Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER
Physics Letters B 626 (2005) 80–85
PHYSICS LETTERS B
www.elsevier.com/locate/physletb

Universal features of the nuclear matrix elements governing the mass sector of the $0\nu\beta\beta$ decay

O. Civitarese [a], J. Suhonen [b]

[a] *Department of Physics, University of La Plata, C.C. 67, 1900 La Plata, Argentina*
[b] *Department of Physics, University of Jyväskylä, P.O. Box 35, FI-40014, Jyväskylä, Finland*

Received 11 February 2005; received in revised form 26 July 2005; accepted 28 August 2005

Available online 15 September 2005

Editor: J.-P. Blaizot

Abstract

In this work we report on manifest universal features found in the nuclear matrix elements which govern the mass sector of the neutrinoless double beta decay. The results are based on the analysis of the calculated matrix elements corresponding to the decays of ^{76}Ge, ^{82}Se, ^{100}Mo, and ^{116}Cd. The results suggest a dominance of few low-lying nuclear states of few multipoles in these matrix elements. Dedicated charge-exchange reactions could be used to probe these key states to determine experimentally the value of the nuclear matrix element.

PACS: 14.60.Pq; 23.40.Bw; 23.40.Hc

The need to know in the most accurate way the values of the nuclear matrix elements which are relevant for studies of the neutrinoless double beta ($0\nu\beta\beta$) decay is far from being a purely academic question. Considering the current efforts devoted to the experimental search of signals of $0\nu\beta\beta$ [1,2] and the implications for particle physics [3], the question about the scope of the involved nuclear-structure calculations and about their predictive power [4–7] cannot be avoided. While the existence of neutrino oscillations is supported by experimental results [8], neutrinoless double beta decay is a unique source of information about the absolute scale of the light-neutrino masses and about the nature of the neutrino [3]. In a recent publication [5], we have analyzed the combined set of data coming from the oscillation experiments and from the limits fixed by double-beta-decay experiments, for the case of the $0\nu\beta\beta$ decay of ^{76}Ge. Similar studies were reported, afterwards, in [7], while recent results of studies, specifically devoted to the neutrino-physics side, can be found in [9].

The adequacy of some of the theoretical assumptions adopted to describe double-beta-decay observ-

E-mail address: suhonen@phys.jyu.fi (J. Suhonen).

0370-2693/$ – see front matter © 2005 Elsevier B.V. All rights reserved.
doi:10.1016/j.physletb.2005.08.107

ables, and the degree of accuracy of the corresponding calculations, can be tested experimentally by means of the single beta decay transitions [10], charge-exchange reactions ((p, n), (^{3}He, t) and (n, p), (d, ^{2}He), see [2, 11]), muon capture [12] and neutrino–nucleus interactions [13]. Theoretical study of these processes allows us to constrain the calculations related to the $0\nu\beta\beta$ decay transitions.

In this Letter we are presenting some evidence which strongly suggests the existence of a certain degree of universality in the calculated nuclear matrix elements, which may indicate that the nuclear-structure sector of the neutrinoless double beta decay problem may be accurately determined by dedicated experiments, in spite of the complexity of the problem [4]. For the sake of completeness, we briefly give the necessary theoretical expressions. Details can be found in [4,6,15]. The mass sector of the half-life, for neutrinoless double-beta decay transitions, is written [4,14]:

$$C^{(0\nu)}_{\mathrm{mm}} = G^{(0\nu)}_1 \big(M^{(0\nu)}_{\mathrm{GT}}(1-\chi_{\mathrm{F}})\big)^2, \tag{1}$$

where $G^{(0\nu)}_1$ is a leptonic phase-space integral and

$$M^{(0\nu)}_{\mathrm{GT}} = (m_{\mathrm{e}}R)^{-2} \times \sum_{ij}\sum_{a}\big\langle 0^+_{\mathrm{F}}\big\| h_+(r_{ij}, E_a) \times \sigma(i)\sigma(j)\tau(i)^-\tau(j)^-\big\|0^+_I\big\rangle \tag{2}$$

is the nuclear matrix element of the two-body Gamow–Teller operator. In the above definition [15] of the Gamow–Teller operator we use the scaling factor $(m_{\mathrm{e}}R)^{-2}$ relative to the one introduced in [14]. This should be kept in mind when comparing our quoted numbers with numbers coming from some other works. The factor χ_{F} is the ratio between the matrix element of the two-body Fermi operator and the two-body Gamow–Teller operator, with I denoting the initial and F the final nuclear state. In these definitions the value of the axial-vector electroweak coupling constant g_{A} is absorbed in the definition of the phase-space integral $G^{(0\nu)}_1$ [4,14] and in the ratio χ_{F}.

