



## Spin squeezing in the presence of dissipation

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### ABSTRACT

The transfer of spin between photons and localized atomic levels, for a chain of two-level atoms, is studied. The Hamiltonian of the system is modelled by a radiation term and by atom–atom and radiation–atom interactions. Effects due to dissipation are accounted for by atom–photon exchange–interactions of complex strength. It is found that the spin-squeezing is suppressed by dissipation. Calculations are performed for arrays of Rb atoms excited by a GaAlAs laser.

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## 1. Introduction

The study of spin systems, as laboratories of quantum mechanical effects, was prompted by the realization of elementary spin-pairs, e.g.: qubits, as primordial units in quantum computing [1]. Another field of interest, in connection with spin systems, is quantum optics [2], where remarkable achievements in the transfer of spin have been reported recently [3–5]. We shall refer to this body of results, as the field of spin squeezing [6,9]. Generally speaking, the interaction between light and localized sources of spin, like atomic levels, produces the transfer of spin [4] and deviations from the expectation values, as compared to the values given by standard commutation and uncertainty relations [6–8]. A review of recent results in this field can be found in [9]. In a previous work we have analyzed the manifestation of squeezing in atomic systems in presence of spin–photon interactions [10]. Concerning the microscopic description of systems which may exhibit squeezing, we have studied the effects associated to asymmetries in the spin–spin interactions [11]. In [11], the competition between the initial conditions and the interactions was analyzed for the case of a  $s = 1/2$  spin-chain, where the dependence of squeezing upon spin-coherent initial conditions becomes evident. Among the open problems in the field of spin transfer between radiation

and atomic levels, one can mention the persistence of the spin orientation, and the revival of the spin alignment [4,13]. A recent work [14] investigates the attenuation of spin transfer in the presence of dissipation. For earlier studies about field dissipation the reader is kindly referred to the work of Bennett and Knight [12]. The description of the phenomena was based in the theory of spin–radiation interactions. The authors of [14] have taken a two-level system coupled to an oscillator and added dissipation by means of a spin–cavity mode interaction with an imaginary coupling constant proportional to the width of the oscillator energy distribution [14]. The results of the work [14] are indeed very interesting, since the authors have shown that the inclusion of dissipation increases the rate of the energy transfer. This effect is reported as to be independent of the model adopted to account for dissipation, and it opens the possibility of several applications, most notably to fusion between light-ions [15]. In this Letter we have taken the results of [14] as a motivation, mainly to test the persistence of the pattern in presence of atom–atom interactions. These interactions have not been considered in [14], and may be of importance, particularly, in view of their effects in dealing with spin transfer mechanisms (see Ref. [4]). Here, we study the conditions and persistence for spin-squeezing in an array of rubidium atoms, a system which was also studied recently [16], under MOT conditions. To this system we apply and extend the method of [14], as described before, to investigate the competition between the above described interactions in a system which resembles an array of atoms in an optical trap [17].

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In addition to the already introduced features, of spin squeezing, the connection between it and entanglement was studied recently [18]. The authors of [18] have considered a system of  $N$  two-level atoms interacting with a radiation field in a dispersive cavity. A very interesting result of the work of [18] is that an almost complete reduction of the noise was found for a system of two atoms, and for a high- $Q$  cavity. In agreement with our previous results [10,11], the authors of [18] have found that the degree of squeezing deteriorates if the number of atoms (spins) in the upper spin-states increases. We aim at the search of similar effects in systems like the one of [17].

Based on these previous findings, most precisely Refs. [14] and [18], we have considered, in the present work, the interplay between atom (spin)-radiation and spin-spin interactions in the presence of dissipation. We have organized the Letter as follows. The formalism is presented in Section 2, where the Hamiltonian which describes the interaction of atoms with a radiation field, and among themselves, is introduced, as well as the method adopted to diagonalize it and to compute the time evolution of the observables. The results of the calculations, performed for an array of rubidium atoms under different model conditions, are presented and discussed in Section 3. Our conclusions are drawn in Section 4.

