Available online at www.sciencedirect.com
SCIENCE DIRECT®
Physics Letters A 357 (2006) 224–228
PHYSICS LETTERS A
www.elsevier.com/locate/pla

Atomic squeezing in three level atoms

O. Civitarese [a,*], M. Reboiro [a,b]

[a] *Department of Physics, University of La Plata c.c. 67 1900, La Plata, Argentina*
[b] *Faculty of Engineering, University of Lomas de Zamora c.c. Km 2, (1836) Lavallol, Argentina*

Received 24 January 2006; received in revised form 18 April 2006; accepted 19 April 2006

Available online 27 April 2006

Communicated by B. Fricke

Abstract

The conditions under which squeezing occurs, in a system of atoms and photons, are modelled by describing atom–photon interactions in three atomic levels. The time evolution of the spin population is calculated for different initial conditions. The effect of the use of coherent states, in the photon sector of the initial condition, is discussed. It is found that (i) the use of coherent states does not suffice for the transfer of spin between the atoms and the laser field, (ii) the interactions between the atomic levels and the radiation field must be non-symmetric, and (iii) that the squeezing is washed-out if the number of atoms is increased.

PACS: 03.65.Fd; 42.50.-p; 42.50.Hz

Keywords: Squeezing; Three level atoms; Laser field; Coherent states

1. Introduction

The conditions under which squeezing may take place, e.g. the transfer of quantum information between atomic states and laser fields, are currently under theoretical and experimental investigations. The following is an overview of the pertinent literature. For a general review see Ref. [1]. Squeezing in quantized electromagnetic fields has received continuous attention, since the first publications appeared more than twenty years ago [2,3]. Ref. [4] describes squeezing generation and revivals in a cavity-ion system in contact with a reservoir. The system consists of a single two-level ion in a harmonic trap, at zero temperature and exposed to the action of two external lasers. The authors of [4] have obtained an analytical solution for the total density operator of the system and shown that squeezing in the motion of the ion and in the cavity field is generated. They have also shown that complete revivals of the states of motion of the ion and of the cavity field occur periodically. Ref. [5] proposes a simple scheme to measure squeezing and phase properties of a harmonic oscillator field to which atoms are exposed. It is shown that by measuring atomic polarizations it may be possible to measure properties of the field. In Ref. [6], the transfer of quantum correlations from atoms to light, by Raman scattering of a strong laser pulse on a spin-squeezed atomic sample, is proposed. It is shown [6] that under adequate conditions the quantum information of collective states of atoms may be transferred to a pulse of light. Atomic squeezing under collective emission was studied in Ref. [7], where a method of governing the temporal behavior of the squeezing factor was developed and the influence of a squeezed effective vacuum on the characteristic of collective emission was investigated. Entanglement and spin squeezing properties for three bosons in two modes were presented in Ref. [8]. The theoretical and experimental aspects of entanglement and squeezing in a two-mode system have been presented in [9]. The study of spin squeezing in nonlinear spin-coherent states is found in [10]. Optimally squeezed spin states have been considered in Ref. [11]. In Ref. [12] the relations between bosonic quadrature and atomic spin squeezing was studied. The spin transfer between photons and atoms was examined by using the Dicke Hamiltonian [13]. Spin squeezing

* Corresponding author.
E-mail address: civitare@fisica.unlp.edu.ar (O. Civitarese).

0375-9601/$ – see front matter
doi:10.1016/j.physleta.2006.04.043

via atom–field interactions was considered within the framework of the Tavis–Cummings model [14] in Ref. [15]. The work of Genes et al. [15] describes an ensemble of N two level atoms interacting with a quantized cavity field. There it is shown that spin squeezing of both the atoms and the field can be achieved provided the initial state of the cavity field has coherence between number states differing by two. Also in this reference an analytic solution was found that is valid in the limit that the number of atoms is greater than the average number of photons in the coherent state of the cavity field. In this limit the degree of spin squeezing increases with increasing values of the average number of photons.