The conventional procedure to evaluate (2) consists of performing the expansion of the neutrino potential $h_+(r_{ij})$ in spherical multipoles, which are then coupled to the spin operators appearing in (2). A suitable way to calculate this expansion consists of introducing, for each multipole, a complete set of states, which span the space of states represented by the sub-index a in (2). These are nuclear states, whose wave functions should be determined to compute the transition densities of the isovector multipole operators [4]

$$\rho^{\mathrm{I(F)}}_{\lambda,\mu}(n,k) = \big\langle J^\pi, n\big|(Y_k\sigma)_{\lambda,\mu} f_k(r)\tau^-\big|0^+_{\mathrm{I(F)}}\big\rangle \tag{3}$$

between the initial (I) and final (F) ground states and the excited states of the intermediate double-odd-mass nucleus, denoted in (3) by their multipolarity J, parity π and eingenvalue index n. The radial function is a Bessel function of the order k.

In the present work the evaluation of (2) has been performed by using the standard proton–neutron Quasiparticle Random-Phase Approximation (pn-QRPA) in conjunction with single-particle states obtained by diagonalizing a Woods–Saxon potential and including the Coulomb interaction, for protons. The monopole pairing effects are accounted for in the Quasiparticle Mean-Field Approximation. Details about this theoretical framework can be found in [4]. The matrix elements of the two-body interaction, used in the calculation, are the ones obtained from the G-matrix treatment of the OBEP [16]. The parameters of the proton–proton and neutron–neutron pairing channels were fixed to fit the observed odd–even mass differences and the energy of low-lying quasiparticle states in the neighborhood of the considered double-beta-decay systems.

It is known [17,18] that in the pn-QRPA calculations the values of the nuclear matrix elements, corresponding to the two-neutrino double-beta ($2\nu\beta\beta$) decay, vary very much as functions of the value of the scaling parameter g_{pp} of the proton–neutron particle–particle channel of the two-body interaction. This fact is well established. Concerning the matrix elements governing the mass sector of the neutrinoless double beta decay it was first shown in [19] that this sensitivity is not that strong there. This conclusion agrees with ours, as we show below.

In the present calculation we have chosen the values of g_{pp} as done in [20], i.e., by making a fit to the matrix element extracted from the recommended two-neutrino double-beta-decay data [21]. We improve on the procedure of [20] by taking into account also the experimental uncertainties in the measured $2\nu\beta\beta$ half-lives and in the value of the axial-vector coupling constant g_{A}. Here we considered the interval $1 \leqslant g_{\mathrm{A}} \leqslant 1.25$ for g_{A}. This procedure yields two bands of possible values of g_{pp}, one for positive and one for negative values of $M_{GT}^{(2\nu)}$. By determining the range of the parameter g_{pp} in this fashion, one guarantees that the variation of the contribution of the $J^\pi = 1^+$ multipole to the $0\nu\beta\beta$ is at its largest, i.e., within this interval that contribution gradually decreases, vanishes and changes its sign, while still keeping the convergence of the pn-QRPA solutions. However, we have to be aware that this procedure does not necessarily yield the best possible value of the coupling constant g_{pp}, since for the $0\nu\beta\beta$ decay the effect of the 1^+ multipole is not that important as it is for the $2\nu\beta\beta$ decay, as we show below.

Our starting point is the calculation of each of the terms entering in the multipole expansion of the matrix elements of Eq. (2). The results are shown in Table 1, where we are listing the values obtained for the considered decays, and for different values of the strength g_{pp} of the renormalized particle–particle channel allowed by the experimental values of the matrix element (the method of extracting the values of g_{pp} was discussed above). In Table 1 we show the values of $M_{GT}^{(0\nu)}$ (third column), the contribution to $M_{GT}^{(0\nu)}$ from the set of $J^\pi = 1^+$ states (fourth column) and the bulk of the matrix element $M_{GT}^{(0\nu)}$, which is obtained by excluding from the multipole expansion the contribution of the $J^\pi = 1^+$ states (fifth column). The contributions of Fermi transitions to the matrix element appearing in (1) are absorbed in the definition of χ_F, and they are shown for two values of the axial-vector coupling constant, g_A, in the last two columns of Table 1.

