iHt/\hbar}. \quad (16)$$

The expectation value $\langle O(t) \rangle$ is then written

$$\begin{aligned} \langle O(t) \rangle &= \text{Tr}(\rho(t) O) \\ &= \sum_{\alpha, \beta} \langle \beta | I \rangle \langle I | \alpha \rangle \langle \alpha | O | \beta \rangle e^{-i(E_\alpha - E_\beta)t/\hbar}, \end{aligned}$$

$$\rho(t) = U^\dagger(t) \rho(0) U(t), \quad \rho(0) = |I\rangle \langle I|, \quad (17)$$

$|I\rangle$ is the initial state of the system, $\{E_\alpha\}$ and $\{|\alpha\rangle\}$ are the α -th eigenvalue and eigenvector of the Hamiltonian and $\rho(t)$ is the density matrix.

The expression (17) can be written in a more compact form in terms of the overlap of the initial state $|I\rangle$ with the eigenvectors $\{|\alpha\rangle\}$, that is

$$\begin{aligned} \langle O(t) \rangle &= \sum_{n, m} T^*(n) \langle n | O | m \rangle T(m), \\ T(m) &= \sum_{\alpha n} c_{\alpha n}^* c_{\alpha m} \langle n | I \rangle e^{iE_\alpha t/\hbar}. \end{aligned} \quad (18)$$

2.3. Squeezing parameter

The basic mechanism of spin squeezing, if appropriate quantum mechanical correlations are established among the elementary spins of a system, consists in partially canceling out the fluctuation of spin in one direction at the expense of enhancement in other direction, while preserving the minimum uncertainty product [21–23].

Different definitions of spin squeezing can be used depending on the context in which squeezing is considered.

For a given pair of operators, P and O , the quantity

$$\begin{aligned} Q(P, O) &= \frac{2\Delta^2 P}{|\langle \phi | [P, O] | \phi \rangle|}, \\ \Delta^2 P &= \langle \phi | P^2 | \phi \rangle - \langle \phi | P | \phi \rangle^2, \end{aligned} \quad (19)$$

is a standard measure of the squeezing of the operator P with respect to the operator O [17].

With this definition, the field squeezing parameters are given by

$$\begin{aligned} Q(q, p) &= 2\Delta^2 q, \\ Q(p, q) &= 2\Delta^2 p. \end{aligned} \quad (20)$$

Thus, we aim at the identification of systems for which the change in population of atomic levels can be determined accurately. That is to say, the related quantum fluctuation should be as small as possible. Particularly, we are interested in the study of the inversion of the population from the ground state of the atoms to the second excited level. As a measure of this, we shall analyze the time evolution of the squeezing parameter [18,24]

$$Q(S_z, S_+) = \frac{2\Delta^2 S_z}{|\langle S_+ \rangle|}. \quad (21)$$

In the previous equation, $S_z = S_z^0$ and $S_+ = S_+^0$. In this scheme, the optimal squeezing is achieved when the quantum fluctuations of the z -component of the spin are minimal. Similar definition of spin squeezing has been advanced in [22]. In [22], the authors

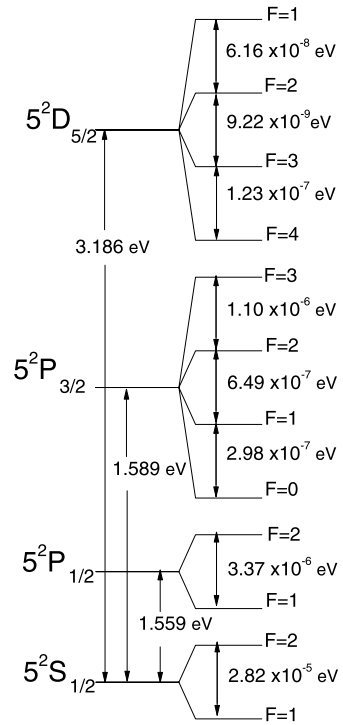


Fig. 2. Level scheme for a ^{87}Rb atom.

have discussed the occurrence of simultaneous squeezing, of two orthogonal spin components, in two level systems.

In Section 3 we shall present some exact results concerning the behavior of the squeezing parameters for the system modeled by the Hamiltonian of Eq. (1).

3. Results and discussion

In what follows, we present the results which we have obtained in the study of the system described in Section 2. We have diagonalized the Hamiltonian of Eq. (1) and we have calculated the time evolution of relevant operators. The adopted energy spacing between the atomic levels corresponds to a system of Rb-atoms [25], see Fig. 2. The energy of the photon sector of the Hamiltonian is fixed to the resonance case [25]. For the initial state, of the photon sector, we have assumed a product coherent state

$$|z_a, z_b\rangle = N e^{z_a a^\dagger} e^{z_b b^\dagger} |0\rangle, \quad (22)$$

with $|z_a|^2 = \langle n_a \rangle$ and $|z_b|^2 = \langle n_b \rangle$, respectively.

