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Isospin-symmetry violation. Collective treatment of the I.A.S.

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Abstract

We study the effects of isospin symmetry violation in the description of the isobaric analogue state (I.A.S.). The sources of such violation are either spurious (like isospin violations induced by the choice of the independent-particle basis), or have a physical origin (as those arising from the presence of isospin violating terms in the residual interactions). We perform a treatment based on the use of collective variables. The restoration of the symmetry is enforced at the collective level, in order to calculate physical isospin violating terms within an isospin conserving basis. The method is illustrated for a schematic model and for realistic single-particle model spaces. In the last situation, we obtain an excellent agreement with data around $A = 208$, both for the energy and width of the I.A.S.

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1. Introduction

In nuclear structure calculations, the treatment of the isospin degree of freedom is frequently hampered by use of single-particle bases which violate the isospin symmetry. This fact has produced ambiguities both in the description of isobaric analogue states (I.A.S.) and in the treatment of Fermi transitions in single- and double-beta decay processes. A common source of violations is to only include terms of the type $T_0.\tau_0$ in the con-

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struction of the basis states, while isospin symmetry would require that the contributions $-T_{\pm}.\tau_{\mp}$ should be taken into account as well. Here T is the total isospin, while τ is the corresponding single-particle operator. An additional source of violations in the double-beta decay problem is the inclusion of only pairing between identical particles, within the BCS formalism. Note that there are also legitimate sources of isospin mixtures, due the presence of physical isospin non-conserving terms in the Hamiltonian, such as Coulomb contributions to the proton single-particle Hamiltonian, or differences between the strength of the isovector proton–proton, neutron–neutron and proton–neutron pairing channels. Therefore, the problem is to disentangle between spurious and legitimate sources of isospin violations, and to prevent unwanted coherence effects between them. Although the existence of this problem was recognized long time ago in studies concerning I.A.S. and isospin impurities [1,2], the problem has remained largely unsolved.[1] A global analysis of the energy and spreading width of the I.A.S., in the range $A = 110$ to 238 can be found in [3]. For a compilation of experimental values for the Coulomb displacement energies between members of isospin multiplets, see Ref. [4].

A collective treatment has been proposed for the pairing case, which not only allows to restore the symmetries in the cases for which this is adequate, but also to disentangle between the real isospin admixtures and the spurious ones [5,6]. This formalism has its own interest, independently of the above mentioned applications. The perfect analogy to this treatment is the use of intrinsic and collective degrees of freedom in the description of nuclear deformations and space rotations [7]. Although such an analogy has been drawn long time ago [7–10], the microscopic theory of gauge and isospin collective phenomena was not discussed before in detail. The present availability of radioactive beams and targets, may also call for the application of the present formalism, in order to explore nuclear structure aspects for which it is relevant a correct treatment of the isospin degree of freedom.

The results obtained in Ref. [5] for the case of purely isovector (spin independent) transitions are in perfect agreement with available exact solutions, in contrast with the behavior of other approximations in the vicinity of the symmetry restoration (see [5,6] for a detailed discussion). The formalism was generalized, presented in detail and applied to particular cases in Ref. [6].

Let us stress the fact that we are facing a different problem in many-body physics. Usually, collective variables are introduced in order to restore, at the macroscopic level, symmetries that are lost in the microscopic description. This implies that there is a symmetry to start with, which is reflected in an invariance property of the original Hamiltonian. However, in the present case, the initial Hamiltonian is in general not scalar in isospin space. Otherwise, neither double-Fermi transitions between states differing in two units of isospin could exist, nor admixtures of I.A.S. with states of the underlying background, carrying one unit less of isospin, would be possible.

Since the previous presentations were confined to superfluid shell-model systems, we here adapt the formalism to the simpler case of normal systems (Section 2). Subsequently

[1] The following entries are taken from Ref. [2]: *a severe limitation of any HF calculation in $N > Z$ nuclei is the appearance of spurious isospin mixing…; this limitation cannot be avoided*; in the section discussion and summary: … *and its* [*the result*] *main limitation is again the isospin mixing.*

we apply it to the study of I.A.S. We do not discuss the spurious sector, for which we refer to [6].

An alternative approach to study this problem is to consider the admixture between the I.A.S. with isospin T and the giant isovector monopole resonance (G.I.M.R.) with isospin T-1 through the Coulomb interaction. In turn, the G.I.M.R. is mixed with doorway states having also T-1. (See, for instance, Ref. [11].) In the present paper we avoid the complication due to the presence of another collective mode, and leave the comparison between the two approaches for a subsequently publication.

In this presentation we consider two cases: (i) a Hamiltonian which conserves isospin for certain values of the parameters associated with the residual two-body interaction, and that may be straightforwardly treated within the present formalism (Section 4); (ii) a realistic single-particle Hamiltonian for which the difference between neutrons and protons single-particle energies proceeds from both an isospin conserving interaction, the isovector Hamiltonian, and from a non-conserving isospin force, the Coulomb interaction (Section 5).

2. The treatment of the isospin degree of freedom with collective coordinates

Let us split the Hamiltonian into single-particle and two-body terms,

$$H = H_{\rm sp} + V, \tag{1}$$

where at least $H_{\rm sp}$ is not invariant against symmetry transformations associated with isospin symmetry. Thus the single-particle basis states, determined by $H_{\rm sp}$, do not carry the quantum numbers corresponding to this symmetry. The symmetry may be restored at the collective level by raising the transformation parameters determining the orientation of an "intrinsic" or "moving" frame of reference, to the status of collective coordinates. The total Hilbert space is thus factorized into an intrinsic and a collective sector. This last one may be labeled by the quantum numbers associated with the original symmetry

$$\Psi = \psi_{\rm intrinsic} \times D^{T}_{MK}(\phi_\alpha, \phi_\beta, \phi_\gamma). \tag{2}$$

Here ϕ_α, ϕ_β and ϕ_γ represent the Euler angles in isospin space. The quantum numbers M and K are the eigenvalues of the isospin projection along the laboratory and intrinsic frames of reference, respectively.

We operate on the intrinsic system, where symmetry violations may still take place (even for a spin symmetric Hamiltonian), since it is an artificial construction. Therefore any operator must first be transformed to the intrinsic frame, before acting on the product states (2). Obviously problems of over-completeness and of constraints should be properly taken into account.

If the total Hamiltonian does not conserve the symmetry, the same procedure may be carried out, yielding an appropriate isospin conserving base for the calculation of matrix elements of iso-multipole operators. Thus we disentangle the physical effects due to non-conservation of isospin in the Hamiltonian, from the spurious effects arising as a consequence of isospin violations within the set of basis states.

2.1. The generators of the transformations and the constraints

The intrinsic generators of the isospin transformations in the intrinsic frame are the operators τ_q ($q = 0, 1$ and $-1 = \bar{1}$). The operators $T_0, T_{\pm 1}$ denote the corresponding collective momenta.

The generators of isospin transformations satisfy the following commutation relations

$$[\tau_0, \tau_{\pm 1}] = \pm\tau_{\pm 1}, \qquad [T_0, T_{\pm 1}] = -\mp T_{\pm 1}, \qquad [\tau_q, T_r] = 0. \tag{3}$$

The over-completeness of the basis poses a problem that is clarified through the observation that a system, described as above, possesses a "gauge" symmetry. This symmetry consists of the group of transformations which simultaneously move the intrinsic frame of reference and the particle system so as to reproduce the same physical situation. It is expressed by the constraints

$$\tau_q - T_q = 0. \tag{4}$$

The conditions (4) may be rigorously derived whenever we consider a Lagrangian corresponding to a description from a moving frame, and we treat both the original coordinates and the coordinates of the moving frame on an equal footing [12]. Such Lagrangian is called singular, meaning that the velocities cannot be inverted as functions of the coordinates and the momenta. As a consequence, the momenta are not independent from each other: there appear relations such as (4) between them, which are called constraints.

At the quantum level, the constraints (4) imply that physical states should be annihilated by them and physical operators should commute with them. Several procedures have been developed in order to enforce these constraints for the case of gauge field theories. In particular, the one based on the BRST invariance [13,14], has been adapted to many-body problems in Ref. [15]. It has been illustrated by means of applications to very simple mechanical models in Ref. [16].

