# Theoretical description of double $\beta$ decay of $^{160}$Gd

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The half-life of the $\beta\beta_{2\nu}$ decay of $^{160}$Gd is estimated by including the pairing interaction within the pseudo-SU(3) model. This process was previously reported as theoretically forbidden in the context of that model, however, the pairing interaction is able to mix different occupations and opens new channels for the decay. Explicit expressions are presented for the mixing induced by the pairing force. Matrix elements for the $\beta\beta_{2\nu}$ and $\beta\beta_{0\nu}$ decays are now calculated by assuming both a dominant component in the wave function of the ground state of $^{160}$Gd and a more general model space. The results, of the calculated $\beta\beta$ half-lives, suggest that the planned experiments would succeed in detecting the $\beta\beta_{2\nu}$ decay of $^{160}$Gd and, eventually, would improve the limits for the zero-neutrino mode.

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## I. INTRODUCTION

The flux of solar and atmospheric neutrinos has been measured with increasing precision, and the data offer direct evidence on neutrino oscillations [1,2]. These findings imply that, at least, neutrinos of some flavor should be massive. The difference between the square of the masses of neutrinos belonging to different families can be extracted from the experimental data, relying on theoretical models of mass hierarchies and textures [3]. On the other hand, their absolute scale cannot be obtained from these experiments.

The neutrinoless double-beta decay ($\beta\beta_{0\nu}$), if detected, would provide the complementary information needed to determine neutrino masses, and would also offer definitive evidence that the neutrino is a Majorana particle, i.e., that it is its own antiparticle [4–6].

Theoretical nuclear matrix elements are needed to translate experimental half-life limits, which are available for many $\beta\beta$-unstable isotopes [7,8], into constraints for the effective Majorana mass of the neutrino and, eventually, for the contribution of right-handed currents to the weak interactions. Thus, the evaluation of these matrix elements is essential to understand the underlying physics.

The two-neutrino mode of the double-beta decay ($\beta\beta_{2\nu}$) is allowed as a second order process in the standard model. It has been detected in ten nuclei [7,8] and has served as a test of a variety of nuclear models [5]. The calculation of the $\beta\beta_{2\nu}$ and $\beta\beta_{0\nu}$ matrix elements requires the use of different theoretical methods. Therefore a successful prediction of the former cannot be considered a rigorous test of the latter [6]. However, in most cases it is the best available proof we can impose to a nuclear model used to predict the $\beta\beta_{0\nu}$ matrix elements. When possible, the test should also include the calculation of the energy spectra, electromagnetic transitions, and particle transfer observables in the neighborhood of the double-beta emitters.

Many experimental groups have reported measurements of $\beta\beta$ processes [7,9]. In direct-counting experiments the analysis of the sum-energy spectrum of the emitted electrons identifies the different $\beta\beta$-decay modes [10].

The pseudo-SU(3) approach has been used to describe many low-lying rotational bands, as well as $B(E2)$ and $B(M1)$ intensities in rare earth and actinide nuclei, both with even-even and odd-mass numbers [11–16]. The $\beta\beta$ half-lives of some of these parent nuclei to the ground and excited states of the daughter were evaluated for the two- and zero-neutrino emitting modes [17–21] using the pseudo-SU(3) scheme. The predictions were found to be in good agreement with the available experimental data for $^{150}$Nd and $^{238}$U.

Based on the selection rules of the simplest pseudo-SU(3) model, the theory predicts the complete suppression of the $\beta\beta$ decay for the following five nuclei: $^{154}$Sm, $^{160}$Gd, $^{176}$Yb, $^{232}$Th, and $^{244}$Pu [17]. It was expected that these forbidden decays would have, in the best case, matrix elements that would be no greater than 20% of the allowed ones, resulting in the increase, by at least one order of magnitude, of the predicted half-life [21]. Experimental limits for the $\beta\beta$ decay of $^{160}$Gd have been reported [22,23]. Recently it was argued that the strong cancellation of the $2\nu$ mode in the $\beta\beta$ decay of $^{160}$Gd would suppress the background for the detection of the $0\nu$ mode [24].

In the present contribution we extend the previous research [17–21] and evaluate the $\beta\beta$ half-lives of $^{160}$Gd using the pseudo-SU(3) model. While the $2\nu$ mode is forbidden when the most probable occupations are considered, states with different occupation numbers can be activated by the pairing interaction. The amount of this mixing is evaluated, and the possibility of observing the $\beta\beta$ decay in $^{160}$Gd is discussed for both the $2\nu$ and $0\nu$ modes. The analysis is performed for two cases: (i) we consider a single active configuration in the initial nucleus, and ii) we take into account four additional configurations for the parent and daughter nuclei.

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The paper is organized as follows. In Sec. II the pseudo-SU(3) formalism and the model Hamiltonian are briefly reviewed. In Sec. III the $^{160}$Gd and $^{160}$Dy ground state wave functions are built. Sections IV and V contain the explicit formulas needed to evaluate, using the pseudo-SU(3) scheme, the $\beta\beta_{2\nu}$ and $\beta\beta_{0\nu}$ matrix elements, respectively. In Sec. VI the $^{160}$Gd $\beta\beta$ nuclear matrix elements and half-lives are presented. Conclusions are drawn in the last section.

## II. THE PSEUDO-SU(3) FORMALISM

In order to obtain a microscopic description of the ground states of $^{160}$Gd and $^{160}$Dy we will use the pseudo-SU(3) model, which successfully describes collective excitations in rare earth and actinide nuclei [11–16,25]. This model has been used to evaluate the half-lives of the $\beta\beta_{2\nu}$ decay to the ground and the excited states and $\beta\beta_{0\nu}$ decay of six heavy deformed nuclei [17–21], and also of the double electron capture decays in other three nuclei [26].

In the pseudo-SU(3) shell model coupling scheme [27] normal parity orbitals with quantum numbers $(\eta,l,j)$ are mapped to orbitals of another harmonic oscillator with $(\tilde{\eta}=\eta-1,\tilde{l},\tilde{j})$. This set of orbitals, with $\tilde{j}=j=\tilde{l}+\tilde{s}$, pseudospin $\tilde{s}=1/2$, and pseudo-orbital-angular-momentum $\tilde{l}$, define the so-called pseudospace. For configurations of identical particles occupying a single $j$ orbital of abnormal parity, a simple characterization of states is made by means of the seniority coupling scheme.

The first step in the pseudo-SU(3) description of a given nucleus consists in finding the occupation numbers for protons and neutrons in the normal and abnormal parity states $n_\pi^N, n_\nu^N, n_\pi^A, n_\nu^A$ [17]. These numbers are determined by filling the Nilsson levels, as discussed in [17].

For even-even heavy nuclei it has been shown that if the residual neutron-proton interaction is of the quadrupole type, regardless of the interaction in the proton and neutron spaces, the most important normal parity configurations are those with highest spatial symmetry $\{\tilde{f}_\alpha\}=\{2^{n_\alpha^N/2}\}$ [25]. This statement is valid for yrast states below the backbending region. It implies that $\tilde{S}_\pi=\tilde{S}_\nu=0$, i.e., only pseudospin zero configurations are taken into account.

Additionally, in the abnormal parity space only seniority zero configurations are taken into account. This simplification implies that $J_\pi^A=J_\nu^A=0$. Although it is a very strong assumption, it is quite useful in order to simplify the calculations.