## 2. Formalism

In this section we shall introduce the Hamiltonian of the system (Section 2.1) and describe a method to solve the eigenvalue problem. In Section 2.2 we shall analyze the evolution of observables, as it is determined by the eigenstates of  $H$ . The dependence upon initial conditions is discussed in Section 2.3. We have considered, as initial states, a product of atomic and radiation field states. For the atomic state we have taken a coherent state [8]. The initial radiation field is taken also as a coherent state. Section 2.4 is devoted to the definition of spin-squeezing [6,9].

### 2.1. The Hamiltonian

The system is composed by a chain of localized atoms, each of them having two atomic levels of spin  $s = 1/2$ . To this atomic system we add a single-frequency radiation field and let it to interact with each of the atoms, separately. The situation which we shall modelled in this section corresponds, roughly, to cold atoms trapped in an optical trap. The spin-spin interaction sector should then be the one of [17].

The atom-atom interaction is described by spin-spin interactions between the atomic levels. The Hamiltonian is written

$$\begin{aligned} H &= H_0 + H_{\text{chain}} + H_{\text{int}}, \\ H_0 &= \hbar\omega \left( a^\dagger a + \frac{1}{2} \right) + \sum_i \epsilon(i) S_z(i), \\ H_{\text{chain}} &= \sum_{i \neq j} \gamma_{ij} S_z(i) S_z(j) + \sum_{i \neq j} \eta_{ij} (S_+(i) S_-(j) + S_-(i) S_+(j)), \\ H_{\text{int}} &= \frac{1}{2} \sum_i \lambda_i (S_+(i) + S_-(i)) (a^\dagger + a). \end{aligned} \quad (1)$$

$H_0$  includes the unperturbed (free) radiation field, of frequency  $\omega$ , and the spin component of the atoms, along the  $z$ -direction. The position of the atoms in the chain is indicated by the index  $i$ .  $H_{\text{chain}}$  is the term which describes the interaction between atomic spins placed in different sites of the chain. The interaction between the  $z$ -components of the spins and the interactions between the  $x$ - and  $y$ -components have different strengths,  $\gamma_{ij}$  and  $\eta_{i,j}$ , respectively.  $H_{\text{int}}$  is the photon-atomic level interaction, weighted by the strength  $\lambda_i$ . To this Hamiltonian we add dissipation by considering a term of the form

$$H_{\text{diss}} = \frac{i}{2} \sum_i \lambda_d(i) (S_+(i) + S_-(i)) (a^\dagger + a), \quad (2)$$

as suggested in [14]. The justification of the term, of Eq. (2), is found in elementary field theory, where the coupling of a particle with a narrow resonance produces, at lowest order, a term of this form. The quantum electrodynamics of a two-level atom interacting with a cavity mode, was developed in Ref. [12]. It leads to coupling terms which are proportional to the width of the cavity mode. In [12] it was shown that the density matrix, in the dissipation picture, is damped in time by exponential factors which are proportional to the width of the cavity mode.

The Hamiltonian of Eq. (1) is diagonalized in the basis

$$\begin{aligned} |k, n\rangle &= |k_1, k_2, \dots, k_i, \dots, k_N, n\rangle \\ &= \prod_{i=1}^N S_+^{k_i}(i) \frac{a^{\dagger n}}{\sqrt{n!}} | \rangle, \quad k_i = 0, 1. \end{aligned} \quad (3)$$

In the above expression  $k$  is a spin array belonging to the set  $\{k\} = \{k_1, k_2, \dots, k_i, \dots, k_N\}$ .

The operators  $S_\pm(i) = S_x(i) \pm i S_y(i)$  are the spin ladder operators of the site  $i$ ; together with  $S_z(i)$  they are the generators of the  $su(2)$  algebra.