From the results shown in Table 1 it is seen that the variation of the bulk matrix element amounts to less than 7% (^{76}Ge), 4% (^{82}Se), 4% (^{100}Mo), and 2% (^{116}Cd), for the considered ranges of g_{pp}. It demonstrates that the bulk of the matrix elements remains practically unaffected by the value of g_{pp}. For all cases the contribution of the 1^+ multipole to $M_{GT}^{(0\nu)}$ represents, at its largest, less than 10% of the sum over all multipoles ($0^+ \to 11^+$, $0^- \to 10^-$) included in (2). Hence, one may think of that estimate as the largest possible theoretical uncertainty in the value of the nuclear matrix element (2) coming from the 1^+ multipole.

To reiterate the basic ideology of our calculations, in all cases we have considered the experimental half-lives of the $2\nu\beta\beta$ decay modes, including the corresponding error bars, to extract the values of g_{pp} which are compatible with the data. For the case of the decay of ^{100}Mo our calculation could not reproduce the small value of the measured matrix element. In this case only one value of g_{pp} is chosen, and it is the one which closest reproduces the two-neutrino double-beta-decay data. More details of our procedure are given in [22].

Table 2 shows the leading multipoles contributing to $M_{GT}^{(0\nu)}$. Also there the variation of the 1 contribu-

Table 1
Value of the matrix element $M_{GT}^{(0\nu)}$ of Eq. (1) as a function of the parameter g_{pp}, across the domain of g_{pp} which best fits the experimental data on $2\nu\beta\beta$. The results of the sum over all multipoles is given in the third column, the contribution of only one set of states ($J^\pi = 1^+$) is shown in the fourth column, and the bulk value, obtained by excluding the $J^\pi = 1^+$ states from the sum, is shown in the fifth column. The last two columns show the results of the ratio χ_F, for two values of the axial-vector coupling constant $g_A = 1.00$ and $g_A = 1.254$, respectively

Case	g_{pp}	$M_{GT}^{(0\nu)}$ (all)	$M_{GT}^{(0\nu)}$ (1^+)	$M_{GT}^{(0\nu)}$ (bulk)	χ_F ($g_A = 1.00$)	χ_F ($g_A = 1.254$)
^{76}Ge	0.89	162.35	19.18	143.17	−0.419	−0.266
	0.96	148.31	8.89	139.42	−0.428	−0.272
	1.00	137.98	1.06	136.92	−0.439	−0.279
	1.05	120.39	−12.79	133.18	−0.470	−0.299
^{82}Se	0.98	114.83	12.23	102.60	−0.378	−0.240
	1.10	103.39	3.07	100.32	−0.374	−0.238
	1.17	95.16	−3.69	98.85	−0.374	−0.238
	1.23	86.70	−10.82	97.51	−0.376	−0.239
^{100}Mo	1.16	142.30	20.44	121.86	−0.373	−0.237
^{116}Cd	1.44	66.12	6.80	59.32	−0.363	−0.231
	1.50	62.77	4.37	58.40	−0.371	−0.236
	1.55	59.06	1.55	57.51	−0.381	−0.242
	1.58	56.01	−0.98	57.99	−0.391	−0.249

Table 2
Leading multipole decomposition of the matrix element $M_{\rm GT}^{(0\nu)}$, as function of the parameter $g_{\rm pp}$. The values of $g_{\rm pp}$ were chosen as it is explained in the captions to Table 1

Case	$g_{\rm pp}$	1^+	2^+	3^+	1^-	2^-	3^-	4^-
^{76}Ge	0.89	−19.183	−9.525	−19.431	−11.553	−41.172	−9.833	−16.343
	0.96	−8.885	−9.314	−19.097	−11.107	−38.789	−9.772	−16.261
	1.00	−1.059	−9.186	−18.891	−10.842	−37.107	−9.734	−16.211
	1.05	12.788	−9.016	−18.614	−10.497	−34.465	−9.683	−16.142
^{82}Se	0.98	−12.231	−5.755	−12.532	−7.043	−35.118	−7.004	−11.689
	1.10	−3.071	−5.597	−12.349	−6.865	−33.744	−6.955	−11.571
	1.17	3.690	−5.498	−12.231	−6.760	−32.845	−6.924	−11.496
	1.23	10.815	−5.408	−12.123	−6.668	−32.009	−6.898	−11.428
^{100}Mo	1.16	−20.436	−9.166	−18.019	−12.485	−29.643	−6.879	−11.128
^{116}Cd	1.44	−6.801	−4.048	−7.916	−7.892	−12.490	−2.744	−5.196
	1.50	−4.375	−4.014	−7.713	−7.765	−12.123	−2.728	−5.151
	1.55	−1.544	−3.982	−7.523	−7.649	−11.755	−2.713	−5.108
	1.58	0.984	−3.966	−7.410	−7.589	−11.524	−2.706	−5.084