A common feature shared by the theoretical models, introduced in the references given above, is the interaction between atomic levels and photons. The general structure of the Hamiltonians which include such type of interactions belongs to the family of couplings presented in Marshalek and Klein in Ref. [16]. These forms are amenable to boson expansions and/or exact boson mappings [17,18]. Boson mapping techniques allow for the generalization of the simple forms of interactions, which have been used so far, like the Dicke Hamiltonian [13] or the Tavis–Cummings Hamiltonian [14], for instance. The adopted scheme of two level atoms interacting with the radiation field was extended to consider squeezing in three level atoms Wodkiewicz et al. [19], by Ficek and Drummond [20,21] and by Javanainen and Gould [22]. In [21], it was found that a significant reduction in the population of the first excite state and a population larger than 0.5 in the second excited state may appear. In [22], the population of three level atoms interacting with two photons is calculated by using correlation function techniques.

In this Letter we investigate the dependence of squeezing with respect to the model parametrization of the interactions between atoms and photons, and with respect to the initial condition imposed to the radiation field. We are considering: (a) three-level atoms, and (b) coherent states of photons to model the initial condition. Concerning point (a) we shall present the algebraic details needed to construct the exact solution of a Hamiltonian describing atomic excitations of A three-level atoms induced by the exchange of photons. Relative to point (b), we shall study the dependence of the solutions upon the average number of photons in the initial state. The details of the formalism are presented in Section 2. The occurrence of squeezing in a system of A three-level atoms and photons is numerically modelled in Section 3, where we present and discuss the solutions for different parameters of the model and different initial conditions. The time evolution of atomic and field squeezing is shown also in Section 3. Conclusions are drawn in Section 4.

2. Formalism

The system consists of A identical three-level atoms in interaction with a radiation field [19]. The atoms and the photons are placed in a cavity. The creation (annihilation) operator for the ith atomic level ($i = 0, 1, 2$), is denoted by $b_i^\dagger$ (b_i), and operators referring to different atomic levels commute. The Hamiltonian of the system reads

$$H = \omega a^\dagger a + \sum_i E_i S^{ii} + g_1\left(a S_+^{01} + a^\dagger S_-^{01}\right) + g_2\left(a S_+^{12} + a^\dagger S_-^{12}\right). \tag{1}$$

Here ω is the energy of the photon, $a^\dagger$ (a) is the one photon-creation (annihilation) operator, E_i is the energy of the ith atomic level, and g_1 and g_2 are coupling constants describing the absorption (emission) of a photon in the presence of an upward (downward) atomic excitation between levels 0 and 1 (term proportional to g_1), and between levels 1 and 2 (term proportional to g_2). The operators

$$S^{ij} = b_j^\dagger b_i, \quad i, j = 0, 1, 2, \tag{2}$$

satisfy the commutation relations

$$\left[S^{ij}, S^{km}\right] = \delta_{im} S^{kj} - \delta_{jk} S^{im}. \tag{3}$$

They are used to define the atomic inversion operators

$$S_z^{ij} = \frac{1}{2}\left(S^{jj} - S^{ii}\right), \tag{4}$$

and the transition operators $S_\pm^{ij}$

$$S_+^{ij} = S^{ij}, \qquad S_-^{ij} = \left(S_+^{ij}\right)^\dagger = S^{ji}, \quad i, j = 0, 1, 2, i < j. \tag{5}$$

The two-photon resonance condition [23] is satisfied by fixing the energies of the atomic levels E_i at the values

$$E_2 - E_0 = 2\omega, \qquad E_1 - E_0 = \omega - \Delta. \tag{6}$$

2.1. The exact solution

The operator

$$\hat{L} = a^\dagger a + 2S_z^{02}, \tag{7}$$

commutes with the Hamiltonian of Eq. (1), which, therefore, can be diagonalized in the basis of states

$$|n_b n_0 n_1 n_2\rangle = \frac{1}{\sqrt{n_b! n_0! n_1! n_2!}} a^{\dagger n_b} b_0^{\dagger n_0} b_1^{\dagger n_1} b_2^{\dagger n_2} |0\rangle. \tag{8}$$