Many-particle states of $n_\alpha^N$ active nucleons in a given normal parity shell $\tilde{\eta}_\alpha$, $\alpha=\nu$ or $\pi$, can be classified by the following chain of groups:

$$\begin{array}{ccccccc} \{1^{n_\alpha^N}\} & \{\tilde{f}_\alpha\} & \{f_\alpha\} \;\; \gamma_\alpha\,(\lambda_\alpha,\mu_\alpha) & \tilde{S}_\alpha & K_\alpha \tilde{L}_\alpha & & J_\alpha^N \\ U(\Omega_\alpha^N) \supset & U(\Omega_\alpha^N/2)\times U(2) \supset & SU(3) & \times SU(2) \supset & SO(3) \times & SU(2) \supset & SU_J(2), \end{array} \tag{1}$$

where above each group the quantum numbers that characterize its irreps are given and $\gamma_\alpha$ and $K_\alpha$ are multiplicity labels of the indicated reductions. The most important configurations are those with the highest spatial symmetry [25,28], namely, those with pseudospin zero.

The model Hamiltonian contains spherical Nilsson single-particle terms for protons ($H_{sp,\pi}$) and neutrons ($H_{sp,\nu}$), the quadrupole-quadrupole ($\tilde{Q}\cdot\tilde{Q}$) and pairing ($V_{pair}$) interactions, as well as three "rotorlike" terms which are diagonal in the SU(3) basis:

$$\begin{aligned} H=&H_{sp,\pi}+H_{sp,\nu}+V_{pair} \\ &-\frac{1}{2}\chi\tilde{Q}\cdot\tilde{Q}+aK_J^2+bJ^2+A_{asym}\hat{C}_2, \end{aligned} \tag{2}$$

where $\tilde{Q}$ denotes the quadrupole operator in the pseudo-SU(3) space. This Hamiltonian can be separated into two parts: the first includes Nilsson single-particle energies, the pairing interaction, and the quadrupole-quadrupole interaction in the pseudospace. They are the basic components of any realistic Hamiltonian [29,30] and have been widely studied in the nuclear physics literature, allowing their respective strengths to be fixed by systematics [29,30]. In the second part, we consider the rotorlike terms used to fine tune the moment of inertia and the position of the different $K$ bands. This part has been studied in detail in previous papers where the pseudo-SU(3) symmetry was used as a dynamical symmetry [25]. In recent works, $a$, $b$, and $A_{asym}$ were the only parameters used to fit the spectra [13,15,16,31,32].

The spherical single-particle Nilsson Hamiltonian is

$$\begin{aligned} H_{sp}&=\hbar\omega_0(\eta+\tfrac{3}{2})-\kappa\hbar\omega_0\{2\vec{l}\cdot\vec{s}+\mu\vec{l}^2\} \\ &=\sum_i \epsilon(\eta_i,l_i,j_i)a_i^\dagger a_i \end{aligned} \tag{3}$$

with parameters [29]

$$\begin{gathered} \hbar\omega_0=41A^{-1/3}\ \text{(MeV)}, \quad \kappa_\pi=0.0637, \quad \kappa_\nu=0.0637, \\ \mu_\pi=0.60, \quad \mu_\nu=0.42. \end{gathered} \tag{4}$$

The pairing interaction is

$$V_{pair}=-\frac{1}{4}G\sum_{j,j'} a_j^\dagger a_{\bar{j}}^\dagger a_{\bar{j}'} a_{j'}, \tag{5}$$

where $\overline{j}$ denotes the time reversal partner of the single-particle state $j$, and $G$ is the strength of the pairing force. In principle, we can start with an isospin invariant pairing force, which contains proton-proton, neutron-neutron, and proton-neutron terms with equal strengths. However, in the limited Hilbert space employed there are no protons and neutrons in the same orbitals, and the proton-neutron pairing term is ineffective. In recent works [13,15,16,31,32], the pairing coefficients $G_{\pi,\nu}$ were fixed following [29,30], with values

$$G_{\pi}=\frac{21}{A}=0.132 \text{ MeV}, \quad G_{\nu}=\frac{17}{A}=0.106 \text{ MeV}. \tag{6}$$

The SU(3) irreps can be mixed, only, by the single-particle and pairing terms in the Hamiltonian. They had to be expressed as one- and two-body pseudo-SU(3) tensors in the normal parity sector, coupled with the seniority expansion of its abnormal parity parts.

## III. THE GROUND STATE OF $^{160}$Gd AND $^{160}$Dy

In this section we shall present the relevant results of the pseudo-SU(3) model for the wave functions of the participant nuclei. We shall assume, in Sec. III A, that the wave function describing the ground state of $^{160}$Gd has only one component and that the corresponding wave function for $^{160}$Dy has two components connected by the pairing interaction. This assumption allow us to perform a simple estimation of the double-beta decay, as shown in Secs. IV and V. In Sec. (III B) we shall include other configurations in the model space, to asses the quality of the simplified description.

### A. The one-component model

Following [17], for a deformation of $\epsilon=0.26$ [33], the most probable occupations for the 14 $^{160}$Gd valence protons are eight particles in normal and six particles in unique parity orbitals, and for the 14 valence neutrons, eight particles in normal and six particles in unique parity orbitals.

After the detailed study of $^{156,158,160}$Gd isotopes performed in [16,32], it becomes clear that the pseudo-SU(3) model is a powerful tool in the description of heavy deformed nuclei. Up to four rotational bands, with their intraband and interband $B(E2)$ transition strengths, as well as $B(M1)$ transitions, were reproduced, with a very good general agreement with the experiment.

The dominant component of the $^{160}$Gd ground state wave function [16] is

$$|^{160}\text{Gd},0^{+}\rangle=|(h_{11/2})^{6}_{\pi},\ J^{A}_{\pi}=0;\ (i_{13/2})^{6}_{\nu},\ J^{A}_{\nu}=0\rangle_{A}$$
$$|\{2^{4}\}_{\pi}(10,4)_{\pi};\{2^{4}\}_{\nu}(18,4)_{\nu};1(28,8)K=1,J=0\rangle_{N}. \tag{7}$$

In a series of papers [17–21], we have reported on the calculation of $\beta\beta$ matrix elements by taking into account only the leading SU(3) coupled proton-neutron irrep. In recent publications it was shown that the leading irrep represents $\approx 60\%$ of the wave function in even-even Dy and Er isotopes [16,31].

Assuming a slightly larger deformation for $^{160}$Dy, the most probable occupations for 16 valence protons are ten particles in normal and six particles in unique parity orbitals, and for the 12 valence neutrons, six particles in normal and six particles in unique parity orbitals. A detailed study of $^{160,162,164}$Dy isotopes has been performed in [32].

The dominant component of the wave function is

$$|^{160}\text{Dy},0^{+}(a)\rangle=|(h_{11/2})^{6}_{\pi},\ J^{A}_{\pi}=0;\ (i_{13/2})^{6}_{\nu},\ J^{A}_{\nu}=0\rangle_{A}$$
$$|\{2^{5}\}_{\pi}(10,4)_{\pi};\{2^{3}\}_{\nu}(18,0)_{\nu};1(28,4)K=1,J=0\rangle_{N}.$$

The two-neutrino double-beta operator annihilates two neutrons and creates two protons with *the same* quantum numbers $\eta,l$. It cannot connect the states $|^{160}\text{Gd},0^{+}\rangle$ and $|^{160}\text{Dy},0^{+}(a)\rangle$ and the transition becomes absolutely forbidden. This is the selection rule found in [17].

However, the pairing interaction allows for the mixing between different occupations. In the deformed single-particle Nilsson scheme it takes an energy $\Delta E$ to promote a pair of protons from the last occupied normal parity orbital to the next intruder orbital. This excited state has eight protons in normal and another eight protons in unique parity orbitals, and its wave function has the form

$$|^{160}\text{Dy},0^{+}(b)\rangle=|(h_{11/2})^{8}_{\pi},\ J^{A}_{\pi}=0;\ (i_{13/2})^{6}_{\nu},\ J^{A}_{\nu}=0\rangle_{A}$$
$$|\{2^{4}\}_{\pi}(10,4)_{\pi};\{2^{3}\}_{\nu}(18,0)_{\nu};\ 1(28,4)K=1,J=0\rangle_{N}.$$

The two-neutrino double-beta decay of $^{160}$Gd can proceed to this state.