tion becomes evident. On the other hand, the other multipoles are practically independent of $g_{\rm pp}$ within the experimentally determined range of values of $g_{\rm pp}$. The contribution of all multipoles to the final matrix element is shown in Fig. 1, for the case of the decay of ^{76}Ge, and in Fig. 2, for the case of the decay of ^{116}Cd. We have taken these cases as illustrative examples of the results shown in Table 2. It shows the clear dominance of the contribution of the $J^\pi = 2^-$ virtual transition, which amounts to roughly 30 percent of the bulk value of the matrix element. This feature may indicate that a mechanism, similar to the single-state dominance [23] found in $2\nu\beta\beta$ decays, could possibly be present, in a softer way, also in the case of $0\nu\beta\beta$ decays. It is interesting to see that out of the high number of multipoles ($0^+ \to 11^+$, $0^- \to 10^-$), only few contribute with significant amounts to $M_{\rm GT}^{(0\nu)}$, and that their contributions are also stable against changes in the value of the parameter $g_{\rm pp}$.

From Table 2 one sees that, with the possible exception of the virtual transitions going by the set of 1^- states, whose contribution may still be reduced by center-of-mass corrections [24], most of the remaining value of $M_{\rm GT}^{(0\nu)}$, after subtracting the 2^- contribution, is given by the transitions going by the set of states with $J^\pi = 3^+$ and 4^-, at the level of 10 to 12 percent, and $J^\pi = 2^+, 5^+$, and 3^-, at the level of 6 to 10 percent.

In view of the found dominance of the contribution of the 2^- multipole, we have analyzed the structure of it, by looking at the corresponding wave functions.

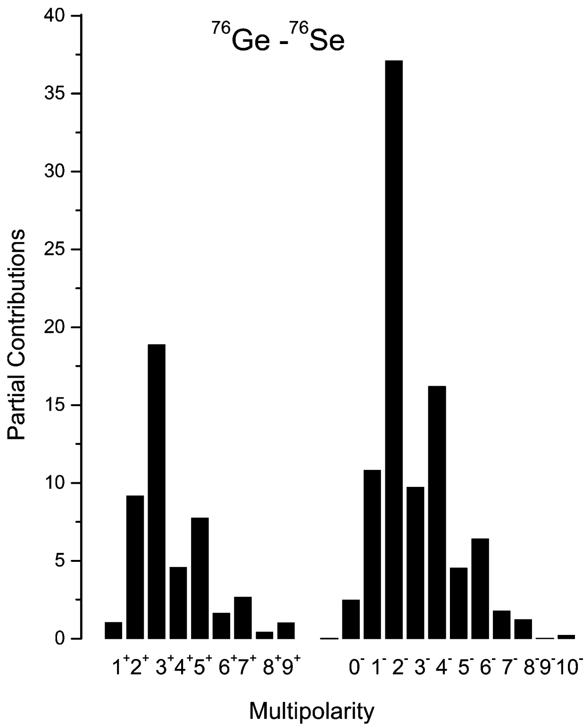

Fig. 1. Multipole contributions to the matrix element $M_{\rm GT}^{(0\nu)}$. The results correspond to the decay of ^{76}Ge (a), with $g_{\rm pp} = 1.00$.

This analysis indicates that, for the case of the decay of ^{76}Ge there is one state that contributes the most and this is the first 2^- state. The wave function of this state is practically a pure configuration, which in-