Let N_0 be the number of photons when all the atoms are in the ground state. To create n_1 atoms in level 1 and n_2 atoms in level 2 one uses $n_1 + 2n_2$ photons, then $n_b + n_1 + 2n_2 = N_0$. Subtract $n_0 + n_1 + n_2 = A$, which is the number of atoms, to obtain

$$n_b + n_2 - n_0 = N_0 - A \equiv N \tag{9}$$

which tell us that N, which is the sum of the number of photons, n_b, and the difference $n_2 - n_0$ between the population of the atomic states $i = 2$ and $i = 0$, is an eigenvalue of $\hat{L}$ in the basis (8). Having established the invariance of $N = n_b + n_2 - n_0$ it follows that $\hat{L}$ is a constant of the motion, as can be verified directly by establishing its commutation with the Hamiltonian (1).

The diagonalization yields the set of eigenvalues, E_α, and eigenvectors

$$|\Psi_\alpha\rangle = \sum_{a \equiv \{n_b, n_0, n_1, n_2\}} c_\alpha(a)|a\rangle. \tag{10}$$

The actual dimension of the configuration space, that is the largest possible value of N for which the exact diagonalization is still feasibly, is fixed by analyzing the stability of the wave function for increasing values of N. The adopted procedure will be discussed in Section 3.

2.2. Relative squeezing of two operators

For a given pair of operators, R and S, the quantity

$$Q(R,S) = \frac{2(\Delta R)^2}{|\langle\phi(t)|[R,S]|\phi(t)\rangle|},$$
$$(\Delta R)^2 = \langle\phi(t)|R^2|\phi(t)\rangle - \langle\phi(t)|R|\phi(t)\rangle^2, \tag{11}$$

measures the squeezing of the operator R with respect to S [1,7,9]. Squeezing means that $Q(R,S) < 1$. The time dependent state $|\phi(t)\rangle$ is obtained by writing the initial state $|\phi(t=0)\rangle$ as a linear combination of eigenstates of the Hamiltonian and introducing the suitable exponential time dependent factor for each eigenstate. In general,

$$\langle\phi(t)|\hat{O}|\phi(t)\rangle = \sum_{\alpha,\beta} D_{\alpha,\beta}(t)\langle\Psi_\alpha|\hat{O}|\Psi_\beta\rangle \tag{12}$$

with

$$D_{\alpha,\beta}(t) = \langle\phi(t)|\Psi_\alpha\rangle\langle\Psi_\beta|\phi(t)\rangle, \tag{13}$$

which depends on the initial condition, $|\phi(0)\rangle$, since

$$|\phi(t)\rangle = e^{-\frac{iHt}{\hbar}}|\phi(0)\rangle, \tag{14}$$

and O is either R, S or $[R,S]$. In the following, we shall write, explicitly, the results for the squeezing $Q(R,S)$ for different initial conditions.

2.3. One atom case

In this section we present the analytic results for the case $A = 1$. The diagonalization of the Hamiltonian of Eq. (1), within the three-dimensional space spanned by the states

$$|a\rangle = |n_b, 1, 0, 0\rangle,$$
$$|b\rangle = |n_b - 1, 0, 1, 0\rangle,$$
$$|c\rangle = |n_b - 2, 0, 0, 1\rangle, \tag{15}$$

yields the eigenvalues λ_α and eigenvectors $|\Psi_\alpha\rangle$, with $\alpha = 1, 2, 3$, for each of the subspaces labelled by a fixed value of $N = n_b + n_2 - n_0$. The basis (15) includes up to three states depending on the values of n_b, n_2 and n_0. For a variable number of photons each subspace of the solutions is labelled by the value N. The matrix of Eq. (13) is determined by the initial condition $|\phi(0)\rangle$. As limiting cases we shall consider two types of initial states: (a) a Fock state of a fixed number of photons and a given initial population of the atomic levels, and (b) a product of a coherent state of photons and a given initial atomic state.