As a first approximation, we shall describe the $^{160}$Dy ground state as a linear combination of these two states:

$$|^{160}\mathrm{Dy},0^+\rangle = a|^{160}\mathrm{Dy},0^+(a)\rangle + b|^{160}\mathrm{Dy},0^+(b)\rangle, \quad (8)$$

with $|a|^2+|b|^2=1$.

The only term in the Hamiltonian (2) which can connect states with different occupation numbers in the normal and unique parity sectors is the pairing interaction. In the present case, the Hamiltonian matrix has the simple form

$$H=\begin{pmatrix} 0 & h_{pair} \\ h_{pair} & \Delta E \end{pmatrix} \quad (9)$$

with

$$h_{pair} = \langle ^{160}\mathrm{Dy},0^+(b)|V_{pair}|^{160}\mathrm{Dy},0^+(a)\rangle$$
$$= \frac{(-1)^{\tilde{\eta}_\pi+1}G_\pi}{4}\sqrt{(n_\pi^A+2)(2j_\pi^A+1-n_\pi^A)}\sum_{\tilde{l}_\pi}\sqrt{2(2\tilde{l}_\pi+1)}\sum_{(\lambda_0\mu_0)}\langle(0\tilde{\eta}_\pi)1\tilde{l}_\pi,(0\tilde{\eta}_\pi)1\tilde{l}_\pi\|(\lambda_0\mu_0)10\rangle_1$$
$$\times\sum_\rho\langle(\lambda^a,\mu^a)1\ \ 0,(\lambda_0\mu_0)1\ \ 0\|(\lambda^b\mu^b)_\sigma 10\rangle_\rho\sum_{\rho_\pi}\begin{bmatrix}(\lambda_\pi^a,\mu_\pi^a) & (\lambda_0,\mu_0) & (\lambda_\pi^b,\mu_\pi^b) & \rho_\pi \\ (\lambda_\nu^a,\mu_\nu^a) & (0,0) & (\lambda_\nu^b,\mu_\nu^b) & 1 \\ (\lambda^a,\mu^a) & (\lambda_0,\mu_0) & (\lambda^b\mu^b)_\sigma & \rho \\ 1 & 1 & 1 & \end{bmatrix}$$
$$\times\langle(\lambda_\pi^a,\mu_\pi^a)|||[\tilde{a}_{(0,\tilde{\eta}_\pi),1/2}\tilde{a}_{(0,\tilde{\eta}_\pi),1/2}]^{(\lambda_0,\mu_0)}|||(\lambda_\pi^b,\mu_\pi^b)\rangle_{\rho_\pi}. \quad (10)$$

In the above formula $\langle\cdots,\cdots\|,,\rangle$ denotes the SU(3) Clebsch-Gordan coefficients [34], the symbol $[\cdots]$ represents a $9-(\lambda\mu)$ recoupling coefficient [35], and $\langle\cdots|||\cdots|||\cdots\rangle$ denotes triple reduced matrix elements [18].

The lowest eigenstate has an energy

$$E=\frac{\Delta E}{2}\left[1-\sqrt{1+\left(\frac{2h_{pair}}{\Delta E}\right)^2}\right] \quad (11)$$

and the components of the $^{160}$Dy ground state wave function are

$$a=\frac{h_{pair}}{\sqrt{E^2+h_{pair}^2}}, \quad b=\frac{E}{\sqrt{E^2+h_{pair}^2}}. \quad (12)$$

It is a limitation of the present model that $\Delta E$ has to be estimated from the deformed Nilsson single-particle mean field, instead of the evaluation of the diagonal matrix element of the Hamiltonian (2). The use of seniority zero states for the description of nucleons in intruder orbits rules out a direct comparison of states with different occupation numbers. A formalism capable of describing nucleons in both normal and unique parity orbitals on the same footing has recently been developed [36]. The effect $\Delta E$ has on the double-beta-decay half-lives is studied in Sec. VI.

### B. The enlarged model space

In addition to the pseudo-SU(3) irreps described in the preceding section, one has, in an enlarged model space, other potential components to the ground state wave functions of $^{160}$Gd and $^{160}$Dy. In Tables I and II we are listing the four configurations included in the description of $^{160}$Gd and those for $^{160}$Dy, respectively.

The wave function of the ground state of $^{160}$Gd is written as

$$|^{160}\mathrm{Gd}\rangle=\sum_k C_k^{(i)}|i_k\rangle, \quad (13)$$

and that of $^{160}$Dy is given by

TABLE I. The four configurations included in the description of $^{160}$Gd. The number of protons and neutrons, in normal and in intruder orbits, and the corresponding pseudo-SU(3) irreps are listed. The last column shows the corresponding amplitudes of the ground state wave function of $^{160}$Gd.

| State | $n_\pi^N$ | $n_\pi^A$ | $n_\nu^N$ | $n_\nu^A$ | $(\lambda_\pi,\mu_\pi)$ | $(\lambda_\nu,\mu_\nu)$ | $(\lambda,\mu)$ | C$(\lambda,\mu)$ |
|---|---|---|---|---|---|---|---|---|
| $i_1$ | 8 | 6 | 8 | 6 | (10, 4) | (18,4) | (28,8) | 0.7253 |
| $i_2$ | 8 | 6 | 10 | 4 | (10, 4) | (20,4) | (30,8) | 0.4848 |
| $i_3$ | 10 | 4 | 8 | 6 | (10, 4) | (18,4) | (28,8) | 0.4066 |
| $i_4$ | 10 | 4 | 10 | 4 | (10, 4) | (20,4) | (30,8) | 0.2713 |

TABLE II. The four configurations included in the description of $^{160}$Dy. The configurations and amplitudes are listed following the notation given in the caption to Table I.

| State | $n_\pi^N$ | $n_\pi^A$ | $n_\nu^N$ | $n_\nu^A$ | $(\lambda_\pi,\mu_\pi)$ | $(\lambda_\nu,\mu_\nu)$ | $(\lambda,\mu)$ | $C(\lambda,\mu)$ |
|---|---|---|---|---|---|---|---|---|
| $f_1$ | 10 | 6 | 8 | 4 | (10, 4) | (18,4) | (28,8) | 0.7639 |
| $f_2$ | 10 | 6 | 6 | 6 | (10, 4) | (18,0) | (28,4) | 0.5446 |
| $f_3$ | 8 | 8 | 8 | 4 | (10, 4) | (18,4) | (28,8) | 0.2690 |
| $f_4$ | 8 | 8 | 6 | 6 | (10, 4) | (18,0) | (28,4) | 0.2181 |

$$|^{160}\mathrm{Dy}\rangle=\sum_k C_k^{(f)}|f_k\rangle, \tag{14}$$

where the amplitudes $C_k^{(i)}$ and $C_k^{(f)}$ are listed in the last column of Tables I and II, respectively.

For the configurations of Tables I and II, we have a $4\times4$ mixing matrix (9), with nondiagonal matrix elements $h_{pair}$ of the form (10). The diagonal matrix elements are the energies needed to promote a pair of protons, neutrons, or both from a normal to an intruder parity orbital or vice versa. They are estimated from the deformed single-particle Nilsson diagrams. The values of $\Delta E$ for the case of $^{160}$Gd are 0.54 MeV, 0.81 MeV, and 1.35 MeV. The corresponding quantities for $^{160}$Dy are of the order of 0.54 MeV, 1.71 MeV, and 2.25 MeV. The diagonalization of the mixing matrix yields the amplitudes $C$, shown in the last column of Tables I and II. As one can see from these tables, although the mixing induced by the pairing interaction is significant, the dominant component of each wave function is still the irrep considered in the restricted model space.