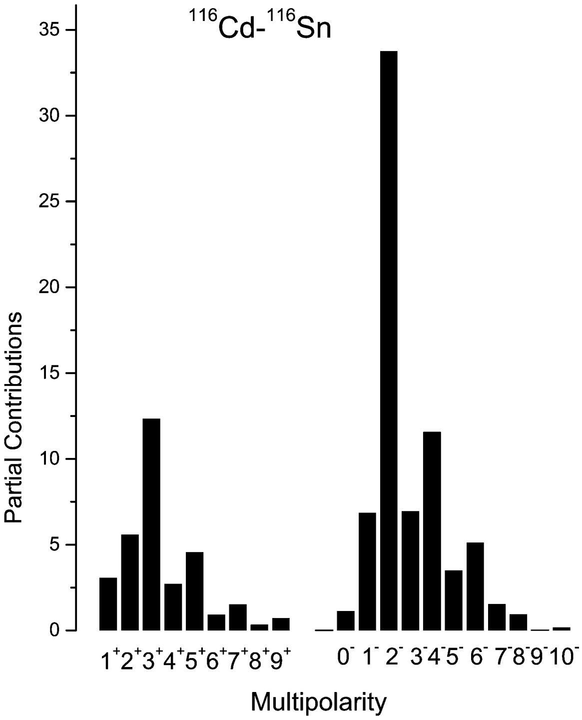

Fig. 2. Multipole contributions to the matrix element $M_{\rm GT}^{(0\nu)}$. The results correspond to the decay of ^{116}Cd (a), with $g_{\rm pp} = 1.55$.

volves the neutron intruder orbit $g_{9/2}$ coupled to the proton orbit $f_{5/2}$. The same situation appears in the case of the decay of ^{82}Se, where the bulk of the contribution is coming from the same configuration. Though the dominance of the 2^- is also clear for the cases of ^{100}Mo and ^{116}Cd, the distribution of intensity is more fragmented since the states have typically two to four components with similar amplitudes, also based on the coupling with the intruder states. These are features which greatly simplify the task of setting theoretical limits on the values of the matrix elements, because they can be absorbed in a sort of polarization factor, once the total matrix element is written as

$$M_{\rm GT}^{(0\nu)} = \left[M_{\rm GT}^{(0\nu)}\right]_{2^-}(1 + f_m), \tag{4}$$

where the factor f_m represents the contribution of all other multipoles. From the results of the present calculations we have extracted the following values: $f(^{76}\text{Ge}) = 2.6$, $f(^{82}\text{Se}) = 2.0$, $f(^{100}\text{Mo}) = 3.2$, and $f(^{116}\text{Cd}) = 3.8$. These values are very stable for the range of $g_{\rm pp}$ values reproducing the $2\nu\beta\beta$ data.

The results of calculations using different interactions can be found in [20], and from there one may conclude that the results are insensitive to the used two-body interaction. The findings of [20] support the notion that the kind of universality reported here will show up in further studies performed with different interactions, too.

Some studies of the multipole decomposition of the $0\nu\beta\beta$ matrix elements have been reported already earlier. These are given, e.g., in Refs. [25,26]. In [25] the plain pn-QRPA and in [26] renormalized and self-consistent renormalized versions of the pn-QRPA were used in the calculations. In both calculations additional refinements were done concerning the basic nucleonic weak current, namely, the short-range correlations between two nucleons and the finite-size effects of the nucleon form factors were taken into account. Both of these contributions were deemed important in these articles. Both [25] and [26] show in their Fig. 2 the decomposition of the mass mode matrix element in terms of multipoles for the $0\nu\beta\beta$ decay of ^{76}Ge. From their Fig. 2, for $g_{\rm pp}$ values near unity, one notices that the leading contribution is, indeed, 2^-. In [25] one can already speak about dominance of the 2^- contribution for the Gamow–Teller operator.

In the present formalism we take into account the finite-size effects by computing the nucleon form factors starting from the quark level [27]. These effects are generally accepted to be important for the neutrinoless double beta decay. However, we have not included the short-range correlations. The matter of short-range correlations is still somewhat open since some authors, like [26], claim to obtain sizable effects from this correction whereas some others not (see, e.g., [28,29]). Obviously, the results vary depending in which way these correlations have been taken into account.

To conclude, the following common features emerge from the analysis of the nuclear matrix elements entering the mass sector of the neutrinoless double beta decay:

(i) Changes in the particle–particle coupling constant $g_{\rm pp}$, around the values which best fit the matrix elements extracted from the available recommended results for $2\nu\beta\beta$ transitions, do not affect the bulk value of the matrix element of Eq. (2).

(ii) The contribution coming from the $J^\pi = 2^-$ set of states dominates, and it represents 30% (or more) of the total value of Eq. (2).

(iii) Of the other multipoles only very few contribute significantly, and their summed contributions amounts to about 50% of (2).

(iv) The theoretical uncertainty in the calculated value of (2), stemming from the variation of the contribution of the set of 1^+ states near the total cancellation of this contribution, can be placed at the 10 percent level, or below. The overall uncertainty of the nuclear matrix elements is not known since the uncertainties of the leading multipoles in the $0\nu\beta\beta$ matrix elements are not known.