The initial condition (a) is written

$$|\phi(0)\rangle = |n_b, 1, 0, 0\rangle, \tag{16}$$

and it represents the product state on n_b photons and one atom in the lower state. Then, if $N = n_b - 1$

$$D_{\alpha,\beta} = d_\alpha^* d_\beta,$$
$$d_\alpha = e^{-i\lambda_\alpha(N)t/\hbar}. \tag{17}$$

With this we shall calculate the time evolution of the operator of atomic inversion, S_z^{02}; that is the time evolution of the difference between the occupations of the upper and lower atomic states. To compute the atomic squeezing, of Eq. (11), one needs also to calculate the time evolution of the ladder operator S_+^{02}. After a rather straightforward calculation one finds that for this initial condition (e.g; a given number of photons in the initial state) atomic squeezing does not appear, no matter how many photons or atomic levels are included in the initial condition.

For the case of condition (b), the calculation leads to the expression

$$D_{\alpha,\beta} = d_\alpha^* d_\beta,$$
$$d_\alpha = e^{-\frac{|z|^2}{2}} \sum_{k=0}^{\infty} \frac{z^k}{\sqrt{k!}} c_\alpha^*(k,1,0,0) e^{-i\lambda_\alpha(k-1)t/\hbar}, \tag{18}$$

where $|z|^2 = n_b$ is the mean value of the number of photons in the coherent state. In this case

$$\langle S_z(t)\rangle = e^{-n_b} \sum_{k=0}^{\infty} \frac{n_b^k}{k!}\langle S_z(k,t)\rangle,$$
$$\langle S_z^2(t)\rangle = e^{-n_b} \sum_{k=0}^{\infty} \frac{n_b^k}{k!}\langle S_z^2(k,t)\rangle,$$
$$\langle S_+(t)\rangle = e^{-n_b} \sum_{k=0}^{\infty} \frac{n_b^{k+1}}{k!} f_0(k)\big(a(k) + ib(k)\big), \tag{19}$$

where the quantities $f_0(k)$, $a(k)$, and $b(k)$, are straightforwardly obtained from the definition of the Hamiltonian and of the exact eigenvectors. The contribution of states with different values of N is a necessary condition for coherence [15]. The time evolution of the ladder operator (19) does not vanish and expression Eq. (11) admits non-diagonal terms in the denominator. It means that, depending on the coupling constants of H, squeezing may appear, that is $Q(S_z, S_+) \leqslant 1$.

3. Results and discussion

The energy spacing between the atomic levels is fixed by Eq. (6), with $\Delta = 0$, thus $E_0 = -\omega$, $E_1 = 0$, $E_2 = \omega$. In all cases we have taken a coherent state in the photon sector. We have considered symmetric, $g_1 = g_2$, and non-symmetric, $g_1 \neq g_2$, couplings in the Hamiltonian. The mean value of the number of photons in the coherent state has been taken as an external variable, and the calculations have been performed for different number of atoms. In Figs. 1–3, we show the results of the present calculations for the time evolution of the atomic inversion, $\langle S_z(t)\rangle$, the atomic squeezing, $Q(S_z, S_+)$, and the field squeezing, $Q(x, p)$ [15]. The atomic initial condition consists of $n_0 = A$ atoms in the ground state, while the parameter z of the photon coherent state is fixed at the value $|z|^2 = n_b$. With