Naturally, and in order to estimate the effect of this mixing upon the double-beta-decay process, we should compute explicitly the corresponding matrix elements, which are the sum of the products of amplitudes and individual matrix elements. It will be done in the following sections.

## IV. THE $\beta\beta_{2\nu}$ HALF-LIFE

The inverse half-life of the two-neutrino mode of the $\beta\beta$ decay, $\beta\beta_{2\nu}$, can be cast in the form [37]

$$[\tau_{2\nu}^{1/2}(0^+\rightarrow0^+)]^{-1}=G_{2\nu}|M_{2\nu}|^2, \tag{15}$$

where $G_{2\nu}$ is a kinematical factor which depends on $Q_{\beta\beta}$, the total kinetic energy released in the decay.

The nuclear matrix element is written as

$$M_{2\nu}\approx M_{2\nu}^{GT}=\sum_N \frac{\langle 0_f^+||\Gamma||1_N^+\rangle\langle 1_N^+||\Gamma||0_i^+\rangle}{\mu_N}, \tag{16}$$

with the Gamow-Teller operator $\Gamma$ expressed as

$$\begin{aligned}\Gamma_m&=\sum_s \sigma_{ms}t_s^-\\ &\equiv\sum_{\pi\nu}\sigma(\pi,\nu)[a^\dagger_{\eta_\pi l_\pi 1/2;j_\pi}\otimes\tilde{a}_{\eta_\nu l_\nu \frac{1}{2};j_\nu}]^1_m,\\ &m=1,0,-1.\end{aligned} \tag{17}$$

The energy denominator is $\mu_N=E_f+E_N-E_i$ and it contains the intermediate ($E_N$), initial ($E_i$), and final ($E_f$) energies. The kets $|1_N^+\rangle$ denote intermediate states.

The mathematical expressions needed to evaluate the nuclear matrix elements of the allowed g.s.→g.s. (g.s. denotes ground state) $\beta\beta$ decay in the pseudo-SU(3) model were developed in [17]. Using the summation method described in [17,38], exploiting the fact that the two-body terms of the $\widetilde{\mathrm{SU(3)}}$ Hamiltonian commute with the Gamow-Teller operator (17) [18], resumming the infinite series, and recoupling the Gamow-Teller operators, the following expression was found:

$$M_{2\nu}^{GT}=\sqrt{3}\sum_{\pi\nu,\pi'\nu'}\frac{\sigma(\pi,\nu)\sigma(\pi',\nu')}{(E_0+\epsilon_\pi-\epsilon_\nu)}\times\langle 0_f^+|[[a^\dagger_\pi\otimes\tilde{a}_\nu]^1\otimes[a^\dagger_{\pi'}\otimes\tilde{a}_{\nu'}]^1]^{J=0}|0_i^+\rangle, \tag{18}$$

where $\pi\equiv(\eta_\pi,l_\pi,j_\pi)$ and $\nu\equiv(\eta_\nu,l_\nu,j_\nu)$, and $E_0=Q_{\beta\beta}/2+m_ec^2$ .

Next we shall analyze the nuclear matrix element (18) for the $\beta\beta_{2\nu}$ decay of the ground state of $^{160}$Gd, Eq. (7), to the ground state of $^{160}$Dy, Eq. (8). Each Gamow-Teller operator (17) annihilates a proton and creates a neutron in the same oscillator shell and with the same orbital angular momentum. In the case of the $\beta\beta$ decay of $^{160}$Gd it means that the operator annihilates two neutrons in the pseudoshell $\eta_\nu=5$ and creates two protons in the abnormal orbit $h_{11/2}$. As a consequence, the only orbitals which in the model space can be connected by the $\beta\beta$ decay are those satisfying $\eta_\pi=\eta_\nu\equiv\eta$, which implies $l_\pi=l_\nu=\eta$, $j_\nu=\eta-\frac{1}{2}$, and $j_\pi=\eta+1/2$.

Under these restrictions, the $\beta\beta_{2\nu}$ decay is allowed only if the occupation numbers obey the following relationships:

$$\begin{aligned}&n^A_{\pi,f}=n^A_{\pi,i}+2,\quad n^A_{\nu,f}=n^A_{\nu,i},\\ &n^N_{\pi,f}=n^N_{\pi,i},\quad n^N_{\nu,f}=n^N_{\nu,i}-2.\end{aligned} \tag{19}$$

When the restricted configuration space is considered in Eq. (18), it follows that only one term in the sum survives and, thus, the nuclear matrix element $M_{2\nu}$, Eq. (18), can be written as

$$M_{2\nu}^{GT}=\frac{\sigma(\pi,\nu)^2}{\mathcal{E}}\langle 0_f^+|[[a^\dagger_\pi\otimes\tilde{a}_\nu]^1\otimes[a^\dagger_\pi\otimes\tilde{a}_\nu]^1]^{J=0}|0_i^+\rangle, \tag{20}$$

where the energy denominator $\mathcal{E}$ is determined by demanding that the energy of the isobaric analog state equals the difference in Coulomb energies $\Delta_C$. It is given by [17]

$$\mathcal{E}=E_0+\epsilon(\eta_\pi,l_\pi,j_\pi=j_\nu+1)-\epsilon(\eta_\nu,l_\nu,j_\nu)$$
$$=E_0-\hbar\omega k_\pi 2j_\pi+\Delta_C,$$

$$\Delta_C=\frac{0.70}{A^{1/3}}[2Z+1-0.76\{(Z+1)^{4/3}-Z^{4/3}\}]\ \text{MeV}. \tag{21}$$

As it was discussed in [17], Eq. (20) has no free parameters, the denominator (21) being a well defined quantity. The reduction to only one term is a consequence of the restricted Hilbert proton and neutron spaces of the model. The initial and final ground states are strongly correlated with a very rich structure in terms of their shell model components.

The calculation of the matrix elements, Eq. (20), can be obtained by using SU(3) Racah calculus in the normal parity part and the quasispin formalism for the abnormal parity sector.

For the one component model case, the Gamow-Teller operators can reach only the second component of the wave function of the ground state of $^{160}$Dy, Eq. (8), and for this reason it is proportional to the amplitude $b$. The expression for the matrix elements of the $\beta\beta_{2\nu}$ channel is given by

$$M^{GT}_{2\nu}(^{160}\text{Gd}\rightarrow {}^{160}\text{Dy})=b\,\frac{2j_\pi-1}{2j_\pi}\left[\frac{(n^A_\pi+2)(2j_\pi+1-n^A_\pi)}{j_\pi-1}\right]^{1/2}\mathcal{E}^{-1}\sum_{(\lambda_0,\mu_0)}\langle(0\tilde{\eta})1\tilde{l},(0\tilde{\eta})1\tilde{l}\|(\lambda_0\mu_0)10\rangle_1$$
$$\times\sum_\rho\langle(28,8)1\ \ 0,(\lambda_0,\mu_0)1\ \ 0\|(28,4)10\rangle_\rho$$
$$\times\sum_{\rho'}\begin{bmatrix}(10,4) & (0,0) & (10,4) & 1\\ (18,4) & (\lambda_0\mu_0) & (18,0) & \rho'\\ (28,8) & (\lambda_0\mu_0) & (28,4) & \rho\\ 1 & 1 & 1 & \end{bmatrix}\langle(18,0)|||[\tilde{a}_{(0\tilde{\eta}),1/2}\tilde{a}_{(0\tilde{\eta}),1/2}]^{(\lambda_0\mu_0)}|||(18,4)\rangle_{\rho'}. \tag{22}$$

