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Nuclear Physics A 705 (2002) 297–334

NUCLEAR PHYSICS A

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Collective motion in iso- and gauge-spaces Application to $2\nu\beta\beta$-decay transitions

D.R. Bes [a,b], O. Civitarese [c,*]

[a] *Departamento de Física, CNEA, Av. Libertador 8250, (1429) Buenos Aires, Argentina*
[b] *Departamento de Física, Universidad Favaloro, Belgrano 1723, (1093) Buenos Aires, Argentina*
[c] *Departamento de Física, UNLP, C.C. 67, (1900) La Plata, Argentina*

Received 22 October 2001; received in revised form 22 January 2002; accepted 29 January 2002

Abstract

The nuclear two-neutrino double-beta decay is a measured process that should be quantitatively understood before the predictions on more exotic, non-standard, double-beta decay processes are fully trusted. In most cases, the current framework for the description of $2\nu\beta\beta$ process includes the quasiparticle and random phase approximation (RPA) procedures, which present instabilities in the region of interest. From the point of view of many-body physics, the problem involved is to disentangle the physical effects associated with the lack of conservation of the isospin symmetry in the Hamiltonian, from those arising from the application of the Bogoliubov–Valatin transformation between identical particles. In the present paper, the separation between both effects is accomplished by introducing the collective subspace in isospin and gauge spaces, and restoring the symmetry within such subspace. Explicit, real, isodipole and isoquadrupole mixing terms are subsequently obtained. The problem of the over-completeness of the basis is solved by isolating the spurious sector via the application of the Becchi–Rouet–Stora–Tyutin (BRST) symmetry. The formalism allows to calculate Fermi double-beta-decay transitions which result—as expected—too small in order to be of significance in the double-beta processes. The same procedure is applied to calculate Gamow–Teller double-beta decay transitions and the already known sensitivity to model parameters is recovered. We have calculated two-neutrino double-beta decay transitions in ^{76}Ge, as an example about the use of the formalism.

PACS: 21.60.Jz; 22.40.Hc; 21.60Fw

* Corresponding author.
E-mail address: civitare@venus.fisica.unlp.edu.ar (O. Civitarese).

0375-9474/02/$ – see front matter
PII: S0375-9474(02)00655-3

1. Introduction

Fine detail nuclear structure calculations have become an issue, particularly those related to the theoretical estimation of nuclear electroweak observables beyond the standard model [1–3]. The interest in such a type of calculations is tied up to the continuous experimental and theoretical efforts devoted to elucidate neutrino properties [4]. If one focuses the attention on the theoretical description of nuclear double-beta decay transitions, and particularizes on the two-neutrino double-beta decay modes [5], one finds that the situation is not settled, in spite of the years elapsed since the first calculations were reported (for details see [6,7]). The predicted matrix elements are still within factors of order two to three from the experimentally extracted values [1]. Moreover, the large uncertainties in the calculations diminish the significance of such discrepancies. The situation is also not very clear in the analysis of the neutrinoless double-beta decay channel, due to the relatively large number of the nuclear matrix elements that are involved [5,8].

The most immediate option for nuclear structure calculations is the shell model. However, the availability of shell model results is restricted to a very small sample of cases and the calculations are, naturally, affected by severe limitations in the single-particle space. The most recent shell model result can be found in [9].

Most of the calculations aimed at the explanation of double-beta decay transitions in medium and heavy mass systems [3,7,10], are based on the use of the quasiparticle random phase approximation (QRPA). Although the theory has been tested extensively in the description of the spectrum of even–even nuclei near shell closures, its validity in the case of double-odd mass nuclei is less established due to the inherent complexity of the spectrum built upon quasiproton–quasineutron pair excitations.

One notorious problem associated with the QRPA description of double-beta decay transitions is the instability found when renormalized components of the interaction between quasiparticles are used in the proton–neutron channel [11]. The cause of this instability has been traced down to the mean field isospin symmetry breaking induced by the separate treatment of pairing correlations among protons and neutrons. For an isospin conserving interaction there would appear an unormalized zero-frequency mode, as explained below. This is an unavoidable fact of the quasiparticle plus RPA approximations and it cannot be cured by supplementing the theory with higher-order corrections. Until now none of the various approaches, including some bizarre ones, has given a satisfactory answer to the problem of calculating proton–neutron pairing interactions near criticality [12].