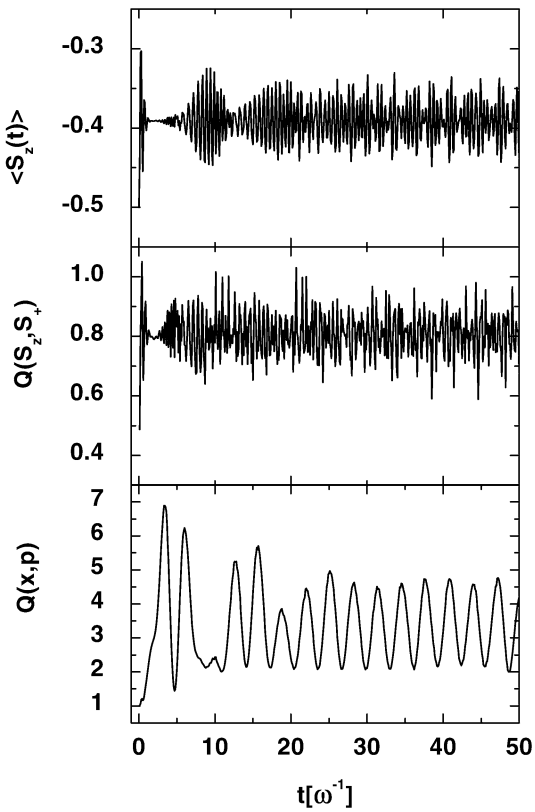

Fig. 1. Mean value of the inversion operator, $\langle S_z(t)\rangle$, atomic squeezing, $Q(S_z, S_+)$, and field squeezing, $Q(x, p)$, as a function of time. The system consists of one atom, initially in its ground state, and a coherent photon field, with mean value of the number of photons $n_b = 10$. The coupling constants are fixed at the values $g_1 = 1$ and $g_2 = 4$.

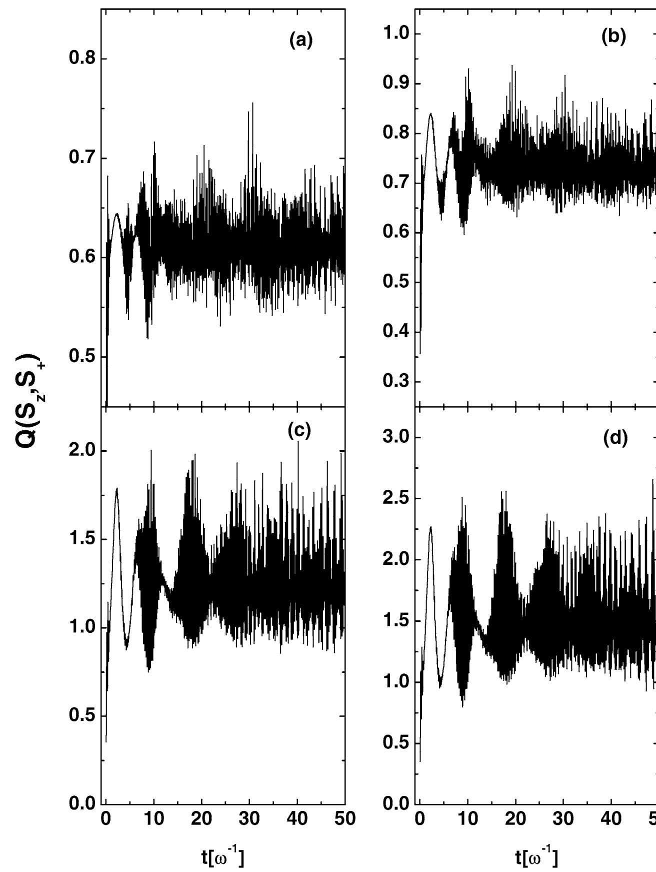

Fig. 2. Atomic squeezing, $Q(S_z, S_+)$, as a function of time. The interaction coupling constants are fixed at the values $g_1 = 1$, $g_2 = 6$. The initial state consists of A atoms in their ground state, and of a coherent state with mean value of the number of photons $n_b = 21$. Insets (a), (b), (c) and (d), correspond to a system with $A = 3$, $A = 6$, $A = 15$ and $A = 18$ atoms, respectively.