The eigenvectors are written as linear combinations of the states of the basis (3)

$$|\alpha\rangle = \sum_{\{k\}, n} c_\alpha(k, n) |k, n\rangle. \quad (4)$$

The term  $H_{\text{diss}}$ , of Eq. (2), is treated perturbatively in the basis of eigenstates (4), at lowest order, that is by correcting the energies and renormalizing the eigenstates  $|\alpha\rangle$ .

### 2.2. Time evolution

In the basis of eigenvectors of  $H$ , constructed as discussed before, the time evolution of a given operator  $O$  is expressed as

$$O(t) = U^\dagger(t) O U(t), \quad U(t) = e^{-iHt/\hbar}. \quad (5)$$

The expectation value  $\langle O(t) \rangle$  is then written

$$\begin{aligned} \langle O(t) \rangle &= \text{Tr}(\rho(t) O) \\ &= \sum_{\alpha, \beta} \langle \beta | I \rangle \langle I | \alpha \rangle \langle \alpha | O | \beta \rangle e^{-i(E_\alpha - E_\beta)t/\hbar}, \end{aligned} \quad (6)$$

where we have defined the density operator  $\rho(t) = U^\dagger(t) \rho(0) U(t)$ , being  $\rho(0) = |I\rangle \langle I|$ ; the state  $|I\rangle$  is the initial state of the system, while  $\{E_\alpha\}$  and  $\{|\alpha\rangle\}$  are the  $\alpha$ th eigenvalue and eigenvector of the Hamiltonian.

The expression (6) can be written in a more compact form in terms of the overlap of the initial state  $|I\rangle$  with the eigenvectors  $\{|\alpha\rangle\}$ , that is

$$\begin{aligned} \langle O(t) \rangle &= \sum_{n, m} T^*(n) \langle n | O | m \rangle T(m), \\ T(m) &= \sum_{\alpha n} c_{\alpha n}^* c_{\alpha m} \langle n | I \rangle e^{iE_\alpha t/\hbar}. \end{aligned} \quad (7)$$

In the above equation  $|n\rangle$  is a short hand notation for the elements  $|k, n\rangle$  of the basis (3) and  $c_{\alpha n} \equiv c_\alpha(k, n)$ , that is the coefficients of the eigenstates  $|\alpha\rangle$  of Eq. (4), with eigenvalue  $E_\alpha$ .

### 2.3. Initial condition

We shall study the time evolution of the system under the action of  $H$ . The initial state, which is not an eigenstate of  $H$ , is a product state of the form

$$|I\rangle = |I\rangle_f \otimes |I\rangle_{at}, \quad (8)$$

where  $|I\rangle_{at}$  is the atomic initial state and  $|I\rangle_f$  is the initial radiation field.

We shall adopt as initial condition for the radiation field a coherent state

$$|I\rangle_f = e^{-|z|^2/2} e^{za^\dagger} |0\rangle, \quad (9)$$

with  $|z|^2 = \langle a^\dagger a \rangle$ .

For the atomic initial state we shall adopt a coherent spin state (CSS). A CSS is defined as an eigenstate of the spin component in the direction  $\vec{n}_0 = (\sin\theta_0 \cos\phi_0, \sin\theta_0 \sin\phi_0, \cos\theta_0)$ , then  $\vec{S} \cdot \vec{n}_0 |CSS\rangle = S |CSS\rangle$ , with eigenvalue  $S$ . The angles  $\theta_0$  and  $\phi_0$  are the polar and azimuth angles of the unit vector  $\vec{n}_0$ . The CSS is written

$$\begin{aligned} |I\rangle_{at} &= e^{z_{at} S_+} |0\rangle, \\ &= (1 + |z_{at}|^2)^{-\frac{N}{2}} \sum_{m=0}^N \frac{z_{at}^m}{m!} S_+^m |0\rangle, \end{aligned} \quad (10)$$