For the enlarged model space, theexpression for the matrix element of the $\beta\beta_{2\nu}$-decay mode is the generalization of the previous one:

$$M^{GT}_{2\nu}(^{160}\text{Gd}\rightarrow {}^{160}\text{Dy})=\sum_{k,l}C^{(i)}_kC^{(l)}_l\frac{2j_{\pi,k}-1}{2j_{\pi,k}}\left[\frac{(n^A_{\pi,k}+2)(2j_{\pi,k}+1-n^A_{\pi,k})}{j_{\pi,k}-1}\right]^{1/2}\mathcal{E}^{-1}\sum_{(\lambda_0,\mu_0)}\langle(0\tilde{\eta})1\tilde{l},(0\tilde{\eta})1\tilde{l}\|(\lambda_0\mu_0)10\rangle_1$$
$$\times\sum_\rho\langle(\lambda_k,\mu_k)1\ \ 0,(\lambda_0,\mu_0)1\ \ 0\|(\lambda_l,\mu_l)10\rangle_\rho$$
$$\times\sum_{\rho'}\begin{bmatrix}(\lambda_{\pi,k},\mu_{\pi,k}) & (0,0) & (\lambda_{\pi,l},\mu_{\pi,l}) & 1\\ (\lambda_{\nu,k},\mu_{\nu,k}) & (\lambda_0\mu_0) & (\lambda_{\nu,l},\mu_{\nu,l}) & \rho'\\ (\lambda_k,\mu_k) & (\lambda_0\mu_0) & (\lambda_l,\mu_l) & \rho\\ 1 & 1 & 1 & \end{bmatrix}\langle(\lambda_l,\mu_l)|||[\tilde{a}_{(0\tilde{\eta}),1/2}\tilde{a}_{(0\tilde{\eta}),1/2}]^{(\lambda_0\mu_0)}|||(\lambda_k,\mu_k)\rangle_{\rho'}. \tag{23}$$

## V. THE $\beta\beta_{0\nu}$ HALF-LIFE

For massive Majorana neutrinos one can perform integration over the four-momentum of the exchanged particle and obtain a neutrino potential that for a light neutrino ($m_\nu < 10$ MeV) has the form

$$H(r,\overline{E})=\frac{2R}{\pi r}\int_0^\infty dq\,\frac{\sin(qr)}{q+\overline{E}}, \tag{24}$$

where $\overline{E}$ is the average excitation energy of the intermediate odd-odd nucleus, and the nuclear radius $R$ has been added to make the neutrino potential dimensionless. In the zero-neutrino case this closure approximation is well justified [39]. The final formula, restricted to the term proportional to the neutrino mass, is [4,37]

$$(\tau^{1/2}_{0\nu})^{-1}=\left(\frac{\langle m_\nu\rangle}{m_e}\right)^2G_{0\nu}M^2_{0\nu}, \tag{25}$$

where $G_{0\nu}$ is the phase space integral associated with the emission of the two electrons. The nuclear matrix elements $M_{0\nu}$ are [37,40,41]

$$M_{0\nu}\equiv\left|M^{GT}_{0\nu}-\frac{g^2_V}{g^2_A}M^F_{0\nu}\right|, \tag{26}$$

with

$$M^{\alpha}_{0\nu}=\langle 0^+_f \| O^{\alpha} \| 0^+_i \rangle, \tag{27}$$

where the kets $|0^+_i\rangle$ and $|0^+_f\rangle$ are denoting the initial and final nuclear states. The quantities $g_V$ and $g_A$ are the dimensionless coupling constants of the vector and axial vector nuclear currents, respectively. In the present work we use the effective value $(g_A/g_V)^2=1.0$ [42]. The operators $O^{\alpha}$ are given by

$$O^{GT}\equiv\sum_{m,n} O^{GT}_{mn}=\sum_{m,n} \vec{\sigma}_m t^-_m \cdot \vec{\sigma}_n t^-_n H(|\vec{r}_m-\vec{r}_n|,\overline{E}),$$

$$O^{F}\equiv\sum_{m,n} O^{F}_{mn}=\sum_{m,n} t^-_m t^-_n H(|\vec{r}_m-\vec{r}_n|,\overline{E}), \tag{28}$$

where the superscript GT denotes the Gamow-Teller operator, while $F$ indicates the Fermi operator. In expressions, (28) $\vec{\sigma}$ is the Pauli spin operator and $t^-$ the isospin lowering operator, which satisfies $t^-|n\rangle=|p\rangle$.

Transforming the transition operators to the pseudo-SU(3) space, we have the formal expression

$$O^{\alpha}=O^{\alpha}_{N_\pi N_\nu}+O^{\alpha}_{N_\pi A_\nu}+O^{\alpha}_{A_\pi N_\nu}+O^{\alpha}_{A_\pi A_\nu}, \tag{29}$$

where the subscript indices $NN$, $NA$, ... indicate the normal $N$ or abnormal $A$ spaces of the nucleon creation and annihilation operators. Given that we use the Nilsson scheme to obtain the occupation numbers, we are considering only nucleon pairs. For this reason only the four types of transitions listed above give a nonvanishing contribution to the $\beta\beta_{2\nu}$ matrix elements.

In a previous work we have restricted our analysis to six potential double-beta emitters, which, within the approximations of the simplest pseudo-SU(3) scheme, are also decaying via the $2\nu$ $\beta\beta$ mode. They include the observed $^{150}$Nd $\rightarrow ^{150}$Sm and $^{238}$U$\rightarrow ^{238}$Pu $\beta\beta_{2\nu}$ decays. In these cases two neutrons belonging to a normal parity orbital decay into two protons belonging to an abnormal parity orbital. The transition is mediated by the operator $O^{\alpha}_{A_\pi N_\nu}$. Under the seniority zero assumption for nucleons in abnormal parity orbitals, only proton pairs coupled to $J=0$ are allowed in the ground state. The matrix element is

$$\begin{aligned}M^{\alpha}_{0\nu}(A_\pi N_\nu)\equiv&\sum_{j_\nu} M^{\alpha}_{0\nu}(A_\pi N_\nu, j_\nu)\\
=&-\sum_{j_\nu} \sqrt{\frac{2j_\nu+1}{2(2\tilde{l}_\nu+1)}}\sqrt{\frac{(n^A_\pi+2)(2j_\pi+1-n^A_\pi)}{2j_\pi+1}}\langle (j^A_\pi j^A_\pi)J=M=0|O^{\alpha}|(j_\nu j_\nu)J=M=0\rangle\\
&\times\sum_{K_\pi L_\pi}\sum_{K_\nu K'_\nu}\langle(\lambda^f_\pi,\mu^f_\pi)K_\pi L_\pi,(\lambda^f_\nu,\mu^f_\nu)K'_\nu L_\pi\|(\lambda^f,\mu^f)10\rangle\langle(\lambda^i_\pi,\mu^i_\pi)K_\pi L_\pi,(\lambda^i_\nu,\mu^i_\nu)K_\nu L_\pi\|(\lambda^i,\mu^i)10\rangle\\
&\times\sum_{(\lambda_0,\mu_0)\rho_0}\langle(\lambda^i_\nu,\mu^i_\nu)K_\nu L_\nu,(\lambda_0,\mu_0)10\|(\lambda^f_\nu,\mu^f_\nu)K'_\nu L_\nu\rangle_{\rho_0}\langle(0,\tilde{\eta}_\nu)1\tilde{l}_\nu,(0,\tilde{\eta}_\nu)1\tilde{l}_\nu\|(\lambda_0,\mu_0)10\rangle\\
&\times\langle(\lambda^f_\nu,\mu^f_\nu)\|\|[\tilde{a}_{(0,\tilde{\eta}_\nu)\tilde{l}_\nu;1/2}\tilde{a}_{(0,\tilde{\eta}_\nu)\tilde{l}_\nu;1/2}]^{(\lambda_0,\mu_0);\tilde{S}=0}\|\||(\lambda^i_\nu,\mu^i_\nu)\rangle_{\rho_0}.\end{aligned} \tag{30}$$

We have implicitly defined $M^{\alpha}_{0\nu}(j_\nu)$ as the contribution of each normal parity neutron state $j_\nu$ to the nuclear matrix element in the transition $(j_\nu)^2\rightarrow(j^A_\pi)^2$.