The work of Ref. [13] has advanced the notion of using collective and intrinsic variables in the treatment of isospin dependent nuclear excitations. This formalism has its own interest, independently of the above mentioned applications. The perfect analogy to this treatment is the use of intrinsic and collective degrees of freedom in the description of nuclear deformations and space rotations [19]. Although such an analogy has been drawn long ago [19–22], the microscopic theory of gauge and isospin collective phenomena has not been discussed before in detail. The present availability of radioactive beams and targets may also call for the application of the present formalism to explore nuclear structure aspects of the excited states in double-odd mass nuclei.

The results obtained in Ref. [13] for the case of purely isovector (spin independent) transitions are in perfect agreement with the available exact solutions, in contrast with

the behavior of other approximations in the vicinity of the symmetry restoration. In the present paper we present a detailed derivation of the formalism applied in Ref. [13].

There is a many-body problem of disentangling symmetry violations effects arising from the original Hamiltonian from those artificially introduced by the formalism. This problem has remained unsolved, although its existence was recognized long ago in studies concerning analogue states and isospin impurities [23]. The neutron excess, $T_z = \frac{1}{2}(Z - N)$, produces a symmetry violating Hartree–Fock Hamiltonian because, in this approximation, only the component $T_z\tau_z$ of the scalar product is taken into account. Moreover, the isospin-violating Coulomb interaction must be introduced between symmetry violating states.[1] In the present paper we solve an analogous problem through the use of the collective formalism. To our knowledge, this was not accomplished before, because collective variables have been usually introduced in order to restore, at the macroscopic level, those symmetries that are lost in the microscopic description. This implies that there are symmetries to start with, which are reflected in invariance properties of the original Hamiltonian. However, as in the present case, the initial Hamiltonian may not be scalar in isospin space (otherwise, there would not be double-Fermi transitions between states differing in two units of isospin). Moreover, the inclusion of pairing between identical particles contributes artificially to the breaking of the isospin symmetry.

We describe the steps that are followed in constructing the Hamiltonian and we extend the formalism presented in [13] to encompass the case of several j-shells. A brief review of the treatment with collective variables is presented in Section 2. The pairing-isospin Hamiltonian is constructed in Section 3, together with the mean field and RPA approximations, which yield the basic set of states. The practically-oriented reader can proceed directly to Section 5. However, a deeper understanding of the method is provided in Section 4, which includes the essence of the procedure due to Becchi, Rouet, Stora and Tyutin (BRST) [14,15], and the nature of the vacuum state and physical excitations that are compatible with the constraints. The same procedure have been applied to many-body physics in [16]. A pedagogical presentation can be found in [17]. A recent application to the treatment of the center of mass problem has been reported in [18].

Calculations based on the results obtained in Section 3 are presented in Section 5, with reference to Fermi and Gamow–Teller double-beta decay transitions. The comparison with the available exact solutions and the result of realistic calculations are also presented in Section 5. Conclusions are drawn in Section 6. All the accessories are presented in the appendices.

2. The treatment with collective coordinates

Many systems can be described in terms of variables which are subject to transformations. Let us assume, for the time being, that the Hamiltonian H is invariant against such

[1] The following are entries taken from Ref. [24]: *A severe limitation of any HF calculation in $N > Z$ nuclei is the appearance of spurious isospin mixing*... ; *this limitation cannot be avoided*; and in their Discussion and Summary: ... *and its* [*the result*] *main limitation is again the isospin mixing.*

transformations, and that it may split into two parts, H_0 and H_r, neither of which is invariant under those transformations. The basic set of eigenstates of H_0 will not carry, in general, the quantum numbers associated with the original symmetry of H.

The symmetry may be restored through the introduction of the collective formalism, i.e., the raising of the transformation parameters determining the orientation of the "intrinsic" or "moving" frame of reference, to the level of collective coordinates. The total Hilbert space is thus factorized into an intrinsic and a collective sector. This last one is labeled by the quantum numbers associated with the original symmetries, which are thus restored at the collective level. The symmetry violations take place in the intrinsic sector.