these parameters the Hamiltonian of Eq. (1) was diagonalized and the density matrix of Eq. (13) was obtained for each subspace, A, N. Fig. 1 shows the results for $A = 1$, $g_1 = 1$, $g_2 = 4$ and $n_b = 10$. This case does exhibit squeezing. The actual central value of the atomic squeezing, $Q(S_z, S_+)$, is of the order of 0.75–0.80. The time evolution of the atomic inversion $\langle S_z(t)\rangle$ is consistent with the central value $\langle S_z(t)\rangle = -0.4$. The couplings have been chosen in this manner because no signal of squeezing was obtained, in spite of the use of a coherent state in the photon sector, when $g_1 = g_2$. This feature persists if the number of atoms is increased. Fig. 2 shows the results corresponding to $A = 3, 6, 15, 18$, $g_1 = 1$, $g_2 = 6$, and $n_b = 21$. It is seen that the atomic squeezing is washed out when the number of atoms is increased. This is understood in terms of the definition of N, see Eq. (9), since for increasing values of the number of atoms and fixed values of N_0 one gets smaller values of N, that is $n_2 \approx n_0$, for a given value of n_b. These results seems to indicate the strong dependence of the atomic squeezing upon the number of atoms. In fact, as it can be deduced from Eq. (9), if one increases the value of A and keeps the value of N_0, the value of N will decrease. It necessarily implies that the difference $n_2 - n_0$ decreases. This change reflects upon the expectation value of S_+, which will decrease. The comparison between the results which we have obtained by using symmetric and non-symmetric couplings points out to the critical dependence of squeezing upon the relative strength of the emission and absorption of photons in transitions involving the ground state, n_0, and the excited state n_2. This is independent of the mean value of the number of photons, see Fig. 3, where the time evolution of squeezing is depicted for $n_b = 400$, $g_1 = 1$, $g_2 = 6$ and $A = 3$ (inset 3.a), $A = 6$ (inset 3.b), $A = 15$ (inset 3.c), and $A = 18$ (inset 3.d). The population of the atomic level 1 works as a gate for the population of the atomic level 2, since $g_1 < g_2$. The opposite condition, $g_1 > g_2$, does not result in squeezing, as we have verified in our calculations. As shown by the cases considered in Fig. 3, revivals may indeed occur. Concerning the choice of Δ of Eq. (6), we have verified that using $\Delta \neq 0$ does not affect the value of $Q(S_z, S_+)$. However, the exchange of the role of g_1 and g_2, that is by setting $g_1 > g_2$, suppresses the squeezing, as we have mentioned before.

4. Conclusions

In this work we have studied the occurrence of squeezing in systems composed of three level atoms and a radiation field. It is found that: (i) the use of initial conditions consisting of a fixed number of photons does not lead to squeezing; instead, it appears if coherent states are considered in the photon sector of the initial condition, (ii) the transfer of spin between the

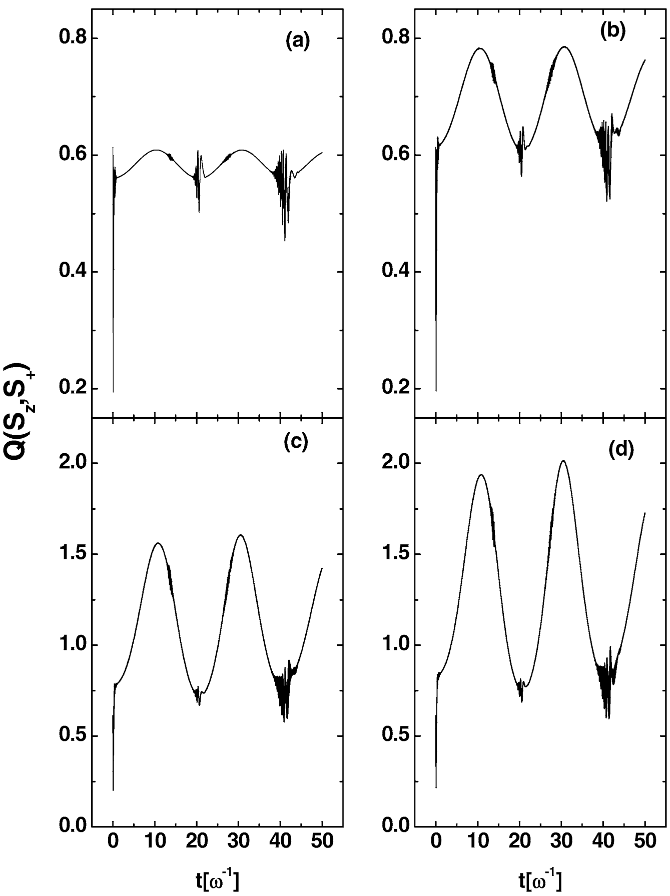

Fig. 3. Idem as Fig. 2, for $n_b = 400$.