2.2. The collective sector of the Hilbert space

The set of basis states within the collective sector may be straightforwardly obtained in two limiting situations, according to whether the Coriolis type of interaction $\frac{1}{\mathcal{I}}(-1)^q T_q \tau_{\bar{q}}$ is taken into account perturbatively or is wholly included in the intrinsic single-particle spectrum (as in the cranking model). The former solution is to be preferred if the magnitude of the isospin T is smaller than other parameters of the problem, while the second solution is adopted for larger values of T. In the present paper we consider this second case.

The assumption of large values of T suggests the use of the Marshalek generalization of the Holstein–Primakoff representation [18]. Thus, the rotational isospin sector in (2) may be expressed in terms of the boson creation operators Υ^+, ξ^+ and ς^+

$$|TMK\rangle = \frac{(\Upsilon^+)^{2T}}{\sqrt{(2T)!}} \frac{(\xi^+)^m}{\sqrt{(m)!}} \frac{(\varsigma^+)^k}{\sqrt{(k)!}} |\rangle, \quad m, k = 0, 1, 2, \ldots, \tag{5}$$

where the quantum numbers $m = \frac{1}{2}(T + M)$ and $k = \frac{1}{2}(T + K)$ substitute the isospin projections M and K, respectively. This representation is specially useful for values of $m/T, k/T \ll 1$, which we assume to be the case.

We may also write down the expression for the matrices $D^{\lambda}_{\mu\delta}$ acting on states (5). Such operators are expanded in powers of $1/T$ in Appendix A for the cases of interest.

Within the same representation, the collective components of the isospin operators may be written in terms of the bosons ς^+, ς

$$T_0 = -T + \varsigma^+\varsigma, \qquad T_{\bar{1}} = \varsigma^+\sqrt{T - \frac{1}{2}\varsigma^+\varsigma} \approx -\sqrt{T}\,\varsigma^+,$$
$$T_1 = -T^+_{\bar{1}} \approx \sqrt{T}\,\varsigma. \tag{6}$$

Therefore, the constraints (4) are written, to leading order in $1/T$, as

$$\tau_0 = -T + \varsigma^+\varsigma, \qquad \tau_1 = -\sqrt{T}\,\varsigma, \qquad \tau_{\bar{1}} = \sqrt{T}\,\varsigma^+. \tag{7}$$

We also define the isospin raising operator β

$$\beta^2|T\rangle = |T+1\rangle \leftrightarrow \beta^2\frac{(\Upsilon^+)^{2T}}{\sqrt{(2T)!}}|\,\rangle = \frac{(\Upsilon^+)^{2T+1}}{\sqrt{(2T+1)!}}|\,\rangle. \tag{8}$$

3. The transformation of a general Hamiltonian

The Hamiltonian (1) displays single-particle and two-body contributions. Each of them may be written in terms of isomultipoles λ with projection $\mu = 0$.

$$H = H_{00} + H_{10} + H_{20},$$
$$H_{00} = H_{\mathrm{sp}00} + V_{00},$$
$$H_{10} = H_{\mathrm{sp}10} + V_{10},$$
$$H_{20} = V_{20}. \tag{9}$$

The isovector single-particle term $H_{\mathrm{sp}10}$ is responsible for the isospin mixtures that are present in the independent-particle basic set of states, and thus for all the evils that we intend to cure through the present work. The $V_{\lambda 0}$ terms represent two-body interactions. The different isospin components may be individualized according to the procedure given in Appendix B.

To lowest order, the constraints (3) imply

$$\langle\tau_0\rangle = -T. \tag{10}$$

Since the calculations are performed in the intrinsic system, any operator should be transformed to this frame. As usual [7], the transformation between laboratory (lab) and intrinsic (int) tensor operators is expressed by[2]

$$\mathcal{O}^{\mathrm{lab}}_{\lambda\mu} = D^{\lambda}_{\mu\nu}\mathcal{O}^{\mathrm{int}}_{\lambda\nu}, \tag{11}$$

[2] We use the Einstein convention that the repetition of an index on a given side of an equation implies a summation over that index (for instance, there is a summation over the index m in the second Eq. (25)). Exceptions: (i) the index is repeated also on the other side of the equation, as for the index j on the second Eq. (25); (ii) whenever there is a limitation on the summation, as for $\sum_{m>0}$.

where $D^{\lambda}_{\mu\nu}$ are the rotational matrices, discussed in the previous section. The operators $\mathcal{O}_{\lambda\mu}$ are irreducible tensor operators carrying isospin λ and isospin-projection μ.

The application of this transformation to the Hamiltonian is a trivial step in cases for which the Hamiltonian displays only scalar terms. In the present situation the Hamiltonian transforms as

$$\begin{aligned} H &= H_0 + H_1 + H_2, \\ H_0 &= H_{\mathrm{sp}00} + V_{00}, \\ H_1 &= D^1_{0\nu} H_{\mathrm{sp}1\nu} + D^1_{0\nu} V_{1\nu}, \\ H_2 &= D^2_{0\nu} V_{2\nu}. \end{aligned} \tag{12}$$

The terms H_1 and H_2 of the Hamiltonian (12) are physical operators, because they commute with the constraints (4). This is not the case for the components H_{10} and H_{20} in (9).

Apparently, we have only succeeded in complicating the problem through the substitution of the Hamiltonian (9) by (12), which must be considered simultaneously with the constraints (4). In the following we show that this is not the case, but rather constitutes the first step of a systematic and simplifying procedure.

3.1. The elementary modes of excitation and the quadratic coupling terms

We replace the rotational functions $D^1_{0\nu}$ and $D^2_{0\nu}$ (Eq. (12)) by the corresponding leading order terms, given in Appendix B

$$\begin{aligned} H = {} & h_0 + \omega_\xi \big(1 + \varsigma^+\varsigma + \xi^+\xi - \beta^{-2}\xi\varsigma - \beta^2\xi^+\varsigma^+\big) \\ & + \frac{1}{\sqrt{T}}\big(H_{\mathrm{sp}11} + V_{11} + \sqrt{3}\,V_{21}\big)\big(\varsigma^+ - \beta^{-2}\xi\big) \\ & - \frac{1}{\sqrt{T}}\big(H_{\mathrm{sp}1\bar{1}} + V_{1\bar{1}} + \sqrt{3}\,V_{2\bar{1}}\big)\big(\varsigma - \beta^2\xi^+\big) + \mathcal{O}\big(T^{-1/2}\big). \end{aligned} \tag{13}$$

We have used the following definitions

$$\begin{aligned} h_0 &\equiv H_{\mathrm{sp}00} + V_{00} + H_{\mathrm{sp}10} + V_{10} + V_{20}, \\ \omega_\xi &\equiv -\frac{1}{T}\big(\langle H_{\mathrm{sp}10}\rangle + \langle V_{10}\rangle + 3\langle V_{20}\rangle\big). \end{aligned} \tag{14}$$

Thus,

$$\begin{aligned} h_1 &= -[\tau_1, h_0] = H_{\mathrm{sp}11} + V_{11} + \sqrt{3}\,V_{21}, \\ h_{\bar{1}} &= [\tau_{\bar{1}}, h_0] = H_{\mathrm{sp}1\bar{1}} + V_{1\bar{1}} + \sqrt{3}\,V_{2\bar{1}}. \end{aligned} \tag{15}$$

If we also replace the operators ς^+, ς by their equivalent values obtained from the constraints (7), we obtain

$$\begin{aligned} H &= W + \omega_\xi\left(\xi^+\xi + \frac{3}{2}\right) + H_{\mathrm{coup}} + \mathcal{O}\big(T^{-1/2}\big), \\ H_{\mathrm{coup}} &= -\frac{1}{\sqrt{T}}\big(\beta^{-2}\xi\,\Xi^+ + \beta^2\xi^+\Xi\big) \end{aligned} \tag{16}$$

where

$$W \equiv h_0 - \frac{\omega_\xi}{2T}[\tau_1, \tau_{\bar{1}}]_+ + \frac{1}{T}(h_1\tau_{\bar{1}} + h_{\bar{1}}\tau_1),$$
$$\Xi^+ \equiv h_1 - \omega_\xi \tau_1,$$
$$\Xi \equiv -h_{\bar{1}} + \omega_\xi \tau_{\bar{1}}. \tag{17}$$

Here W is an effective Hamiltonian, yielding the normal modes q, carrying energy ω_q and isospin $T-1$, through the TDA or RPA. The particle-hole creation (annihilation) operators Ξ^+ (Ξ) are expressed in terms of normal modes q. The operators Ξ^+, Ξ and W are independent of $\tau_{\pm 1}$ to leading order[3] in T. Therefore, W yields a zero-energy root. The corresponding degree of freedom is thus eliminated from the spectrum, being replaced by the collective degree of freedom ξ^+, i.e., by the sequence of analogue states. All the successive I.A.S. carry the isospin T. The renormalization procedure is able to eliminate the spurious effects associated with the badly behaved operator τ_1, in particular the divergence appearing in RPA treatments.