(v) In view of Eq. (4) and the discussion in point (iv) above, dedicated experiments to look at the charge-exchange reactions to few lowest 2 states in the intermediate nuclei of double beta decays are called for. Work along these lines has been started recently (see, e.g., [2,11]).

Acknowledgements

This work has been partially supported by the National Research Council (CONICET) of Argentina and by the Academy of Finland under the Finnish Centre of Excellence Programme 2000-2005 (Project No. 44875, Nuclear and Condensed Matter Programme at JYFL). One of the authors (O.C.) gratefully thanks the warm hospitality extended to him at the Department of Physics of the University of Jyväskylä, Finland. The authors acknowledge fruitful discussions with the late Professor Yuri Zdesenko, and with Profs. Alexander Barabash, Hiro Ejiri and Frank Avignone III about the material presented in the manuscript.

References

[1] S.R. Elliott, P. Vogel, Annu. Rev. Nucl. Part. Sci. 52 (2002) 115.
[2] H. Ejiri, Phys. Rep. 338 (2000) 265.
[3] J.D. Vergados, Phys. Rep. 361 (2002) 1.
[4] J. Suhonen, O. Civitarese, Phys. Rep. 300 (1998) 123.
[5] O. Civitarese, J. Suhonen, Nucl. Phys. A 729 (2003) 867.
[6] A. Faessler, F. Simkovic, J. Phys. G 24 (1998) 2139.
[7] S.M. Bilenky, A. Faessler, F. Simkovic, Phys. Rev. D 70 (2004) 033003.
[8] SNO Collaboration, Q.R. Ahmad, et al., Phys. Rev. Lett. 89 (2002) 011301;
SNO Collaboration, Q.R. Ahmad, et al., Phys. Rev. Lett. 89 (2002) 011302;
Super-Kamiokande Collaboration, S. Fukuda, et al., Phys. Rev. Lett. 86 (2001) 5651;
KamLAND Collaboration, K. Eguchi, et al., Phys. Rev. Lett. 90 (2003) 021802;
M. Appollonio, et al., Phys. Lett. B 466 (1999) 415;
C.L. Bennet, et al., astro-ph/0302207.
[9] S. Pascoli, S.T. Petcov, Phys. Lett. B 580 (2004) 280.
[10] J. Suhonen, Phys. Lett. B 607 (2005) 87.
[11] S. Rakers, et al., Phys. Rev. C 70 (2004) 054302.
[12] M. Kortelainen, J. Suhonen, Europhys. Lett. 58 (2002) 666;
M. Kortelainen, J. Suhonen, Nucl. Phys. A 713 (2003) 501;
M. Kortelainen, J. Suhonen, J. Phys. G 30 (2004) 2003.
[13] C. Volpe, J. Phys. G 31 (2005) 903.
[14] M. Doi, T. Kotani, E. Takasugi, Prog. Theor. Phys. Suppl. 83 (1985) 1.
[15] J. Suhonen, S.B. Khadkikar, A. Faessler, Nucl. Phys. A 529 (1991) 727.
[16] K. Holinde, Phys. Rep. 68 (1981) 121.
[17] P. Vogel, M. Zirnbauer, Phys. Rev. Lett. 57 (1986) 3148.
[18] O. Civitarese, A. Faessler, T. Tomoda, Phys. Lett. B 194 (1987) 11.
[19] T. Tomoda, A. Faessler, Phys. Lett. B 199 (1987) 475.
[20] V.A. Rodin, A. Faessler, F. Simkovic, P. Vogel, Phys. Rev. C 68 (2003) 044302.
[21] A.S. Barabash, Czech. J. Phys. 52 (2002) 567.
[22] O. Civitarese, J. Suhonen, Nucl. Phys. A, in press.
[23] J. Abad, et al., Ann. Fis. Ser. A 80 (1984) 9.
[24] D.R. Bes, O. Civitarese, Phys. Rev. C 63 (2001) 044323.
[25] K. Muto, E. Bender, H.V. Klapdor, Z. Phys. A 334 (1989) 187.
[26] A. Bobyk, W.A. Kaminski, F. Simkovic, Phys. Rev. C 63 (2001) 051301.
[27] J. Suhonen, S.B. Khadkikar, A. Faessler, Phys. Lett. B 237 (1990) 8.
[28] F. Krmpotic, J. Hirsch, H. Dias, Nucl. Phys. A 542 (1992) 85.
[29] C. Barbero, F. Krmpotic, A. Mariano, D. Tadic, Nucl. Phys. A 650 (1999) 485.