The transitions that are forbidden for the $\beta\beta_{2\nu}$ decay are instead allowed for the zero-neutrino mode, due to presence of the neutrino potential. In the simplest model space, the $\beta\beta_{0\nu}$ of $^{160}$Gd has finite contributions for the two components with different occupation numbers in the $^{160}$Dy final state. There are two terms in the $\beta\beta_{0\nu}$ decay: one for the basis state that has allowed $\beta\beta_{2\nu}$ decay, and one for the state with forbidden $\beta\beta_{2\nu}$ decay. In the first case the above equation must be used. The second case involves the annihilation of two neutrons in normal parity orbitals, and the creation of two protons in normal parity orbitals. This transition is mediated by the operator $O^{\alpha}_{N_\pi N_\nu}$. The $\beta\beta_{0\nu}$ matrix element has the form

$$\begin{aligned}M^{\alpha}_{0\nu}(N_\pi N_\nu)\equiv&\sum_{J} M^{\alpha}_{0\nu}(N_\pi N_\nu, J)\\
=&-\frac{1}{4}\sum_{j_\pi j_{\pi'} j_\nu j_{\nu'}} \sqrt{(2j_\pi+1)(2j_{\pi'}+1)(2j_\nu+1)(2j_{\nu'}+1)}\\
&\times\sum_J \sqrt{2J+1}\langle (j_\pi j_{\pi'})J|O^{\alpha}|(j_\nu j_{\nu'})J\rangle W\left(\tilde{l}_\pi J\frac{1}{2}j_{\pi'},\tilde{l}_{\pi'}j_\pi\right)W\left(\tilde{l}_\nu J\frac{1}{2}j_{\nu'},\tilde{l}_{\nu'}j_\nu\right)\end{aligned}$$

$$\times \sum_{(\lambda_0^\pi,\mu_0^\pi)K_0^\pi} \langle(\tilde{\eta}_\pi,0)1\tilde{l}_\pi,(\tilde{\eta}_\pi,0)1\tilde{l}_{\pi'}\|(\lambda_0^\pi,\mu_0^\pi)K_0^\pi J\rangle \sum_{(\lambda_0^\nu,\mu_0^\nu)K_0^\nu} \langle(0,\tilde{\eta}_\nu)1\tilde{l}_\nu,(0,\tilde{\eta}_\nu)1\tilde{l}_{\nu'}\|(\lambda_0^\nu,\mu_0^\nu)K_0^\nu J\rangle$$

$$\times \sum_{(\lambda_0,\mu_0)\rho_0} \langle(\lambda_0^\pi,\mu_0^\pi)K_0^\pi J,(\lambda_0^\nu,\mu_0^\nu)K_0^\nu J\|(\lambda_0,\mu_0)10\rangle_{\rho_0} \sum_\rho \langle(\lambda^i,\mu^i)10,(\lambda_0,\mu_0)10\|(\lambda^f,\mu^f)10\rangle_\rho$$

$$\times \sum_{\rho_\pi\rho_\nu} \begin{bmatrix} (\lambda_\pi^i,\mu_\pi^i) & (\lambda_0^\pi,\mu_0^\pi) & (\lambda_\pi^f,\mu_\pi^f) & \rho_\pi \\ (\lambda_\nu^i,\mu_\nu^i) & (\lambda_0^\nu,\mu_0^\nu) & (\lambda_\nu^f,\mu_\nu^f) & \rho_\nu \\ (\lambda^i,\mu^i) & (\lambda_0,\mu_0) & (\lambda^f\mu^f) & \rho \\ 1 & \rho_0 & 1 & \end{bmatrix} \langle(\lambda_\pi^f,\mu_\pi^f)|||[a^\dagger_{(\tilde{\eta}_\pi,0),1/2}a^\dagger_{(\tilde{\eta}_\pi,0),1/2}]^{(\lambda_0^\pi,\mu_0^\pi);\tilde{S}=0}|||(\lambda_\pi^i,\mu_\pi^i)\rangle_{\rho_\pi}$$

$$\times\langle(\lambda_\nu^f,\mu_\nu^f)|||[\tilde{a}_{(0,\tilde{\eta}_\nu)\tilde{l}_\nu;1/2}\tilde{a}_{(0,\tilde{\eta}_\nu)\tilde{l}_\nu;1/2}]^{(\lambda_0^\nu,\mu_0^\nu);\tilde{S}=0}|||(\lambda_\nu^i,\mu_\nu^i)\rangle_{\rho_\nu}. \qquad (31)$$

In the above expression the $W(\ldots,\ldots)$ are Racah coefficients [43]. The two-body matrix element can be expanded in its $L,S$ components,

$$\langle(j_\pi j_{\pi'})J|O^\alpha|(j_\nu j_{\nu'})J\rangle=\sum_{LS}\chi\begin{Bmatrix} l_\pi & l_{\pi'} & L \\ \frac{1}{2} & \frac{1}{2} & S \\ j_\pi & j_{\pi'} & J\end{Bmatrix}\chi\begin{Bmatrix} l_\nu & l_{\nu'} & L \\ \frac{1}{2} & \frac{1}{2} & S \\ j_\nu & j_{\nu'} & J\end{Bmatrix}\langle(l_\pi l_\pi)L\|H(r,\overline{E})\|(l_\nu l_\nu)L\rangle\ \langle(\tfrac{1}{2}\tfrac{1}{2})S\|\Gamma\cdot\Gamma(\alpha)\|(\tfrac{1}{2}\tfrac{1}{2})S\rangle, \qquad (32)$$

where the $\chi\{\cdots\}$ are Jahn-Hope coefficients [43] and

$$\Gamma\cdot\Gamma(GT)\equiv\vec{\sigma}_1\cdot\vec{\sigma}_2,\quad \Gamma\cdot\Gamma(F)\equiv 1,\quad \alpha=GT \text{ or } F. \qquad (33)$$

In order to evaluate the spatial matrix elements, we have used the Bessel-Fourier expansion of the potential [44], which gives

$$\langle(l_1l_2)LM|H(r)|(l_3l_4)LM\rangle$$
$$=\sum_l (-1)^{l_1+l_4+L}(2l+1)(l_1\|C_l\|l_3)$$
$$\times(l_2\|C_l\|l_4)W(l_1l_2l_3l_4;Ll)R^l(l_1l_2,l_3l_4), \qquad (34)$$

where $(l_i\|C_l\|l_j)$ are the reduced matrix elements of the unnormalized spherical harmonics

$$C_{lm}(\Omega)\equiv\sqrt{4\pi/(2l+1)}Y_{lm}(\Omega)$$

and $R^l(l_1l_2,l_3l_4)$ are the radial integrals described in Appendix A of Ref. [18]. They include the effects of the finite nucleon size and the short range correlations, as explained in Appendix C of Ref. [18].