We have considered, for the atom–photon sector of the Hamiltonian, the values $g_1 = 0.025$ eV and $g_2 = 0.1$ eV for the Λ - and V-schemes, and $g_1 = 0.025$ eV and $g_2 = 0.3$ eV for the ladder case, respectively [23]. The calculations has been performed for systems with $N = 2, 3$, and 5 atoms.

In Figs. 3–5, we show the results of the present calculations for the time evolution of the atomic squeezing parameter $Q(S_z, S_+)$, for the different type of photon–atom interactions, with and without the inclusion of the dipole–dipole term of the interaction (3).

Fig. 3 shows the results obtained for the Λ -interaction scheme [26]. This configuration is realized in rubidium by adopting the levels $^5S_{1/2}$ ($F = 2$), $^5S_{1/2}$ ($F = 1$), and $^5P_{3/2}$ ($F = 2$), of Fig. 2, as the states $|0\rangle$, $|2\rangle$, and $|1\rangle$, of Fig. 1(a). Insets (a) and (b) correspond to a system of $N = 2$ atoms, insets (c) and (d) correspond to a system with $N = 3$ atoms, and inset (e) and (f) to $N = 5$ atoms, respectively. The initial photon-state corresponds to $\langle n_a \rangle = \langle n_b \rangle = 2$, for the insets (a), (c), and (e), and $\langle n_a \rangle = \langle n_b \rangle = 6$ for the insets (b), (d), and (f). The curves shown in Fig. 3 are labeled by the dipole–

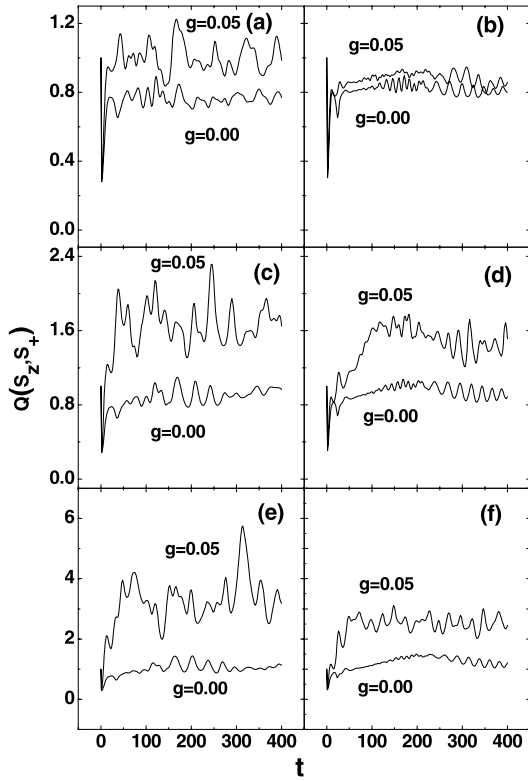


Fig. 3. Squeezing parameters $Q(S_z, S_+)$ as a function of time. The photons and atoms interact via a Λ -scheme. For the coupling constants of the photon–atom sector of the Hamiltonian, we have adopted the values $g_1 = 0.025$ eV and $g_2 = 0.1$ eV. The value of the coupling constant for the dipole–dipole sector of the Hamiltonian, g [MeV], is indicated in the figure. Insets (a) and (b) show the results for a system of two atoms, insets (c) and (d) show results for a system of three atoms, and insets (e) and (f) for five atoms. In all cases the atoms are initially in their ground state. The mean value of photons in the initial state was fixed at $\langle n_a \rangle = \langle n_b \rangle = 2$ (cases (a), (c), and (e)) and $\langle n_a \rangle = \langle n_b \rangle = 6$ (cases (b), (d), and (f)).

dipole strength g ($g = 0.0$ eV, and $g = 0.05$ eV). The inclusion of the dipole–dipole interaction yields larger values of $Q(S_z, S_+)$. The effect is relatively larger for smaller values of the average number of photons.