atoms and the photons is enhanced if the interactions between the atomic levels and the photons are non-symmetric ($g_2 > g_1$ since the choice $g_1 > g_2$ does not lead to squeezing); the use of coherent states *does not* lead to squeezing unless the interactions are non-symmetric, and, (iii) the squeezing is washed-out if the number of atoms is increased.

Acknowledgements

This work was partially supported by the National Research Council of Argentine (CONICET).

References

[1] P.D. Drummond, Z. Ficek, Quantum Squeezing, Springer, Berlin, 2004.
[2] D.F. Walls, Nature 306 (1983) 141.
[3] C.K. Hong, L. Mandel, Phys. Rev. Lett. 54 (1985) 323.
[4] R. Rangel, R. Carvalho, N. Zagury, quant-ph/0412055.
[5] G.T. Rubin-Linares, H. Moya-Cessa, quant-ph/0501158.
[6] U.V. Poulsen, K. Mølmer, Phys. Rev. Lett. 87 (2001) 123601.
[7] V.I. Yukalov, E.P. Yukalova, Phys. Rev. A 70 (2004) 053828.
[8] B. Zeng, D.L. Zhou, Z. Xu, L. You, quant-ph/0501082.
[9] V. Josse, A. Dantan, A. Bramatti, E. Giacobino, J. Opt. B: Quantum Semiclass. Opt. 6 (2004) S532.
[10] X. Wang, J. Opt. B: Quantum Semiclass. Opt. 3 (2001) 93.
[11] A.G. Rojo, Phys. Rev. A 68 (2003) 013807.
[12] X. Wang, B.C. Sanders, Phys. Rev. A 68 (2003) 033821.
[13] R.H. Dicke, Phys. Rev. 93 (1954) 99.
[14] M. Tavis, F.W. Cummings, Phys. Rev. 170 (1968) 379;
E.T. Jaynes, F.W. Cummings, Proc. IEEE 51 (1963) 89;
E.R. Buley, F.W. Cummings, Phys. Rev. 134 (1964) A1454;
F.W. Cummings, Phys. Rev. 140 (1965) A1051.
[15] C. Genes, P.R. Berman, A.G. Rojo, Phys. Rev. A 68 (2003) 043809.
[16] A. Klein, E.R. Marshalek, Rev. Mod. Phys. 63 (1991) 375.
[17] O. Civitarese, M. Reboiro, Phys. Rev. C 57 (1998) 3055.
[18] O. Civitarese, M. Reboiro, Phys. Rev. C 60 (1999) 034302.
[19] K. Wodkiewicz, P.L. Knight, S.J. Buckle, S.M. Barnett, Phys. Rev. A 35 (1987) 2567.
[20] Z. Ficek, P. Drummond, Phys. Rev. A 43 (1991) 6247.
[21] Z. Ficek, P. Drummond, Phys. Rev. A 43 (1991) 6258.
[22] J. Javanainen, P.L. Gould, Phys. Rev. A 41 (1990) 5088.
[23] P.L. Knight, Phys. Scr. T 12 (1986) 51;
D. Phoenix, P.L. Knight, J. Opt. Soc. Am. B 4 (1990) 116;
R.R. Puri, R.K. Bullough, J. Opt. Soc. Am. 5 (1998) 116;
L.A. Lugiato, P. Galatola, L.M. Narducci, Opt. Commun. 76 (1990) 276;
R.R. Puri, G.S. Agarval, Phys. Rev. A 37 (1988) 3879;
L.-S. He, J. Opt. Soc. Am. B 6 (1989) 1915.