The coupling term $H_{\rm coup}$ mixes the I.A.S. with the q modes. It creates (annihilates) the modes q simultaneously with the annihilation (creation) of the I.A.S. mode and the lowering (rising) of the isospin T by one unit.

3.2. Transition probabilities

The Fermi operator is written

$$\beta^{(F-)} = \sqrt{2}\,\tau_1. \tag{18}$$

As we proceeded in the case of the Hamiltonian, we must transform the operator $\beta^{(F-)}$ to the intrinsic frame

$$\beta^{(F-)} \to \sqrt{2}\left(D^1_{11}\tau_1 + D^1_{10}\tau_0 + D^1_{1(-1)}\tau_{-1}\right) = \sqrt{2}\,D^1_{10}\langle\tau_0\rangle + O\left(T^{-1/2}\right)$$
$$= -\sqrt{2T}\,\xi^+ + O\left(T^{-1/2}\right). \tag{19}$$

The badly-behaved operator τ_1 is again replaced by the well-behaved collective operator ξ^+.

Thus the Fermi operator would only populate the I.A.S., but for the presence of the coupling term $H_{\rm coup}$. We diagonalize this term following the procedure advocated in Ref. [17], i.e., we neglect the residual matrix element between the I.A.S. itself and between the q-modes. The energies E_k are given by the dispersion relation

$$(\omega_\xi - E_k) = \frac{\langle q|H_{\rm coup}|\xi\rangle^2}{\omega_q - E_k}, \tag{20}$$

while the eigenstates

$$|k\rangle = c_k(\xi)|\xi\rangle + c_k(q)|q\rangle \tag{21}$$

[3] $[\tau_{\pm 1}, \Xi] = \mathcal{O}(1)$ and $[\tau_{\pm 1}, W] = \mathcal{O}(T^{-1/2})$, while they could be expected to be one order of magnitude larger in T.

display the amplitudes

$$c_k(\xi) = \left(1 + \frac{\langle q|H_{\rm coup}|\xi\rangle^2}{(E_k - \omega_q)^2}\right)^{-1/2},$$
$$c_k(q) = c_k(\xi)\frac{\langle q|H_{\rm coup}|\xi\rangle}{E_k - \omega_q}. \tag{22}$$

The matrix elements of the Fermi operator between the ground state and any state k is given by the product

$$\langle k|\beta^{F-}|\text{g.s.}\rangle = -\sqrt{2T}\, c_k(\xi). \tag{23}$$

The strength of this transition is centered at an energy $\bar{E}$ and has a spread σ given by

$$\bar{E} = \frac{E_k|c_k(\xi)|^2}{|c_k(\xi)|^2},$$
$$\sigma = \left(\frac{(E_k - \bar{E})^2|c_k(\xi)|^2}{|c_k(\xi)|^2}\right)^{1/2}. \tag{24}$$

4. Model I

This model is analytically tractable but non-trivial. We expect to learn from it how the above formalism works, rather than to extract, at this point, physical predictions. The single-particle levels are labeled by the isospin component (p, n) and the angular momentum (j, m) quantum numbers

$$\tau_0 = \sum_j \tau_{0j}, \qquad \tau_{0j} = \frac{1}{2}\left(c^+_{pjm}c_{pjm} - c^+_{njm}c_{njm}\right),$$
$$\tau_1 = \sum_j \tau_{1j}, \qquad \tau_{1j} = -\frac{1}{\sqrt{2}}c^+_{pjm}c_{njm},$$
$$\tau_{\bar{1}} = -\tau_1^+. \tag{25}$$

We assume that every single-particle state is completely filled with neutrons and completely empty of protons. Therefore

$$\langle\tau_{0j}\rangle = -\hat{j}^2, \quad T = \hat{j}^2, \quad \hat{j} \equiv \sqrt{j + \frac{1}{2}}. \tag{26}$$

The Hamiltonian of this model is

$$H = H_{\rm sp} + H_{\rm TD} + H_{\rm sc},$$
$$H_{\rm sp} = (\epsilon_{aj} + \epsilon_{0j})\tau_{0j},$$
$$H_{\rm TD} \equiv \alpha\langle k|\hat{V}|j\rangle\tau_{1k}\tau_{\bar{1}j},$$
$$H_{\rm sc} = -\beta\frac{\langle k|\hat{V}|j\rangle}{2}\tau'_{0k}\tau'_{0j}, \tag{27}$$

where

$$\epsilon_{0j} \equiv s_j + r_j, \quad s_j \equiv \langle j|\hat{V}|k\rangle \hat{k}^2,\ r_j \equiv \frac{1}{2}\langle j|\hat{V}|j\rangle. \tag{28}$$

This Hamiltonian is an isoscalar provided $\alpha = \beta = 1$, since

$$[\tau_1, H] = -\big((1-\alpha)s_j + (1-\beta)r_j\big)\tau_{1j} - (\alpha-\beta)\langle k|\hat{V}|j\rangle\tau_{ik}\tau_{0j}. \tag{29}$$

We also define the large quantities of the problem,

$$S \equiv \hat{j}^2 s_j, \qquad R \equiv \hat{j}^2 r_j \tag{30}$$

and we assume that S is of order T (cf. Eqs. (26), (28) and (30)). Thus R is an order of magnitude smaller, in powers of T^{-1}. The matrix elements $\langle k|\hat{V}|j\rangle$ are of $\mathcal{O}(T^{-1})$. From here on we keep only the leading order terms of an expansion in powers of T^{-1}.

The Hamiltonian displays in general isoscalar, isovector and isoquadrupole terms. The transformation to the intrinsic system (11) yields

$$\begin{aligned}
h_0 &= \epsilon_{0j}\tau_{0j} + \alpha\langle k|\hat{V}|j\rangle\tau_{1k}\tau_{\bar{1}j} - \beta\frac{1}{2}\langle k|\hat{V}|j\rangle\tau_{0k}\tau_{0j} + \mathcal{O}\big(T^{-1}\big),\\
h_1 &= (\epsilon_{0j} - \alpha s_j)\tau_{1j} - \beta\langle k|\hat{V}|j\rangle\tau_{1k}\tau_{0j} + \mathcal{O}\big(T^{-1/2}\big),\\
h_{\bar{1}} &= (\epsilon_{0j} - \alpha s_j)\tau_{\bar{1}j} - \beta\langle k|\hat{V}|j\rangle\tau_{0k}\tau_{\bar{1}j} + \mathcal{O}\big(T^{-1/2}\big),\\
\omega_\xi &= \frac{S(1-\alpha)}{T} + \mathcal{O}\big(T^{-1}\big).
\end{aligned} \tag{31}$$

4.1. TDA

For the sake of simplicity, we further assume that the interaction matrix elements can be factorized

$$\langle k|\hat{V}|j\rangle = v_k v_j. \tag{32}$$

Under this approximation, we write

$$\epsilon_{0j} = s_j = V v_j, \qquad S = V^2, \tag{33}$$

where $V \equiv v_j \hat{j}^2$. The TDA Hamiltonian reads

$$H_{\mathrm{TDA}} = \epsilon_{0j}\tau_{0j} + \alpha v_k v_j \tau_{1k}\tau_{\bar{1}j} + V(v_j\tau_{1j}\tau_{\bar{1}} + v_j\tau_{\bar{1}j}\tau_1) - \frac{(1-\alpha)V^2}{2T^2}[\tau_1, \tau_{\bar{1}}]_+ \tag{34}$$

and the creation operators for the normal bosons are

$$\Gamma_q^+ = \lambda_{qj}\gamma_j^+, \quad \gamma_j^+ = -\frac{1}{\hat{j}}\tau_{1j}. \tag{35}$$