Fig. 4 displays the results obtained when the ladder configuration (see Fig. 1(b)) is considered [27]. The effective level scheme includes the state $^5S_{1/2}$, as the lower state ($|0\rangle$), the state $^5P_{3/2}$ as intermediate state ($|1\rangle$), and $^5D_{5/2}$ as upper state ($|2\rangle$). We have assumed that, initially, all atoms are in their ground state. Notice that, in order to obtain atomic squeezing, we have to increase the asymmetry in the photon–atom interaction ($g_1 = 0.025$ eV, $g_2 = 0.3$ eV), as compared with the Λ -configuration ($g_1 = 0.025$ eV, $g_2 = 0.1$ eV). It can be seen that the inclusion of the dipole–dipole interaction, for this scheme, has a minor effect, while the increase in the average number of photons, in the initial state, tends to wash-out the effect of the dipole sector of the interaction, and improves slightly the atomic squeezing.

In Fig. 5, we investigate the response in the V-scheme [16,28] (see Fig. 1(c)). We have taken the $^5S_{1/2}$ as lower state, and the states $^5P_{3/2}$ and $^5P_{5/2}$ as the upper ones [16]. We have assumed that, initially, all atoms are in the $^5P_{3/2}$ -state.¹ Insets (a)–(f) show the results obtained with the same set of parameters of the previous Figs. 3–4. In this case the appearance of atomic squeezing is restricted only to a short interval of time. The inclusion of the dipole interaction produces the same effects that in the previ-

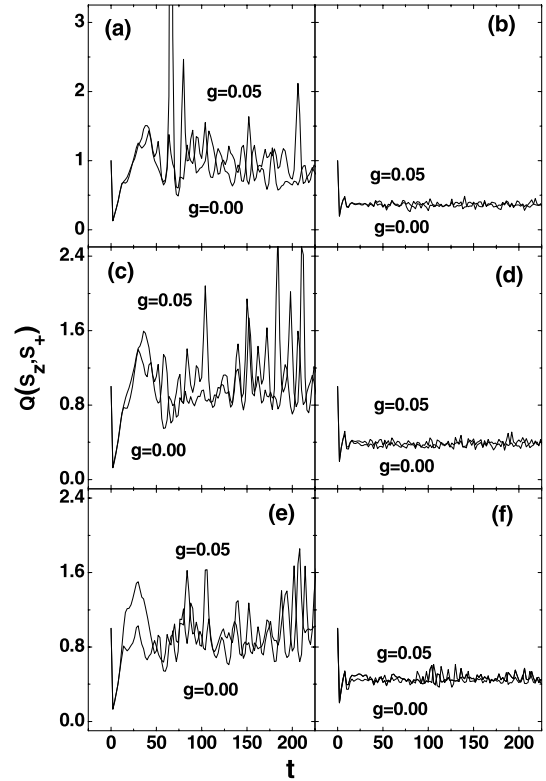


Fig. 4. Squeezing parameters $Q(S_z, S_+)$ as a function of time, for the ladder configuration. The values of the coupling constants of the atom–photon sector of the Hamiltonian were fixed at $g_1 = 0.025$ eV and $g_2 = 0.3$ eV. The results are presented in the order explained in the captions to Fig. 3.

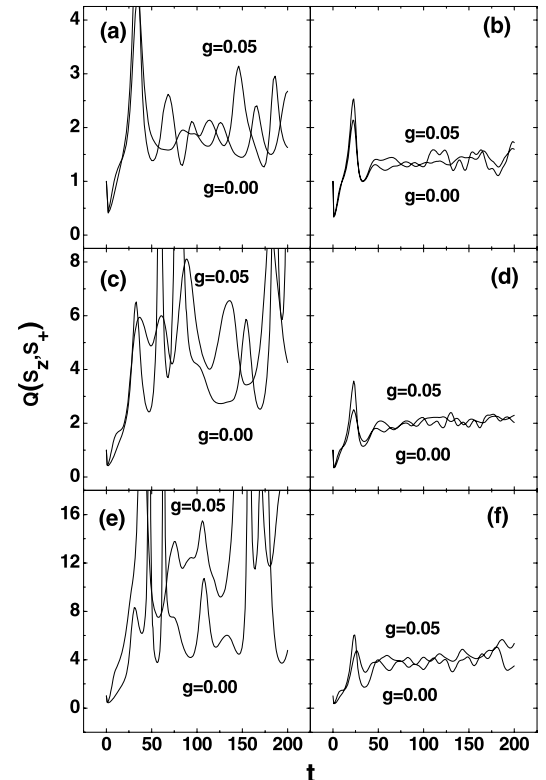


Fig. 5. Squeezing parameters $Q(S_z, S_+)$ as a function of time for a V-configuration. The values for coupling constants were fixed at $g_1 = 0.025$ eV and $g_2 = 0.1$ eV. The results are presented as in Fig. 3.

¹ Noticed that, if the initial state consists of all atoms in the $^5S_{1/2}$ -state, no squeezing is obtained.

ous configurations, so does the increase in the average number of photons in the initial state.