with

$$z_{at} = -e^{-i\phi_0} \tan\left(\frac{\theta_0}{2}\right). \quad (11)$$

### 2.4. Atomic squeezing

Atomic spin-squeezed-states are quantum correlated systems with reduced fluctuations in one of the collective spin components. For a component along a general unit vector  $\vec{n} \equiv (\sin\theta \cos\phi, \sin\theta \sin\phi, \cos\theta)$ , of the spin vector  $\vec{S}$ , we shall define the squeezing factor as  $2(\Delta S_n)^2 / |\langle \vec{S}_\perp \rangle|$ , where  $\vec{S}_\perp$  is the spin component in the direction perpendicular to  $\vec{n}$ . We shall study the spin-squeezing of the components of  $\vec{S}$  in a direction perpendicular to the mean spin of the system,  $\langle \vec{S} \rangle$ . Therefore, we shall adopt as an indicator of the spin squeezing the quantity [8]

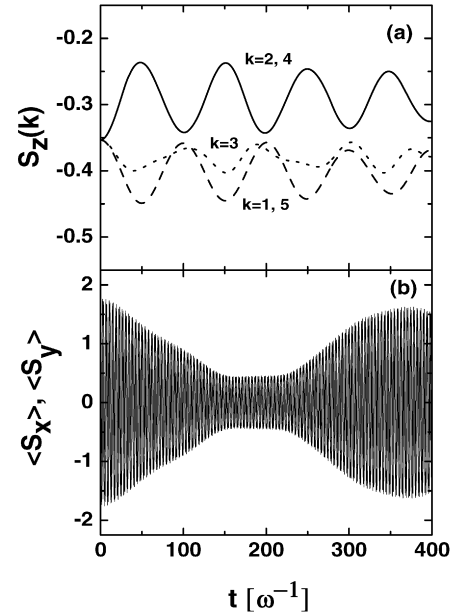
$$\zeta^2 = \frac{2(\Delta S_n)^2}{|\langle \vec{S} \rangle|}. \quad (12)$$

Consequently,  $S_n$  is squeezed if  $\zeta^2 < 1$  [8].

## 3. Results and discussion

We have diagonalized the Hamiltonian of Eq. (1), and studied the time evolution of spin observables of the system. We have taken an atomic open chain consisting of  $N = 5$  sites. The values of  $\epsilon(i)$  are taken from the atomic S and P levels of  $^{87}\text{Rb}$ , that is  $\epsilon(i) = 1.559$  eV, if  $i$  is odd, and  $\epsilon(i) = 1.589$  eV, if  $i$  is even, respectively. The energy of the photon was fixed at the value  $\hbar\omega = 1.582$  eV, corresponding to a GaAlAs laser. The coupling constants  $\gamma_{i,j}$  and  $\eta_{i,j}$ , of the spin-spin sector of the Hamiltonian, are fixed by considering nearest and near-neighbors interactions ( $\gamma_{i,j} = 0.01$  eV if  $|i - j| \leq 2$ , otherwise  $\gamma_{i,j} = 0$ , and  $\eta_{i,j} = 2\gamma_{i,j}$ ). The coupling constants  $\lambda_i$  and  $\lambda_d(i)$  are fixed at the values  $\lambda_i = 0.022$  eV and  $\lambda_d(i) = 0.002$  eV. The initial condition for atoms is represented by a coherent spin state, CSS, of Eq. (10), with  $\phi_0 = \theta_0 = \pi/4$ .

These values are to be taken as very rough estimations of the values to be encountered in a realistic situation. However, they are consistent with the experimental information about the energy difference between S and P levels in Rb atoms, and with the



**Fig. 1.** Spin expectation values, with a purely atomic Hamiltonian. The results corresponding to a chain of 5 sites are shown as a function of the time  $t$  ( $t$  is given in arbitrary units). The expectation value of the z-component of the total spin,  $S_z$ , is given for each of the sites, separately (inset (a)). The time dependence of the x- and y-components of the total spin  $\mathbf{S}$  is shown in inset (b). The Hamiltonian includes only the atomic (spin) components of the system and it does not include photons. The initial state is a CSS (see Eq. (10)), and the parameters of the Hamiltonian are given in the text.