The linearization equation

$$\big[H_{\mathrm{TDA}}, \Gamma_q^+\big] = \omega_q \Gamma_q^+ \tag{36}$$

yields the transformation amplitudes

$$\lambda_{qj} = \frac{1}{\epsilon_{0j} - \omega_q}\left[\Lambda_q \frac{(1-\alpha)V}{T}\left(v_j\hat{j} - \frac{V}{T}\hat{j}\right) + \Xi_q\left(\alpha v_j \hat{j} - \frac{(1-\alpha)V}{T}\hat{j}\right)\right], \tag{37}$$

where

$$\Lambda_q \equiv \lambda_{qj}\hat{j} = \Lambda_q \frac{(1-\alpha)V}{T}\left(X_1 - \frac{V}{T}X_0\right) + \Xi_q\left(\alpha X_1 + \frac{(1-\alpha)V}{T}X_0\right),$$

$$\Xi_q \equiv \lambda_{qj} v_j \hat{j} = \Lambda_q \frac{(1-\alpha)V}{T}\left(X_2 - \frac{V}{T}X_1\right) + \Xi_q\left(\alpha X_2 + \frac{(1-\alpha)V}{T}X_1\right),$$

$$X_\nu = \frac{v_j^\nu \hat{j}^2}{\epsilon_{0j} - \omega_q}. \tag{38}$$

Eq. (38) constitute a homogeneous system of equations in the amplitudes Λ_q, Ξ_q. The vanishing of the determinant of the coefficients of this system yields the roots ω_q.

As a checkout, we verify that there is a root ($q = 0$) corresponding to the eigenvalue $\omega_0 = 0$, and for it

$$X_0 = \frac{\hat{j}^2}{V v_j}, \qquad X_1 = \frac{T}{V}, \qquad X_2 = 1. \tag{39}$$

Using these values, the second Eq. (38) becomes an identity. Thus the root $\omega_0 = 0$ is always present. The first Eq. (38) determines the ratio

$$\Xi_q/\Lambda_q = -V/T. \tag{40}$$

According to Eq. (16), the matrix elements of the Hamiltonian between the I.A.S. state $\xi^+|\,\rangle$ and the states $\Gamma_q^+|\,\rangle$ are given by

$$\langle q|H_{\rm coup}|\xi\rangle = \frac{1}{\sqrt{T}}\lambda_{qj}\hat{j}(\epsilon_{0j} - \alpha s_j - \omega_\xi). \tag{41}$$

4.2. Numerical results

We remind that this schematic model does not represent quantitatively any physical situation. We rather use it as a qualitative check of the formalism in situations where isospin invariance may be invoked.

We have solved the system of Eq. (38) using the single-particle states j and factors v_j in matrix elements, listed in Table 1. There are $\Omega = 6$ unperturbed configurations, and the

Table 1
Single particle levels used for model I. The single-particle energies e_{0j} are obtained from Eq. (33), and the values v_j are the matrix elements of the separable interaction. The values of the energies are given in units of MeV

#	j	e_{0j}	v_j
1	9/2	0.630	0.100
2	7/2	1.260	0.200
3	13/2	1.890	0.300
4	3/2	2.520	0.400
5	5/2	3.150	0.500
6	1/2	3.780	0.600

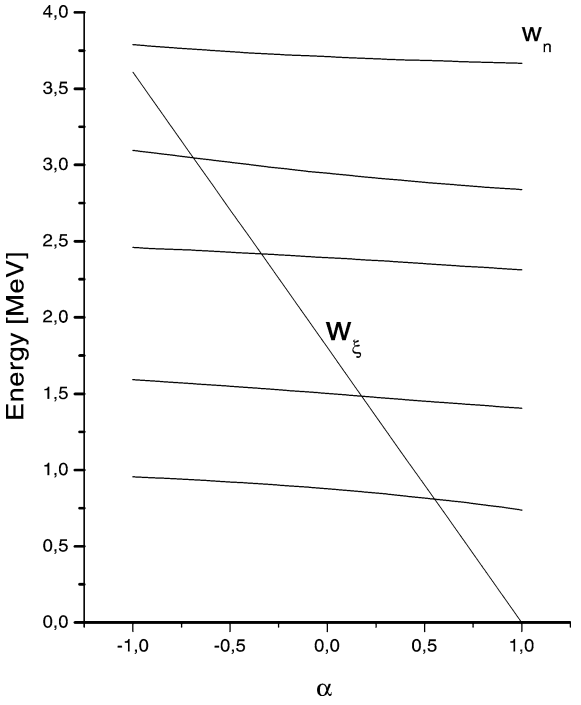

Fig. 1. RPA frequencies ω_q and the collective energy ω_ξ, for model I. The single particle energies and the matrix elements of the residual two-body interaction are given in Table 1. The RPA equations (38) yield one root at zero energy and five non-zero intrinsic excitations ω_q. The collective energy ω_ξ is defined in Eq. (31). The spectrum is given for each value of the parameter α. The energies are given in units of MeV.

isospin of the g.s. has the value $T = 22$ (as in ^{208}Pb). The parameter α is varied within the interval $(-1, 1)$. The isospin symmetry is fully restored for $\alpha = 1$. The TDA intrinsic, $\Omega - 1 = 5$, roots ω_q are shown in Fig. 1 as a function of α, together with the energy ω_ξ of the collective I.A.S. The root $\omega_0 = 0$ is present for any value of α. It must not be confused with the root ω_ξ, which vanishes only for $\alpha = 1$. This behavior clearly illustrates the well known but frequently ignored fact that the presence of a zero frequency eigenvalue does not imply the restoration of the symmetry.

However, the admixture between the I.A.S. and the states q cannot be inferred from the results displayed in Fig. 1. One must diagonalize the matrix (41). The results are shown in Fig. 2, where the physical roots E_k (20) are represented as a function of α. For comparison, we have included also the value of ω_ξ, which has no meaning now, since it has been included already in the diagonalization. Several consequences are extracted from the results shown in Fig. 2, namely: (i) there is no $E_k = 0$ root, except for $\alpha = 1$; (ii) in contrast with the smooth behavior shown in Fig. 1, every energy eigenvalue is affected by the coupling between the collective and intrinsic states; (iii) the number of roots is again the original number of j-states, $\Omega = 6$; (iv) in the symmetry limit $\alpha = 1$ the lowest root corresponds to the collective I.A.S.

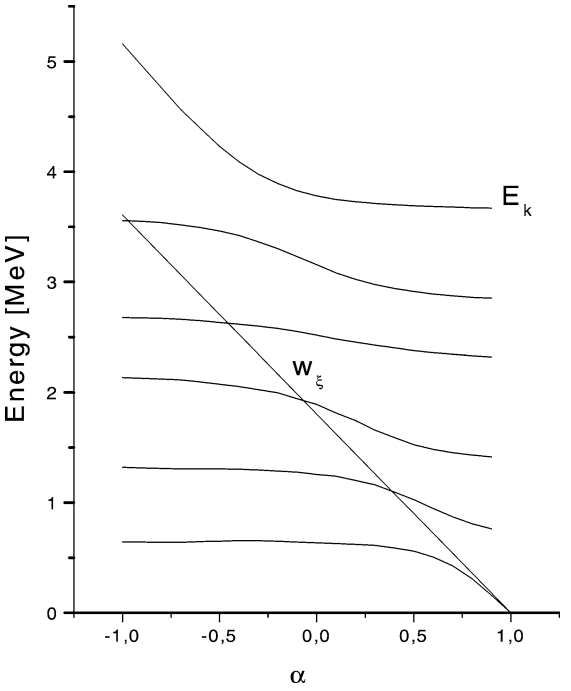

Fig. 2. Solutions of the system of equations (20), E_k. The diagonalization yields six eigenvalues. The energy ω_ξ is shown for completeness. Note that, as $\alpha \to 1$ (full symmetry restoration) $\omega_\xi = E_1 = 0$.