The above results suggest that, for the considered three-level configurations, the appearance of spin squeezing depends on several physical conditions. To distinguish among them we can, as a first instance, separate the configurations where squeezing does indeed become evident: that is the case of the ladder- (or cascade) and lambda-configurations. Spin-squeezing does not appear so clearly in the V-configuration, regardless the choice of the couplings, number of atoms and number of photons.

The gross features emerging from the Λ and ladder configurations seemingly indicate that, in the absence of dipole–dipole interactions, variations in the number of atoms may not affect the spin observables. In the same condition ($g = 0$) the increase in the number of photons tends to favor the appearance of squeezing in the ladder-scheme, and it smears-out the time evolution of the spin-observables in the Λ -scheme. Concerning the effect of the dipole interaction, it definitely does not affect the squeezing in the ladder-scheme but it becomes crucial in destroying the squeezing pattern in the Λ -scheme.

To gain some physical insight into the situation, we may go back to the analysis of the symmetry aspects involved in the calculations. The conserved numbers in the basis are linear combinations of the occupation of the atomic levels, the number of photons and the number of atoms, as it was explained in Section 2.1. If we restrict the analysis to the two cases where we find squeezing, that is for the ladder and Λ schemes, it is somehow evident, from the numerical analysis, that for the ladder configuration the relation $n_0 > n_1 \geq n_2$ changes to $n_0 > n_2 \geq n_1$ when the number of photons increases. For the Λ -scheme the relation $n_0 > n_2 > n_1$ remains when the number of photons increases.

One can translate these relations between level-occupations in terms of the time dependence of the expectation value of the z-component of the spin and its fluctuations. For the ladder-scheme, the increase in the number of photons produces an increase of the absolute value of the z-component of the spin, independently of dipole–dipole effects, and the spin-fluctuation decreases. For the Λ -scheme, under the same conditions, the absolute value of the z-component of the spin decreases, and its fluctuation increases slightly.

Concerning the absolute value of the spin-raising operator, it increases (ladder-scheme) or it remains nearly constant (Λ -scheme), with and without turning on dipole–dipole interactions, but increasing the number of photons. All of these reflects upon the time dependence of the spin-squeezing factor displayed in Figs. 3 and 4. In each case, the trend is enhanced by the asymmetry of the interaction between spin-components of the atomic levels. The modifications of the initial conditions will then affect these features in the way which has been explained at the beginning of this section.

4. Conclusions

In this work we have studied the appearance of atomic squeezing in a system of three-level atoms interacting with a radiation field. We have considered as possible schemes Λ -, ladder- and V-configurations. We have performed calculations, with and without, including the dipole–dipole interaction between the atoms. From the results, one may conclude that: (i) concerning the photon–atom interaction, it seems that atomic squeezing is obtained for

the Λ and ladder schemes; (ii) the inclusion of dipole–dipole interactions acts against the appearance of atomic squeezing; (iii) the increase in the mean value of photons of the initial state smears out the effect of the dipole–dipole interactions.

Acknowledgement

This work was partially supported by the National Research Council of Argentine (CONICET).