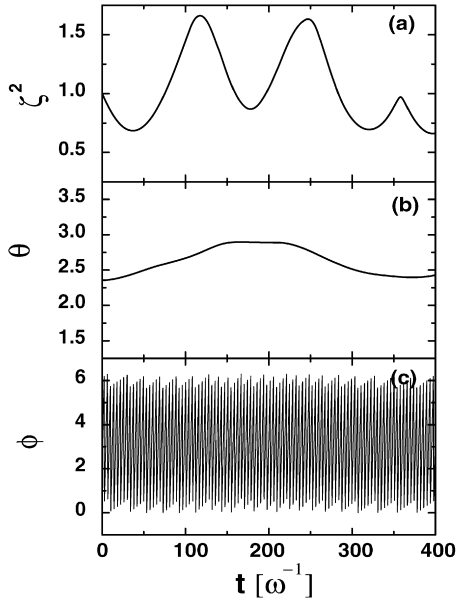
properties of a known laser field, like the 1.582 eV line of a GaAlAs laser. Particularly, the value of  $\lambda_i$  and  $\lambda_d(i)$  have been chosen in a way consistent with the known field-theoretical treatment of the interaction between radiation and atomic levels.

In order to assess the order of magnitude of the effects associated with the different terms of the Hamiltonian of Eq. (1), we have performed calculations by gradually turning on these terms. In the following, we shall present and discuss our results for the expectation value of the z-component of the spin, for each of the sites of the chain, the mean value of the components of the total spin ( $\langle S_x \rangle$ ,  $\langle S_y \rangle$ ,  $\langle S_z \rangle$ ), and the spin squeezing parameter  $\zeta^2$ .

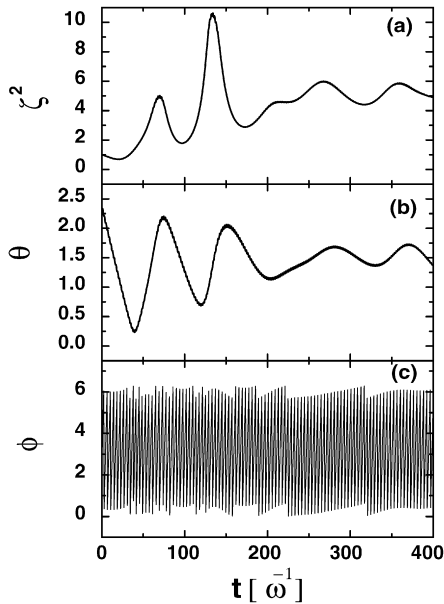
As a first step we have verified that, for a restricted Hamiltonian with purely spin-spin interactions between near sites, the z-component of the total spin and the z-component of the spin of each atom are constants of motion. The inclusion of nearest site interactions, like in the present case, breaks the symmetry. The results of the calculations are shown in Fig. 1, which illustrates (a) the time dependence of  $\langle S_z \rangle$  for each atom, and (b) the revival, in time, of the values associated to the expectation value of the x and y components of the spin.

Fig. 2 shows the results obtained by using, like in Fig. 1, only the atomic sector of Eq. (1). In the insets (a) to (c) of Fig. 2 we show the time dependence of the expectation value of the spin squeezing factor  $\zeta^2$ , and the time dependence of the orientation angles of the mean value of the total spin (see Eq. (12)). As seen from the curves of Fig. 2, the atomic squeezing persists with time, shows features of revival, and reaches a minimum value of the order of  $\zeta^2 \approx 0.7$ .