Fig. 3 shows the energy centroid $\bar{E}$ of the states populated by the Fermi transition from the ground state and the spread σ (24), as a function of the parameter α. Note that $\bar{E} \approx \omega_\xi$. The distribution narrows as α approaches the symmetry point ($\alpha = 1$).

5. Model II

5.1. The single-particle states

Since model II is supposed to be realistic, we start discussing with some detail the single-particle states. They are created by the operators $b^+_{p\omega jm}$ for protons and by the $c^+_{n\nu jm}$ for neutrons. Here ω, ν stand for the sequential number of times that the state with the same j and l appears (the label j includes the orbital angular momentum l). We allow for the fact that the proton and the neutron bases may be different. For instance, they may be obtained from Woods–Saxon potentials with different parameters. We denote by $c^+_{p\nu jm}$ the proton creation operator that is obtained from the $c^+_{n\nu jm}$ through the charge conjugation. It may be expanded in terms of the $b^+_{p\omega jm}$ operators with the same values of j, m

$$c^+_{p\nu jm} = x_{\nu\omega j} b^+_{p\omega jm}. \tag{42}$$

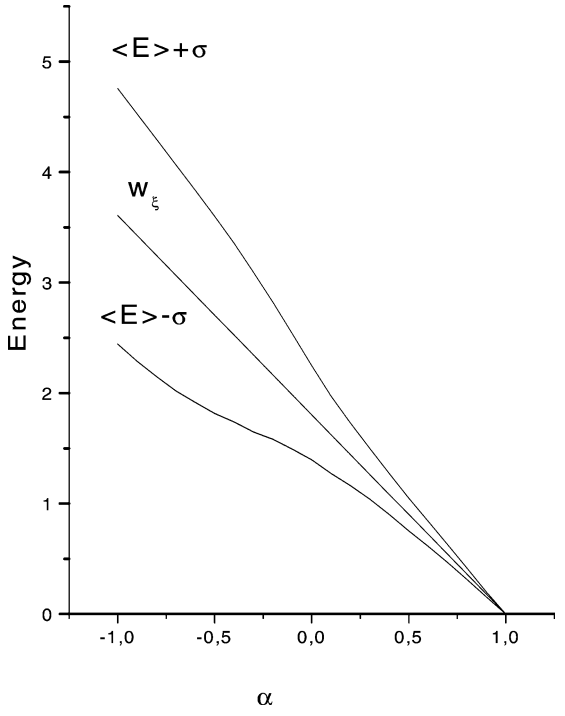

Fig. 3. Average excitation energy, $\bar{E}$, and its root mean square deviation, σ, as a function of the parameter α (see Eq. (24)). The value of the collective energy ω_ξ is shown for completeness. As $\alpha \to 1, \sigma \to 0$ and $\bar{E} \to \omega_\xi$.

The isospin operators should take into account the differences between the proton and neutron bases

$$\tau_1 = -\frac{1}{\sqrt{2}} c^+_{p\nu jm} c_{n\nu jm} = -\frac{1}{\sqrt{2}} x_{\nu\omega j} b^+_{p\omega jm} c_{n\nu jm},$$
$$\tau_{-1} = \frac{1}{\sqrt{2}} c^+_{n\nu jm} c_{p\nu jm} = \frac{1}{\sqrt{2}} x_{\nu\omega j} c^+_{n\nu jm} b_{p\omega jm},$$
$$\tau_0 = \frac{1}{2} x_{\omega\sigma j} x_{\sigma\nu j} \left(b^+_{p\omega jm} b_{p\nu jm} - c^+_{n\omega jm} c_{n\nu jm} \right). \tag{43}$$

5.2. The Hamiltonian

We assume total and Coulomb single-particle Hamiltonians

$$H^{(e)}_{\rm sp} = e_{p\omega j} b^+_{p\omega jm} b_{p\nu jm} + e_{n\nu j} c^+_{n\nu jm} c_{n\nu jm}, \tag{44}$$
$$H^{(q)}_{\rm sp} = \langle p\omega j | V_{\rm coul} | p\sigma j \rangle b^+_{p\omega jm} b_{p\sigma jm}. \tag{45}$$

$H^{(e)}_{\rm sp}$ may display realistic single-particle energies. Both single-particle Hamiltonians include isoscalar $H_{\rm sp00}$ and isovector $H_{\rm sp10}$ components.

If the Coulomb term is the only isospin symmetry breaking contribution, the difference

$$H_{\rm sp}^{(\epsilon)} = H_{\rm sp}^{(e)} - H_{\rm sp}^{(q)} \tag{46}$$

should be attributed to the Hartree–Fock contribution of an (unknown) isospin conserving two-body interaction. We may replace this interaction with a contribution that restores the isospin symmetry. To do so, we follow the treatment[4] of the motion of the center of mass developed in [19]. Thus we add counterterms

$$H^{(\epsilon)} = H_{\rm sp}^{(\epsilon)} - \mathcal{T}_1\tau_{\bar{1}} - \mathcal{T}_{\bar{1}}\tau_1. \tag{47}$$

The operators $\mathcal{T}_{\pm 1}$ are determined by the requirement that $H^{(\epsilon)}$ is invariant under isospin transformations. To leading order, this requirement leads to the expressions

$$\mathcal{T}_\nu = -\frac{1}{T}H_{{\rm sp}1\nu}^{(\epsilon)} - \frac{1}{2T^2}\langle H_{\rm sp10}^{(\epsilon)}\rangle\tau_\nu. \tag{48}$$

The counterterms (47) display isoscalar and isoquadrupole terms

$$\begin{aligned} V_{00} &= -\frac{2}{3}\mathcal{T}_1\tau_{\bar{1}} - \mathcal{T}_{\bar{1}}\tau_1 + \frac{2}{3}\mathcal{T}_0\tau_0, \\ V_{20} &= -\frac{1}{3}\mathcal{T}_1\tau_{\bar{1}} - \mathcal{T}_{\bar{1}}\tau_1 - \frac{2}{3}\mathcal{T}_0\tau_0, \end{aligned} \tag{49}$$

where we have used expressions of Appendix B. The total Hamiltonian is

$$H = H_{\rm sp00}^{(e)} + H_{\rm sp10}^{(e)} + V_{00} + V_{20}. \tag{50}$$

We perform the calculations indicated in Section 3 for this Hamiltonian and we obtain the expressions

$$\begin{aligned} \omega_\xi &= -\frac{1}{T}\big(\langle H_{\rm sp10}^{(e)}\rangle - \langle H_{\rm sp10}^{(\epsilon)}\rangle\big), \\ h_0 &= H_{\rm sp}^{(e)} - (\mathcal{T}_1\tau_{-1} + \mathcal{T}_{-1}\tau_1), \\ h_{\pm 1} &= H_{{\rm sp}1t}^{(e)} - H_{{\rm sp}1t}^{(\epsilon)} = H_{{\rm sp}1t}^{(q)}, \\ W &= H_{\rm sp}^{(e)} + \frac{1}{T}\big(H_{\rm sp11}^{(e)}\tau_{-1} + H_{{\rm sp}1(-1)}^{(e)}\tau_1\big) + \frac{1}{2T^2}\langle H_{\rm sp10}^{(e)}\rangle[\tau_1,\tau_{-1}]_+, \\ \Xi^+ &= H_{\rm sp11}^{(q)} + \frac{1}{T}\langle H_{\rm sp10}^{(q)}\rangle\tau_1. \end{aligned} \tag{51}$$

If we perform the isospin decomposition of Appendix B by means of the isospin operators (43), we obtain

$$\begin{aligned} H_{\rm sp11}^{(e)} &= -\frac{1}{\sqrt{2}}E_{\omega\nu j}^{(e(\epsilon))}b_{p\omega jm}^+ c_{n\nu jm}, \\ H_{\rm sp10}^{(e)} &= -\frac{1}{2}E_{\omega\sigma j}^{(e)}x_{\sigma\nu j}b_{p\omega jm}^+ b_{p\nu jm} + \frac{1}{2}E_{\nu\sigma j}^{(e)}x_{\omega\sigma j}c_{n\omega jm}^+ c_{n\nu jm}, \end{aligned}$$