References

- [1] P. Kok, W.J. Munro, K. Nemoto, T.C. Ralph, J.P. Dowling, G.J. Milburn, *Rev. Modern Phys.* 79 (2007) 135; H. Imai, M. Hayashi (Eds.), *Quantum Computation and Information: From Theory to Experiment*, Topics in Applied Physics, Springer-Verlag, Heidelberg, 2006.
- [2] G.K. Brennen, C.M. Caves, P.S. Jessen, I.H. Deutsch, *Phys. Rev. Lett.* 82 (1999) 1060; D. Jaksch, H.-J. Briegel, J.I. Cirac, W. Gardiner, P. Zoller, *Phys. Rev. Lett.* 82 (2000) 1975; J. Ahn, T.C. Weinacht, P.H. Bucksbaum, *Science* 287 (2000) 463.
- [3] H.-J. Briegel, T. Calarco, D. Jaksch, J.I. Cirac, P. Zoller, *J. Mod. Opt.* 47 (2000) 415.
- [4] D. Jaksch, J.I. Cirac, P. Zoller, S.L. Rolston, R. Côté, M.D. Lukin, *Phys. Rev. Lett.* 85 (2000) 2208; D.D. Yavuz, P.B. Kulatunga, E. Urban, T.A. Johnson, T. Henage, T.G. Walker, M. Saffman, *Phys. Rev. Lett.* 96 (2008) 063001-1.
- [5] M. Saffman, T.G. Walker, *Phys. Rev. A* 72 (2005) 0233347; M. Saffman, K. Mølmer, *Phys. Rev. A* 78 (2008) 012336-1.
- [6] M.D. Lukin, M. Fleishhauer, R. Côté, L.M. Duan, D. Jaksch, J.I. Cirac, P. Zoller, *Phys. Rev. Lett.* 87 (2001) 037901.
- [7] E. Brion, K. Mølmer, M. Saffman, *Phys. Rev. Lett.* 99 (2007) 260501.
- [8] M. Saffman, K. Mølmer, *Phys. Rev. A* 78 (2008) 012336; T.G. Walker, M. Saffman, *Phys. Rev. A* 77 (2008) 032723.
- [9] T.A. Johnson, E. Urban, T. Henage, L. Isenhower, D.D. Yavuz, T.G. Walker, M. Saffman, *Phys. Rev. Lett.* 100 (2008) 113003.
- [10] M. Saffman, K. Mølmer, *Phys. Rev. Lett.* 102 (2009) 240502.
- [11] M. Macovei, J. Evers, C.H. Keitel, *Phys. Rev. A* 71 (2005) 033802; M. Kiffner, J. Evers, C.H. Keitel, *Phys. Rev. A* 75 (2006) 032313.
- [12] C.L. Wang, Z.H. Kang, S.C. Tian, Y. Jiang, J.Y. Gao, *Phys. Rev. A* 79 (2009) 043810.
- [13] A. Dantan, M. Pinard, V. Josse, N. Nayak, P.R. Berman, *Phys. Rev. A* 67 (2003) 045801.
- [14] W. Vogel, R. Blatt, *Phys. Rev. A* 45 (1992) 3319; B.J. Dalton, Z. Ficek, P.L. Knight, *Phys. Rev. A* 50 (1994) 2646; Z. Ficek, B.J. Dalton, P.L. Knight, *Phys. Rev. A* 51 (1995) 4062.
- [15] G.V. Varada, G.S. Agarwal, *Phys. Rev. A* 45 (1991) 6721.
- [16] G.S. Agarwal, A.K. Patnaik, *Phys. Rev. A* 63 (2001) 043805.
- [17] P.D. Drummond, Z. Ficek, *Quantum Squeezing*, Springer, Berlin, 2004; N.J. Cerf, G. Leuchs, E.S. Polzik (Eds.), *Quantum Information with Continuous Variables of Atoms and Light*, Imperial College Press, London, 2007.
- [18] O. Civitarese, M. Reboiro, *Phys. Lett. A* 357 (2006) 224; M. Reboiro, *Phys. Scr.* 78 (2008) 045006.
- [19] A.B. Klimov, J. Negro, R. Farias, S.M. Chumakov, *J. Opt. B: Quantum Semiclass. Opt.* 1 (1999) 562.
- [20] C. Cohen-Tannoudji, J. Dupont-Roc, G. Grynberg, *Atom–Photon Interactions*, Wiley–VCH Verlag GmbH, Weinheim, 2004.
- [21] M. Kitagawa, M. Ueda, *Phys. Rev. A* 47 (1993) 5138.
- [22] H. Prakash, R. Kumar, *J. Opt. B: Quantum Semiclass. Opt.* 7 (2005) S747.
- [23] M. Reboiro, O. Civitarese, L. Rebón, *Phys. Lett. A* 366 (2007) 241.
- [24] Ö.E. Müstecapoglu, W. Zhang, L. You, *Phys. Rev. A* 75 (2007) 023605.
- [25] S. Brattke, U. Kallmann, W.-D. Hartmann, *Eur. J. Phys. D* 3 (1998) 159; M.J. Snadden, A.S. Bell, E. Riis, A.I. Ferguson, *Opt. Commun.* 125 (1996) 70.
- [26] Y. Zhu, J. Lin, *Phys. Rev. A* 53 (1996) 1767; J. Wang, Y. Zhu, K.J. Jiang, M.S. Zhan, *Phys. Rev. A* 68 (2003) 063810; D.A. Cardimona, P.M. Alsing, H. Mozer, C. Rhodes, *Phys. Rev. A* 79 (2009) 063817; P. Barberis-Blostein, M. Bienert, *Phys. Rev. A* 79 (2009) 063824.
- [27] H. van Kampen, V.A. Sautenkov, E.R. Eliel, J.P. Woerdman, *Opt. Commun.* 180 (2000) 81.
- [28] A. Kuzmich, K. Mølmer, E.S. Polzik, *Phys. Rev. Lett.* 79 (1997) 4782; H. Tan, H. Xias, G. Li, *Phys. Rev. A* 79 (2009) 063805.