The effect of the inclusion of the interaction with the photon field is displayed in Fig. 3. The Hamiltonian used to perform the calculations is the one of Eq. (1), except for the term corresponding to dissipation, which has been omitted in this set of calculations. The initial condition for the photon sector is given by a coherent state (see Eq. (9)) with  $|z|^2 = 10$ . The choice of a coherent state, as initial condition in the photon sector, is justified by the fact



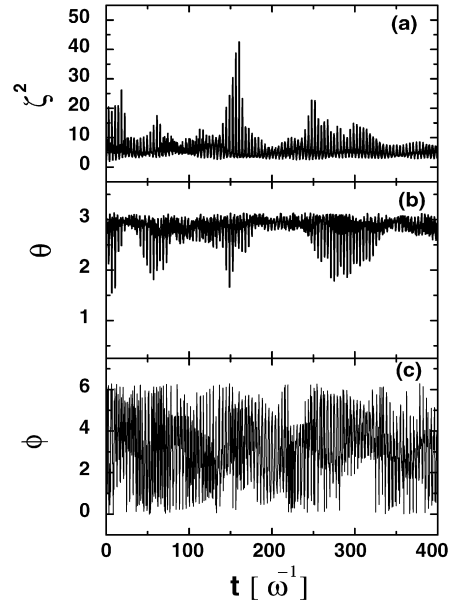
**Fig. 2.** Time evolution of the spin-squeezing factor  $\zeta^2$  (inset (a)), and of the orientation angles of the mean value of the total spin ( $S$ ), ( $\theta, \phi$ ) (insets (b) and (c)). We have considered only the atom–atom terms of (1). The initial state is a coherent state  $|l\rangle_{at}$  (see Eq. (10)), with  $\phi_0 = \theta_0 = \pi/4$ .



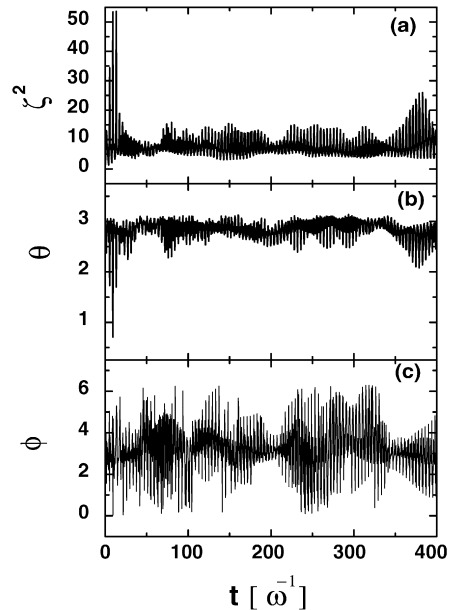
**Fig. 3.** Time dependence of the spin squeezing (inset (a)), and orientation angles  $\theta$  (inset (b)) and  $\phi$  (inset (c)) of the mean value of the total spin, in presence of atom–photon and atom–atom interactions, without dissipation. The initial state, for photons, is a coherent state (see Eq. (9)) with 10 photons ( $n_b = |z|^2 = 10$ ), and, for the atoms, it is a coherent atomic state  $|l\rangle_{at}$  (see Eq. (10)), with  $\phi_0 = \theta_0 = \pi/4$ . The parameters of the Hamiltonian are given in the text.

that this class of states gives non-vanishing matrix elements of  $H_{int}$ , regardless the actual value of  $n_b$ . The other parameters of the Hamiltonian are given above. In this case the atomic squeezing  $\zeta^2$  does not persists with time, and it reaches a minimum at  $\theta = \pi/2$ .