[4] The commonly used procedure of adjusting constants to ensure the appearance of a zero-energy root, may introduce significant errors in the population of the q-modes through the operator τ_1, and thus in the calculation of the width of the I.A.S.

$$H^{(q)}_{\rm sp11} = -\frac{1}{\sqrt{2}} Q_{\omega\nu j} b^{+}_{p\omega jm} c_{n\nu jm}, \tag{52}$$

where

$$E^{(e)}_{\omega\nu j} = (e_{p\omega j} - e_{n\nu j}) x_{\nu\omega j},$$
$$Q_{\omega\nu j} = \langle p\omega j | H_{\rm coul} | p\sigma j \rangle x_{\nu\sigma j}. \tag{53}$$

5.2.1. RPA

The unperturbed particle-hole creation operators γ^{+}_{taj} include a quantum number $t = \pm 1$, according to whether they increase/decrease the isospin projection and a label $a = a(\omega, \nu) = 1, 2, \ldots$ specifying the proton and neutron configuration

$$\gamma^{+}_{1aj} = \frac{1}{\hat{j}\sqrt{2}} b^{+}_{p\omega jm} c_{n\nu jm},$$
$$\gamma^{+}_{\bar{1}aj} = \frac{1}{\hat{j}\sqrt{2}} c^{+}_{n\nu jm} b_{p\omega jm}. \tag{54}$$

The possible particle-hole creation operators are listed in Table 3 of Appendix C, together with their unperturbed energies Δ_{taj}, amplitudes η_{taj}, and Coulomb matrix elements Q_{taj}.

The RPA expressions for the isospin components and for the single-particle Hamiltonian components carrying $t = \pm 1$ are

$$\tau_t = -t\hat{j}\left(\eta_{taj}\gamma^{+}_{taj} + \eta_{\bar{t}aj}\gamma_{\bar{t}aj}\right),$$
$$H^{(e)}_{{\rm sp}1t} = -\hat{j}\left(\Delta_{taj}\eta_{taj}\gamma^{+}_{taj} - \Delta_{\bar{t}aj}\eta_{\bar{t}aj}\gamma_{\bar{t}aj}\right). \tag{55}$$

The uncoupled boson creation operators Γ^{+}_{tq} carry also the quantum number t and they are also labelled by the sequential number $q = 1, 2, \ldots$. They are written as the linear combination

$$\Gamma^{+}_{tq} = \lambda_{tqaj}\gamma^{+}_{taj} - \mu_{\bar{t}qaj}\gamma_{\bar{t}aj}. \tag{56}$$

As usual, the linearization equation $[W, \Gamma^{+}_{tq}] = \omega_{tq}\Gamma^{+}_{tq}$ determines both the finite frequencies ω_{tq} and the amplitudes $\lambda_{tqaj}, \mu_{tqaj}$ (see Appendix C). The frequencies are fixed through the vanishing of the determinant

$$\begin{vmatrix} -1 + \frac{1}{T}X_{t1} + \frac{\langle H_{\rm sp10}\rangle}{T^2}X_{t0} & \frac{1}{T}X_{t0} \\ \frac{1}{T}X_{t2} + \frac{\langle H_{\rm sp10}\rangle}{T^2}X_{t1} & -1 + \frac{1}{T}X_{t1} \end{vmatrix} = 0, \tag{57}$$

where

$$X_{t2} = \hat{j}^2\left(\frac{\Delta^2_{taj}\eta^2_{taj}}{\Delta_{tjm} - \omega_{tq}} + \frac{\Delta^2_{\bar{t}aj}\eta^2_{\bar{t}aj}}{\Delta_{\bar{t}jm} + \omega_{tq}}\right),$$
$$X_{t1} = t\hat{j}^2\left(\frac{\Delta_{taj}\eta^2_{taj}}{\Delta_{tjm} - \omega_{tq}} - \frac{\Delta_{\bar{t}aj}\eta^2_{\bar{t}aj}}{\Delta_{\bar{t}jm} + \omega_{tq}}\right),$$
$$X_{t0} = \hat{j}^2\left(\frac{\eta^2_{taj}}{\Delta_{tjm} - \omega_{tq}} + \frac{\eta^2_{\bar{t}aj}}{\Delta_{\bar{t}jm} + \omega_{tq}}\right). \tag{58}$$

The determinant (57) has also a zero-frequency root $\omega_0 = 0$, as one may verify through the replacements

$$\lim_{\omega_0 \to 0} X_{t2} = -\langle H_{\rm sp10} \rangle,$$
$$\lim_{\omega_0 \to 0} X_{t1} = T \tag{59}$$

in the expressions given in Appendix C. The coupling between the I.A.S. and the intrinsic phonons $(1q)$ is obtained by means of expressions (51) and (52)

$$h_t = -\hat{j}\left(Q_{taj}\gamma^+_{taj} - Q_{\bar{t}aj}\gamma_{\bar{t}aj}\right)$$
$$= Q_{tq}\Gamma^+_{tq} - Q_{\bar{t}q}\Gamma_{\bar{t}aj} + \frac{1}{T}\langle H^{(q)}_{\rm sp10}\rangle\tau_t, \tag{60}$$

where

$$Q_{tq} = -\hat{j}(Q_{taj}\lambda_{tqaj} - Q_{\bar{t}aj}\mu_{\bar{t}qaj}) \tag{61}$$

and therefore

$$\langle q|H_{\rm coup}|\xi\rangle = Q_{1q}. \tag{62}$$

5.3. Numerical results

We have calculated the expressions obtained from model II, starting from the separate diagonalization of a Woods–Saxon potential for protons and neutrons. The parameters have been taken from Ref. [7]. The central part of the potential has the strength

$$V_0^{(p)} = -51\ \text{MeV}, \qquad V_0^{(n)} = \left(-51 + 33\frac{N-Z}{A}\right)\ \text{MeV}, \tag{63}$$

while the spin orbit strength is

$$V_{\rm so}^{(q)} = -0.44V_0^{(q)}. \tag{64}$$

The radius R_0 and diffuseness a_0 are fixed at the values $1.27A^{1/3}$ fm and 0.67 fm, respectively. We have parametrized the Coulomb potential at the interior as

$$V_c(r) = \left(2.16\frac{Z}{R_0} - 0.72\frac{Z}{R_0^3}r^2\right)\ \text{MeV fm}. \tag{65}$$

The coefficients $x_{v\omega j}$ in Eq. (42) have been obtained as follows: firstly, we have performed a diagonalization of the W.S. neutron potential in a harmonic oscillator basis, for each value of (l, j). The radial part of the states is

$$\phi^{(n)}_{\alpha,lj}(r) = a^{(\alpha,n)}_{Nlj}\psi_{Nlj}(r), \tag{66}$$

where N is the principal quantum number, and ψ_{Nlj}, harmonic oscillator wave functions. Similarly for protons

$$\phi^{(p)}_{\beta,lj}(r) = a^{(\beta,p)}_{Nlj}\psi_{Nlj}(r). \tag{67}$$

Table 2
The eigenvalues e_{nlj} (pure WS potential) for neutrons, and e_{plj} (WS plus Coulomb potential) for protons. The energies are in MeV. These are the states belonging to the region of neutron excess

#	lj	e_{nlj}	e_{plj}
1	$h_{9/2}$	−10.514	2.375
2	$f_{7/2}$	−9.971	3.359
3	$i_{13/2}$	−8.209	3.668
4	$f_{7/2}$	−7.794	5.693
5	$p_{3/2}$	−7.752	6.089
6	$p_{1/2}$	−6.937	6.917

The inclusion of the Coulomb interaction requires an additional diagonalization in the basis of eigenstates $\phi^{(p)}_{\beta,lj}(r)$. The resulting eigenstates may be expressed in terms of the $\phi^{(p)}_{\beta,lj}(r)$'s

$$\Phi^{(p)}_{\gamma,lj}(r) = b^{(\gamma,p)}_{\beta,lj}\phi^{(p)}_{\beta,lj}(r). \tag{68}$$

Replacing the eigenstates of the WS potential (protons without Coulomb) and using the completeness of the harmonic oscillator basis, the proton states may be expressed as linear combinations (42) of neutron states, where

$$\eta_{\gamma,\alpha,lj} = b^{(\gamma,p)}_{\beta,lj}a^{(\beta,p)}_{Nlj}\left(a^{(\alpha,n)}_{Nlj}\right)^*. \tag{69}$$

The calculations have been performed for $A = 208$ (0^+ states in ^{208}Bi as proton (particle)–neutron (hole) excitations on the ground state of ^{208}Pb). We have included seven major harmonic oscillator shells in the calculation. This is the space which exhausts all possible transitions, connecting with the neutron excess region, to be included in the description of the isospin dependent monopole excitations. A sample of single-particle energies, corresponding to the region of neutron excess, is listed in Table 2.