The zero-neutrino $\beta\beta$ matrix element has, as mentioned above, contributions from the two components of the $^{160}$Dy wave function:

$$M_{0\nu}^\alpha=aM_{0\nu}^\alpha(N_\pi N_\nu)+bM_{0\nu}^\alpha(A_\pi N_\nu). \qquad (35)$$

As said before, these expressions are to be supplemented by other contributions, $M_{0\nu}^\alpha(N_\pi A_\nu)$ and $M_{0\nu}^\alpha(A_\pi A_\nu)$, when the large model space (of Sec. III B) is used to construct the initial and final wave functions. The expressions of the additional terms are similar to those of the above equations and they are omitted for the sake of brevity. The matrix element of the $\beta\beta_{0\nu}$ mode is thus given by

$$M_{0\nu}^\alpha=\sum_{k,l} C_k^{(i)}C_l^{(f)}M_{0\nu}^\alpha(k,l), \qquad (36)$$

where the indices $k$ and $l$ denote the set of quantum numbers needed to specify the occupations and irreps included in the wave functions.

## VI. THE $\beta\beta$ DECAY OF $^{160}$Gd

In this section we study the two-neutrino and zero-neutrino modes of the double-beta decay of $^{160}$Gd to the ground state of $^{160}$Dy.

In the restricted configuration space of Sec. III A, the ground state of $^{160}$Dy, defined in Eq. (8), is a linear combination of two states having different occupation numbers. The energy difference between these states, estimated from the difference in their deformed Nilsson single-particle energies, is $\Delta E=1.71$ MeV. The pairing mixing between them, using the interaction strength $G_\pi=21/A$ MeV, is $h_{pair}=0.865$ MeV. With these matrix elements, the diagonalization of Eq. (9) yields the amplitudes $a=0.923$, $b=0.385$ of the wave function of the ground state of $^{160}$Dy.

The $\beta\beta_{2\nu}$ matrix element is suppressed by a factor $b$, compared with the *allowed* decays. It implies that the $\beta\beta_{2\nu}$ half-life is an order of magnitude $(1/b^2)$ larger than in other nuclei with similar $Q_{\beta\beta}$ values. The energy denominator takes the value $\mathcal{E}=12.19$ MeV. The two-neutrino $\beta\beta$ matrix element is $M_{2\nu}^{GT}(^{160}\text{Gd}\rightarrow{}^{160}\text{Dy})=0.0455$ MeV$^{-1}$. The phase space integral is $G_{GT}=8.028\times10^{-20}$ MeV$^2$ yr$^{-1}$, using $g_A/g_V=1.0$. The estimated $\beta\beta_{2\nu}$ half-life is

TABLE III. $M^{\alpha}_{0\nu}(N_\pi N_\nu, J)$ for the $\beta\beta_{0\nu}$ of $^{160}$Gd.

| $J$ | $F$ | GT |
|---|---|---|
| 0 | −0.117 46 | 0.255 88 |
| 2 | 0.037 92 | −0.050 16 |
| 3 | −0.001 31 | −0.000 11 |
| 4 | 0.020 90 | 0.004 54 |
| 6 | 0.003 12 | 0.005 01 |
| Sum | −0.056 83 | 0.215 16 |
| | $M_{0\nu}(N_\pi N_\nu)=0.27199$ | |

$$\tau^{1/2}_{2\nu}(^{160}\mathrm{Gd}\rightarrow{}^{160}\mathrm{Dy})=6.02\times10^{21}\ \mathrm{yr}. \qquad (37)$$

The contributions of the different angular momentum $J$ to the matrix elements $M^{\alpha}_{0\nu}(N_\pi N_\nu, J)$, in Eq. (35), are shown in Table III.

It is remarkable that the $J=0$ channel largely dominates both the Fermi ($F$) and Gamow-Teller (GT) transitions. The $J=2$ channel tends to reduce the transition matrix elements by about 20–30 %. For the Fermi matrix elements, the $J=4$ channel also contributes significantly. Fermi and Gamow-Teller matrix elements add up coherently due to the sign inversion in Eq. (26).

The transition matrix elements from the different neutron single-particle angular momentum $j_\nu$ in the matrix elements $M^{\alpha}_{0\nu}(A_\pi N_\nu, j_\nu)$, are presented in Table IV. In this case two neutrons are annihilated in the normal parity orbitals $p_{1/2}, p_{3/2}, f_{5/2}, f_{7/2}, h_{9/2}$ and two protons are created in the intruder orbit $h_{11/2}$.

As seen in Table IV, both for the Fermi and Gamow-Teller matrix elements the terms add up coherently. While in the Gamow-Teller case the $h_{9/2}\rightarrow h_{11/2}$ transition clearly dominates, in the Fermi case all transition amplitudes are comparable. As in Table III, Fermi and Gamow-Teller final matrix elements add up coherently. In absolute value, the *allowed* $\beta\beta_{0\nu}$ transition matrix element $M_{0\nu}(A_\pi N_\nu)$ is a factor 6 larger than the *forbidden* one $M_{0\nu}(N_\pi N_\nu)$. The final $\beta\beta_{0\nu}$ matrix element is

$$M_{0\nu}(^{160}\mathrm{Gd}\rightarrow{}^{160}\mathrm{Dy})=a\,M_{0\nu}(N_\pi N_\nu)+b\,M_{0\nu}(A_\pi N_\nu)$$
$$=0.251+0.668=0.919. \qquad (38)$$

The zero-neutrino phase space integral for the $\beta\beta_{0\nu}$ of $^{160}$Gd is $G_{0\nu}=1.480\times10^{-14}\ \mathrm{yr}^{-1}$. Expressing the Majorana mass of the neutrino $\langle m_\nu\rangle$ in units of eV, the calculated zero neutrino $\beta\beta$ half-life is

TABLE IV. $M^{\alpha}_{0\nu}(A_\pi N_\nu, j_\nu)$ for the $\beta\beta_{0\nu}$ of $^{160}$Gd.

| $j_\nu$ | $F$ | GT |
|---|---|---|
| $p_{1/2}$ | 0.015 52 | −0.067 92 |
| $p_{3/2}$ | 0.021 05 | −0.053 59 |
| $f_{5/2}$ | 0.021 68 | −0.161 33 |
| $f_{7/2}$ | 0.063 41 | −0.118 47 |
| $h_{9/2}$ | 0.043 10 | −1.165 98 |
| Sum | 0.164 77 | −1.567 29 |
| | $M_{0\nu}(A_\pi N_\nu)=-1.73206$ | |

TABLE V. The mixing parameter $b$, the double-beta-decay matrix elements and half-lives are listed as functions of the parameter $\Delta E$.

| $\Delta E$ (MeV) | $b$ | $M^{GT}_{2\nu}$ (MeV$^{-1}$) | $\tau^{1/2}_{2\nu}$ ($10^{21}$ yr) | $M_{0\nu}$ | $\tau^{1/2}_{0\nu}$ ($10^{25}$ yr) |
|---|---|---|---|---|---|
| 1.10 | 0.481 | 0.0568 | 3.86 | 1.072 | 1.53 |
| 1.20 | 0.464 | 0.0547 | 4.16 | 1.044 | 1.62 |
| 1.30 | 0.447 | 0.0527 | 4.48 | 1.017 | 1.70 |
| 1.40 | 0.431 | 0.0508 | 4.82 | 0.992 | 1.79 |
| 1.50 | 0.415 | 0.0490 | 5.18 | 0.967 | 1.89 |
| 1.60 | 0.401 | 0.0473 | 5.57 | 0.943 | 1.98 |
| 1.70 | 0.387 | 0.0456 | 5.98 | 0.921 | 2.08 |
| 1.71 | 0.385 | 0.0455 | 6.02 | 0.919 | 2.09 |
| 1.80 | 0.374 | 0.0441 | 6.41 | 0.900 | 2.18 |
| 1.90 | 0.361 | 0.0426 | 6.86 | 0.879 | 2.28 |
| 2.00 | 0.349 | 0.0412 | 7.34 | 0.860 | 2.38 |
| 2.10 | 0.338 | 0.0399 | 7.83 | 0.841 | 2.49 |
| 2.20 | 0.327 | 0.0386 | 8.36 | 0.824 | 2.60 |
| 2.30 | 0.317 | 0.0374 | 8.90 | 0.807 | 2.71 |
| 2.40 | 0.307 | 0.0363 | 9.47 | 0.791 | 2.82 |
| 2.50 | 0.298 | 0.0352 | 10.06 | 0.776 | 2.93 |

$$\tau^{1/2}_{0\nu}(^{160}\mathrm{Gd}\rightarrow{}^{160}\mathrm{Dy})\langle m_\nu\rangle^2=2.09\times10^{25}\ \mathrm{yr}. \qquad (39)$$

This half-life is a factor 20 larger than that reported in [40]. The difference is easily understood, given the fact that the present model takes explicitly into account the nuclear deformation, while in [40] the spherical quasiparticle random-phase approximation (QRPA) was used to obtain a crude estimation of the half-lives of all potential $\beta\beta$ emitters.