We have investigated the effects associated with the number of photons ( $n_b = |z|^2$ ) in the initial state, by performing calculations with  $n_b = 2, 4, 6$  and  $8$ , and by comparing the results with the ones obtained with  $n_b = 10$ . It is evident, from the curves shown in Fig. 3, that the atom–photon interaction destroys the squeez-



**Fig. 4.** Time dependence of the spin squeezing (inset (a)), and orientation angles  $\theta$  (inset (b)) and  $\phi$  (inset (c)) of the mean value of the total spin, in presence of atom–photon and atom–atom interactions, with dissipation. The initial states, for photons and atoms, are coherent states. The parameters of the Hamiltonian are given in the text, and the initial photon state is defined as a two photons state ( $n_b = |z|^2 = 2$ ).



**Fig. 5.** Time dependence of the spin squeezing (inset (a)), and orientation angles  $\theta$  (inset (b)) and  $\phi$  (inset (c)) of the mean value of the total spin, in presence of atom–photon and atom–atom interactions, with dissipation. The initial states, for photons and atoms, are coherent states. The parameters of the Hamiltonian are given in the text. The initial photon state is a ten photons state ( $n_b = |z|^2 = 10$ ).

ing induced by the spin–spin interactions. This feature persists if the number of photons in the initial state is changed, that is the number of photons in the initial state does not influence much the time dependence of the squeezing of the atomic spin.

The results of our calculation for the complete Hamiltonian, of Eq. (1), are presented in Fig. 4, for  $n_b = 2$ . The effects due to dissipation are notorious, since the atomic squeezing is washed out on the whole domain of time considered in the calculations, and the orientation of the expectation value of the total spin oscillates between  $\theta = \pi/2$  and  $\theta = \pi$ . The spin transfer from the photon field

to each site of the chain differs drastically, as compared with the results shown in Fig. 2, since for this case the levels of each atomic site are equally populated. Same conclusion concerning the effect of the photon number can be drawn here, as it is supported by the results shown in Fig. 5, which corresponds to the case  $n_b = 10$ . In Figs. 4(a) and 5(a) it is seen that the squeezing disappears at longer times.

Concerning the dependence of the results upon the choice of the initial atomic condition, we have repeated the above presented calculations by replacing the atomic CSS state by an initial condition consisting of all atoms in the ground state. The results of the calculations, omitted here for reasons of space, show that the use of the CSS is as important as the use of coherent states for the photon sector.

To gain further insights about the above presented results, we have compared them to the ones of Ref. [16]. Basically, we have obtained similar results concerning the effects of dissipation upon spin squeezing. Both for Ref. [16] and the present work, squeezing is suppressed by dissipation. In the case of [16] dissipation is accounted for by the scattering of photons, while in our case it is due to the coupling of the atomic spin to the finite width of the photon field.

#### 4. Conclusions

In this work we have calculated the time dependence of spin observables in presence of atom–atom and atom–radiation interactions. We have also included dissipation by means of an atom–photon interaction of complex strength. In this respect we have taken the results of the work of the M.I.T. group, Refs. [14], as initial step and added to the formalism a localized atom–atom interaction. Our results, regarding spin observables, are the following:

- (a) The inclusion of atom–atom interactions produces the enhancement of the squeezing factor and, eventually, is responsible for revival effects;
- (b) The presence of dissipation washes-out the spin transfer from the radiation field to the atomic levels;
- (c) The atomic initial state, in our case a coherent spin state (CSS), is as important as the initial state of the radiation field (also a coherent state) in producing the spin alignment;
- (d) The main effect concerning the interplay between atom–photon and atom–atom interactions is that they add incoherently regardless the number of photons involved;
- (e) The dependence of the spin transfer upon the atomic couplings is evident and it opens the possibility of modelling

atom–radiation spin-transfer effects by means of atomic lattices, and/or optical traps.

Work is in progress concerning the distribution of spin in a spin-chain with site-dependent interactions.