To start with, we have performed calculations by considering only particle-hole excitations in the region of neutron excess. This implies neglecting the differences between the neutron and proton wave functions. All the expressions derived above apply as well, with the amplitudes $x_{\nu\omega j} = \delta_{\nu\omega}$ (Eq. (42)). In this case, the RPA calculation reduces to a TDA calculation, similar to the one performed for model I in Section 5. The resultant values are: $\omega_\xi = 17.85$ MeV; $\bar{E} = 18.11$ MeV; $\sigma = 0.29$ MeV. Although the average energy $\bar{E}$ does not differ much from the experimental value of the I.A.S. relative to the ground state of ^{208}Pb [4], the width is about 3–4 times larger than the experimental value, which is of the order of 78 KeV. The coupling shifts upwards the collective energy by 0.26 MeV.

In a second calculation we have taken into account the difference between proton and neutron wave functions and the complete single-particle basis, which includes states belonging to two mayor shells above and below the region of neutron excess.

The solution to the RPA equations yields positive and negative eigenvalues, corresponding to isospin decreasing and increasing modes, respectively. The situation is similar to the case of pairing interactions, where the negative eigenvalues are associated to pair-removal modes. Negative eigenvalues are associated to backward going transitions (see Appendix C, Table 3). Since the negative eigenvalues do not couple with the I.A.S., they do not con-

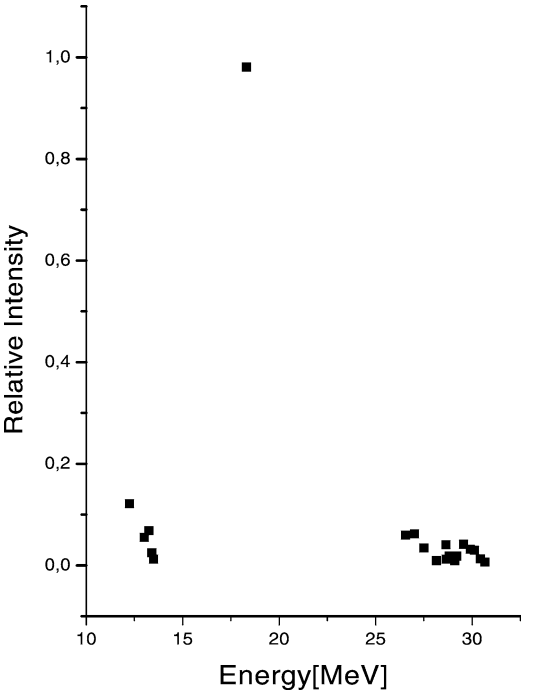

Fig. 4. Strength distribution, in percentage, for Fermi transitions from the ground state of ^{208}Pb(g.s.) to ^{208}Bi, see Eq. (23). The values are the square of the coefficients $c_k(\xi)$, since the total strength is equal to $2T$. The energies are measured from the ground state of ^{208}Pb. The largest contribution corresponds to the excitation of the I.A.S. The results correspond to the realistic calculation described in Section 5.

tribute to the physical strength distribution. They should nonetheless be included in the overall normalization of the intrinsic states.

For the considered single-particle basis we have obtained the value $w_\xi = 18.17$ MeV, a value which is indeed already quite comparable with the experimental value $E_{\text{I.A.S.}} = 17.852$ MeV [3]. Note that we are given the value of the I.A.S. energy respect to the ground state of ^{208}Pb. The experimental value $E_{\text{I.A.S.}} = 15.172$ MeV, which is the energy measured from the ground state of ^{208}Bi, is obtained after substraction of the neutron-proton mass difference (1.29 MeV) and the difference in the energies of the ground state of ^{208}Bi and ^{208}Pb (1.39 MeV). As said in the previous section, the RPA spectrum contains negative energy eigenvalues, a zero energy root and the non-zero energy eigenvalues, which include the excitations involving the neutron excess as well as the ones with relates single particle states two shells above and below the active ones (see Table 3, Appendix C). Like it has been done, for the case of the schematic model I, we have solved the system of equations which couples the I.A.S. with the RPA excitations. The resulting spectrum is shown in Fig. 4. The average excitation energy, determined from the eigenvalues and matrix elements for transitions between the perturbed states and the ground state, is equal to $E_{\text{average}} = 18.17$ MeV and the spreading width is of the order of 84 keV. Again, these results agree rather nicely with the measured values [3].

As a final test about the dependence of the I.A.S. energy and strength with the Coulomb interaction, we have performed calculations with the renormalized Coulomb interaction, in a manner similar to the one described in the case of model I, where the parameter α was taken as an effective coupling constant. The dependence of the I.A.S. energy, with respect to a renormalization of the Coulomb interaction is rather strong. If one takes a renormalization of 0.9, the energy of the I.A.S. is obtained at 16 MeV and a renormalization of 1.2 produces a value of 22 MeV. This shows that the actual value of the energy of the I.A.S., calculated by means of an empirical single-particle basis, gives a good indication about the degree of violation of the isospin symmetry inherent to the choice of the single-particle states.

6. Conclusions

In this paper we have studied the effects of isospin symmetry violation in the description of the I.A.S. The formalism, based on the introduction of collective and intrinsic variables, allows for the treatment of both spurious and physical isospin symmetry violations. The formalism has been applied to two model situations, to illustrate the steps which we have followed in constructing the theory. The results of realistic calculations, performed for the case of $A = 208$, show that the predictions of the theory are in excellent agreement with data. Work is in progress concerning the application of the present formalism to the calculation of the structure of Fermi beta decay transitions between mirror nuclei, and on the comparison between the present method and other approaches predicting both the displacement and the width of the I.A.S. [20].

Acknowledgements

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Appendix A. The collective operators

In this appendix we express the collective operators $D^{\lambda}_{\nu\delta}$ in terms of creation and annihilation operators. A more detailed derivation, following Ref. [18], is given in Ref. [6]. Therein the meaning of the operators entering in the following definitions is given.

(a) Hamiltonian dipole operators

$$
\begin{aligned}
D^{(1)}_{0,1} &= \big(\varsigma^{\dagger} - \beta^{-2}\xi\big)T^{-1/2} + \mathcal{O}\big(T^{-3/2}\big),\\
D^{(1)}_{0,0} &= 1 - \big(1 + \varsigma^{\dagger}\varsigma + \xi^{\dagger}\xi - \beta^{-2}\varsigma\xi - \beta^{2}\varsigma^{\dagger}\xi^{\dagger}\big)T^{-1} + \mathcal{O}\big(T^{-2}\big),\\
D^{(1)}_{0,\bar{1}} &= -\big(\varsigma - \beta^{2}\xi^{+}\big)T^{-1/2} + \mathcal{O}\big(T^{-3/2}\big);
\end{aligned}
\tag{A.1}
$$