As mentioned above, the parameter $\Delta E$, which strongly influences the pairing mixing, is taken from the deformed Nilsson single-particle energies. To estimate up to which extent changes in this parameter affect the predicted double-beta-decay half-lives, we have allowed it to vary from 1.1 MeV to 2.5 MeV, covering most of the physically reasonable range. The results are listed in Table V.

The mixing parameter $b$ and the double-beta-decay matrix elements and half-lives are very smooth functions of $\Delta E$. The two-neutrino double-beta-decay half-life varies between $4\times10^{21}$ yr and $10\times10^{21}$ yr, around the predicted value of $6\times10^{21}$ yr. The calculated half-life of the neutrinoless double-beta decay is less dependent upon the mixing induced by pairing and it varies in the range $(1.5\text{–}2.9)\times10^{25}$ yr.

Concerning the results obtained in the enlarged space of Sec. III B, we are presenting them in Table VI. The partial contributions to the $\beta\beta_{2\nu}$ and $\beta\beta_{0\nu}$ decay processes, for transitions between the components of the initial and final wave functions, are listed in this table. We are indicating, also, the character of each of the transitions, with regard to the type of orbital, abnormal ($A$) or normal ($N$), of the neutrons and protons involved in the decay.

For the case of the $\beta\beta_{2\nu}$ decay mode, there are four active configurations and each of them add up up coherently to the final matrix element. They are partially suppressed by

TABLE VI. Matrix elements for the $\beta\beta_{2\nu}$ and $\beta\beta_{0\nu}$ decay modes. Each arrow shows the participant configurations, of the initial and final wave functions, which are listed in Tables I and II. The transitions not included in the table are trivially forbidden because they imply the change of the states of more than two nucleons. The $\beta\beta_{2\nu}$ transitions listed as ''not allowed,'' are those which are forbidden by the selection rules discussed in Sec. IV. The matrix elements, of the $\beta\beta_{2\nu}$ mode are given in units of $\mathrm{MeV}^{-1}$.

| Transition | Channel | $2\nu\beta\beta$ mode | $0\nu\beta\beta$ |
|---|---|---|---|
| $i_1\to f_1$ | $N_\pi A_\nu$ | Not allowed | -0.231 |
| $i_1\to f_2$ | $N_\pi N_\nu$ | Not allowed | 0.272 |
| $i_1\to f_3$ | $A_\pi A_\nu$ | Not allowed | -2.122 |
| $i_1\to f_4$ | $A_\pi N_\nu$ | 0.118 | 1.732 |
| $i_2\to f_1$ | $N_\pi N_\nu$ | Not allowed | 0.315 |
| $i_2\to f_3$ | $A_\pi N_\nu$ | 0.122 | 1.787 |
| $i_3\to f_1$ | $A_\pi A_\nu$ | Not allowed | -2.122 |
| $i_3\to f_2$ | $A_\pi N_\nu$ | 0.118 | 1.732 |
| $i_4\to f_1$ | $A_\pi N_\nu$ | 0.122 | 1.787 |

their amplitudes and the final matrix element is of the order of 0.086 $\mathrm{MeV}^{-1}$. This value is about twice that obtained by using the small configuration space of Sec. III A. Consequently, the calculated half-life, which is of the order of $1.68\times10^{21}$ yr, is shorter than that obtained in the small space. Concerning the $\beta\beta_{0\nu}$ decay mode, the results shown in Table VI indicate that there is an interference between configurations where both nucleons are in intruder orbits and those where the proton occupies an intruder orbit. For this channel the resulting matrix element is of the order of 0.293, a value which is about a factor of 3 smaller than that obtained in the small configuration space. The predicted half-life

$$\tau_{0\nu}^{1/2}(^{160}\mathrm{Gd}\to{}^{160}\mathrm{Dy})\langle m_\nu\rangle^2=2.05\times10^{26}\ \ \mathrm{yr} \qquad (40)$$

is an order of magnitude larger than that obtained in the small configuration space.

The above presented results can be summarized by noticing that the effect of including occupations other than the most probable one is less crucial for the two-neutrino mode than for the case of the neutrinoless double-beta decay. Nevertheless, the predicted two-neutrino double-beta decay of $^{160}$Gd is still suppressed, as compared to other double-beta-decay emitters. In this respect, the results of the present calculations are an improvement over earlier ones [17], where claims about a suppression of the two-neutrino mode have been made. Here, we have used a larger configuration space, as explained before, instead of a single configuration. The two-neutrino double-beta decay in $^{160}$Gd is hindered by nuclear structure effects, and the predicted half-life is of the order of $10^{21(22)}$ yr, depending upon the model space. The zero-neutrino double-beta-decay half-life is at least three to four orders of magnitude larger. In view of these predicted values, we are confident that the planned experiments using cerium-doped gadolinium silicate (GSO) crystals [24] would definitely be able to detect the $\beta\beta_{2\nu}$ decay of $^{160}$Gd, and could establish competitive limits to the $\beta\beta_{0\nu}$ decay. The background suppression due to a large $\beta\beta_{2\nu}$ half-life would be effective, although not as noticeably as was optimistically envisioned in [24].

Results regarding selection rules in other deformed double-beta-decay emitters are reported in [45].

## VII. CONCLUSIONS

In the present paper we have studied the $\beta\beta_{2\nu}$ and $\beta\beta_{0\nu}$ decay modes of $^{160}$Gd to the ground states of $^{160}$Dy. The transitions have been analyzed in the context of the pseudo-SU(3) model.

The energy spectrum and electromagnetic transitions in $^{160}$Gd and $^{160}$Dy have been studied in detail, in previous works, using the pseudo-SU(3) model and a realistic Hamiltonian. Ground state wave functions were built as linear combinations of the pseudo-SU(3) irreps associated with the larger quadrupole deformations, in a model space with fixed occupation numbers in normal and unique parity orbitals. Nucleons occupying intruder orbits were frozen. The pseudo-SU(3) leading irrep typically carries 60% of the total wave function.

In the present contribution the mixing of different occupation numbers in the $^{160}$Dy ground state wave function was studied. Only leading irreps, for each occupation, were considered in the calculations. The mixing induced by the pairing interaction makes possible the two-neutrino double-beta decay of $^{160}$Gd, which is forbidden when only the most probable occupation numbers are used.

Explicit expressions are presented for the pairing mixing, and for the $\beta\beta_{2\nu}$ and $\beta\beta_{0\nu}$ nuclear matrix elements in the present pseudo-SU(3) approach. The estimated $\beta\beta$ half-lives are larger than those obtained using a spherical QRPA model, and the results suggest that the planned experiments would succeed in detecting the $\beta\beta_{2\nu}$ decay in $^{160}$Gd, and in setting competitive limits for the zero-neutrino mode.

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