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#### References

- [1] P. Kok, W.J. Munro, K. Nemoto, T.C. Ralph, J.P. Dowling, G.J. Milburn, *Rev. Mod. Phys.* 79 (2007) 135;
- H. Imai, M. Hayashi (Eds.), *Quantum Computation and Information: From Theory to Experiment (Topics in Applied Physics)*, Springer-Verlag, Heidelberg, 2006.
- [2] M.O. Scully, M.S. Zubairy, *Quantum Optics*, Cambridge Univ. Press, Cambridge, 1997;
- J.S. Peng, G.X. Lee, *Introduction to Modern Quantum Optics*, World Scientific, Singapore, 1999;
- J.R. Klauder, E.C.G. Sudarshan, *Fundamentals of Quantum Optics*, Dover Publications, Dover, 2006.
- [3] A. Furusawa, N. Taki, *Phys. Rep.* 443 (2007) 97;
- N. Taki, et al., *Phys. Rev. A* 76 (2005) 042304;
- J. Sherson, *Nature* 443 (2006) 557;
- H. Yonezawa, S.L. Braunstein, A. Furusawa, *Phys. Rev. Lett.* 99 (2007) 110503.
- [4] A.O. Lyakhov, D. Braun, C. Bruder, *Phys. Rev. A* 76 (2007) 0222321;
- L. Brey, H.A. Fertig, S. Das Sarma, *Phys. Rev. Lett.* 99 (2007) 116802;
- P. Rabl, P. Zoller, *Phys. Rev. A* 76 (2007) 042308;
- L. Campos Venuti, C. Degli Esposti Boschi, M. Roncaglia, *Phys. Rev. Lett.* 99 (2007) 060401;
- O. Romero-Isart, K. Eckert, S. Sanpera, *Phys. Rev. A* 75 (2007) 050303R.
- [5] J. Joo, Y.L. Lim, A. Beige, P.L. Knight, *Phys. Rev. A* 74 (2006) 042344;
- R. Fermani, S. Scheel, P.L. Knight, *Phys. Rev. A* 75 (2007) 062905.
- [6] D.F. Walls, P. Zoller, *Phys. Rev. Lett.* 47 (1981) 709.
- [7] H. Prakash, R. Kumar, *J. Opt. B: Quantum Semiclass. Opt.* 7 (2005) S747.
- [8] M. Kitagawa, M. Ueda, *Phys. Rev. A* 47 (1993) 5138.
- [9] P.D. Drummond, Z. Ficek, *Quantum Squeezing*, Springer, Berlin, 2004;
- N.J. Cerf, G. Leuchs, E.S. Polzik (Eds.), *Quantum Information with Continuous Variables of Atoms and Light*, Imperial College Press, London, 2007.
- [10] O. Civitarese, M. Reboiro, *Phys. Lett. A* 357 (2006) 224.
- [11] O. Civitarese, M. Reboiro, L. Rebon, *Phys. Lett. A* 366 (2007) 241.
- [12] S.M. Barnett, P.L. Knight, *Phys. Rev. A* 33 (1986) 2444.
- [13] I. Sainz, A.B. Klimov, L. Roa, *Phys. Rev. A* 73 (2006) 032303;
- S.K. Lee, C.K. Law, *Phys. Rev. A* 76 (2007) 033809.
- [14] P.L. Hagelstein, I.U. Chaudhary, *cond-mat/0612306*, (2006).
- [15] P.L. Hagelstein, M.C.H. McKubre, D.J. Nagel, T.A. Chubb, R.J. Hekman, in: J.P. Biberian (Ed.), *Condensed Matter Nuclear Science, Proceedings of the 11th International Conference on Cold Fusion, Marseilles, France, November 2004*, World Scientific, Singapore, 2006, p. 23.
- [16] S.R. Echaniz, et al., *J. Opt. B: Quantum Semiclass. Opt.* 7 (2005) S548.
- [17] A. Sorensen, K. Molmer, *Phys. Rev. Lett.* 83 (1999) 2274.
- [18] R.N. Deb, M. Sebawe Abdalla, S.S. Hassan, N. Nayak, *Phys. Rev. A* 73 (2006) 053817.