(b) Hamiltonian quadrupole operators

$$
\begin{aligned}
D^{(2)}_{0,2} &= \sqrt{\frac{3}{2}}\big((\varsigma^\dagger)^2 - 2\beta^{-2}\varsigma^\dagger\xi + \beta^{-4}\xi^2\big)T^{-1} + \mathcal{O}\big(T^{-2}\big),\\
D^{(2)}_{0,1} &= \sqrt{3}\,\big(\varsigma^\dagger - \beta^{-2}\xi\big)T^{-1/2} + \mathcal{O}\big(T^{-3/2}\big),\\
D^{(2)}_{0,0} &= 1 - 3\big(1 + \varsigma^\dagger\varsigma + \xi^\dagger\xi - \beta^{-2}\varsigma\xi - \beta^2\varsigma^\dagger\xi^\dagger\big)T^{-1} + \mathcal{O}\big(T^{-2}\big),\\
D^{(2)}_{0,\bar{1}} &= -\sqrt{3}\,\big(\varsigma - \beta^2\xi^\dagger\big)T^{-1/2} + \mathcal{O}\big(T^{-3/2}\big),\\
D^{(2)}_{0,\bar{2}} &= \sqrt{\frac{3}{2}}\big(\varsigma^2 - 2\beta^2\varsigma\xi^\dagger + \beta^4\big(\xi^\dagger\big)^2\big)T^{-1} + \mathcal{O}\big(T^{-2}\big);
\end{aligned}
\tag{A.2}
$$

(c) Transition dipole operators

$$
\begin{aligned}
D^{(1)}_{1,1} &= \beta^{-2} - \big(\beta^{-2}\big(1 + \varsigma^\dagger\varsigma + \xi^\dagger\xi\big) - \varsigma^\dagger\xi^\dagger\big)T^{-1} + \mathcal{O}\big(T^{-2}\big),\\
D^{(1)}_{1,0} &= \big(\xi^\dagger - \beta^{-2}\varsigma\big)T^{-1/2} + \mathcal{O}\big(T^{-3/2}\big),\\
D^{(1)}_{1,\bar{1}} &= \frac{1}{2}\big(\beta^{-2}\varsigma^2 - 2\varsigma\xi^\dagger + \beta^2\big(\xi^\dagger\big)^2\big)T^{-1} + \mathcal{O}\big(T^{-2}\big).
\end{aligned}
\tag{A.3}
$$

Appendix B. The isomultipole decomposition of the operators

$$
\begin{aligned}
&H_{\mathrm{sp}11} = [H_{\mathrm{sp}}, \tau_1], && H_{\mathrm{sp}10} = -[H_{\mathrm{sp}11}, \tau_{\bar{1}}],\\
&V_{22} = \frac{1}{\sqrt{6}}\big[[V, \tau_1], \tau_1\big], && V_{21} = -\frac{1}{\sqrt{2}}[V_{22}, \tau_{\bar{1}}], && V_{20} = -\frac{1}{\sqrt{3}}[V_{21}, \tau_{\bar{1}}],\\
&V_{11} = [V, \tau_1] - \sqrt{3}V_{21}, && V_{10} = -[V_{11}, \tau_{\bar{1}}], && V_{00} = V - V_{10} - V_{20}.
\end{aligned}
$$

Appendix C. The RPA for the realistic model

If we limit[5] the expansion (42) to terms $\omega = \nu, \nu \pm 1$, there are five types of relevant particle-hole creation operators γ^+_{taj} (see Table 3).

The commutation $[W, \Gamma^+_{tq}]$ yields

$$
\begin{aligned}
\big[W, \Gamma^+_{tq}\big] = {} & \lambda_{tqaj}\Delta_{taj}\gamma^+_{taj} + \mu_{\bar{t}qaj}\Delta_{\bar{t}aj}\gamma_{\bar{t}aj}\\
& + \Lambda_{tq}\left(H_{\mathrm{sp}1t} + \frac{\langle H_{\mathrm{sp}10}\rangle}{T}\tau_t\right) + \Xi_{tq}\tau_t,
\end{aligned}
\tag{C.1}
$$

where

[5] This is equivalent to the $\Delta N = 0, \pm 2$ for the oscillator potential.

Table 3
The particle-hole creation operators. The labels ω_p, ν_n indicate the last proton and the last neutron filled states with angular momentum and parity j

$\gamma^+_{11j} = -\frac{1}{\sqrt{2}\hat{j}} b^+_{p\nu_n jm} c_{n\nu_n jm}$	$\eta_{11j} = x_{\nu_n\nu_n j}$
$\gamma^+_{12j} = -\frac{1}{\sqrt{2}\hat{j}} b^+_{p(\nu_n+1)jm} c_{n\nu_n jm}$	$\eta_{12j} = x_{\nu_n(\nu_n+1)j}$
$\gamma^+_{13j} = -\frac{1}{\sqrt{2}\hat{j}} b^+_{p\nu_n jm} c_{n(\nu_n-1)jm}$	$\eta_{13j} = x_{(\nu_n-1)\nu_n j}$
$\gamma^+_{14j} = -\frac{1}{\sqrt{2}\hat{j}} b^+_{p(\omega_p+1)jm} c_{n\omega_p jm}$	$\eta_{14j} = x_{(\omega_p)(\omega_p+1)j}$
$\gamma^+_{\bar{1}1j} = \frac{1}{\sqrt{2}\hat{j}} c^+_{n(\omega_p+1)jm} b_{p\omega_p jm}$	$\eta_{\bar{1}1j} = x_{(\omega_p+1)\omega_p j}$
$\Delta_{11j} = e_{p\nu_n j} - e_{n\nu_n j}$	$Q_{11j} = \langle p\nu_n j\|H_{\rm coul}\|p\sigma j\rangle x_{\nu_n\sigma j}$
$\Delta_{12j} = e_{p(\nu_n+1)j} - e_{n\nu_n j}$	$Q_{12j} = \langle p(\nu_n+1)j\|H_{\rm coul}\|p\sigma j\rangle x_{\nu_n\sigma j}$
$\Delta_{13j} = e_{p\nu_n j} - e_{n(\nu_n-1)j}$	$Q_{13j} = \langle p\nu_n j\|H_{\rm coul}\|p\sigma j\rangle x_{(\nu_n-1)\sigma j}$
$\Delta_{14j} = e_{p(\omega_p+1)j} - e_{n\omega_p j}$	$Q_{14j} = \langle p(\omega_p+1)j\|H_{\rm coul}\|p\sigma j\rangle x_{\omega_p\sigma j}$
$\Delta_{\bar{1}1j} = -e_{p\omega_p j} + e_{n(\omega_p+1)j}$	$Q_{\bar{1}1j} = -\langle p(\omega_p)j\|H_{\rm coul}\|p\sigma j\rangle x_{(\omega_p+1)\sigma j}$

$$\Lambda_{tq} = \frac{\hat{j}t}{T}(\lambda_{tqaj}\eta_{taj} + \mu_{\bar{t}qaj}\eta_{\bar{t}aj}),$$
$$\Xi_{tq} = \frac{\hat{j}}{T}(\lambda_{tqaj}\Delta_{taj}\eta_{taj} - \mu_{\bar{t}qaj}\Delta_{\bar{t}aj}\eta_{\bar{t}aj}). \tag{C.2}$$

Therefore

$$\lambda_{tqaj} = \frac{\hat{j}}{\Delta_{taj} - \omega_{tq}}\left(\Lambda_{tq}\left(\frac{\langle H_{\rm sp10}\rangle t}{T}\eta_{taj} + \Delta_{taj}\eta_{taj}\right) + \Xi_{tq}t\eta_{taj}\right),$$
$$\mu_{\bar{t}qaj} = \frac{\hat{j}}{\Delta_{\bar{t}aj} + \omega_{tq}}\left(\Lambda_{tq}\left(\frac{\langle H_{\rm sp10}\rangle t}{T}\eta_{\bar{t}aj} - \Delta_{\bar{t}aj}\eta_{\bar{t}aj}\right) + \Xi_{tq}t\eta_{\bar{t}aj}\right). \tag{C.3}$$

The self-consistency on the values of the constants Λ_{tq}, Ξ_{tq} requires

$$0 = \Lambda_{tq}\left(-1 + \frac{1}{T}X_{t1} + \frac{\langle H_{\rm sp10}\rangle}{T^2}X_{t0}\right) + \Xi_{tq}\frac{1}{T}X_{t0},$$
$$0 = \Lambda_{tq}\left(\frac{1}{T}X_{t2} + \frac{\langle H_{\rm sp10}\rangle}{T^2}X_{t1}\right) + \Xi_{tq}\left(-1 + \frac{1}{T}X_{t1}\right), \tag{C.4}$$

where the X_{ts} are given in Eq. (58).

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