Since the calculations are carried out in the intrinsic frame of reference, any operator must be first transformed to this frame before acting on the basic set of product states.

The overcompleteness of the basis poses a problem that is clarified through the observation that a system, described as above, possesses a "gauge" symmetry. This symmetry consists of the group of transformations which simultaneously move the intrinsic frame of reference and the particle system so as to reproduce the same physical situation. It is expressed by the constraints

$$\tau_k - T_k = 0, \tag{1}$$

where the τ_k and T_k are the particle and collective generators of the transformations, respectively. They satisfy the following commutation relations

$$[\tau_k, \tau_l] = i c_{klj} \tau_j, \qquad [T_k, T_l] = -i c_{klj} T_j, \qquad [\tau_k, T_l] = 0, \tag{2}$$

where c_{klj} are the structure constants of the Lie group associated with the transformations. The conditions (1) may be rigorously derived whenever we consider a Lagrangian corresponding to a description from a moving frame, and we treat both the original coordinates and the coordinates of the moving frame on an equal footing [25]. Such Lagrangian is called singular, meaning that the velocities cannot be inverted as functions of the coordinates and the momenta. As a consequence, the momenta are not independent from each other: there appear relations (1) between them, which are called constraints.

At the quantum level, the constraints (1) imply that physical states should be annihilated by them and physical operators should commute with them. Several procedures have been developed in order to enforce these constraints for the case of gauge field theories. In particular, the one based on the BRST invariance [14,15], has been adapted to many-body problems in Ref. [16].

3. The pairing-isospin problem

The intrinsic generators of the transformations in gauge and isospin spaces are: (a) the operator measuring the number of pairs of particles[2]

[2] We use the Einstein convention that the repetition of an index on a given side of an equation implies a summation over that index (for instance, there is a summation over the index m in the second Eq. (3)). Exceptions: (i) the index is repeated also on the other side of the equation, as for the index j on the second Eq. (3); (ii) whenever there is a limitation on the summation, as for the $\sum_{m>0}$ on Eqs. (8).

$$\tau_a = \sum_j \tau_{aj}, \qquad \tau_{aj} = \frac{1}{2}\left(c^+_{pjm}c_{pjm} + c^+_{njm}c_{njm}\right) \tag{3}$$

and (b) the isospin operators

$$\begin{aligned}
&\tau_0 = \sum_j \tau_{0j}, \qquad \tau_{0j} = \frac{1}{2}\left(c^+_{pjm}c_{pjm} - c^+_{njm}c_{njm}\right),\\
&\tau_1 = \sum_j \tau_{1j}, \qquad \tau_{1j} = -\frac{1}{\sqrt{2}}\, c^+_{pjm}c_{njm},\\
&\tau_{\bar{1}} = -\tau_1^+.
\end{aligned} \tag{4}$$

Hereon we use the notation $k = a, q$, where $q = 0, \pm 1$ denotes the spherical labeling of the vector components. Moreover, $\bar{1} = -1$. Alternatively, instead of the pair of quantum numbers $(q = 0, a)$, we may use the linear combinations v $(v = p, n)$, where the indices (p, n) stand for proton and neutrons, respectively. Therefore

$$\tau_p = \tau_a + \tau_0, \qquad \tau_n = \tau_a - \tau_0 \tag{5}$$

represent the number of protons and neutrons, respectively.

The structure constants of the pairing-isospin problem are obtained from the commutation relations

$$[\tau_a, \tau_q] = 0, \qquad [\tau_0, \tau_{\pm 1}] = \pm\tau_{\pm 1}, \qquad [\tau_{\bar{1}}, \tau_1] = \tau_0. \tag{6}$$

The operators $T_a, T_0, T_{\pm 1}$ denote the collective momenta corresponding to (3). More details of the notation are given in Appendix A.

In the remaining part of this section we select a pairing Hamiltonian and we transform it to the moving system (Subsection 3.1). Subsequently we discuss the collective Hilbert space and the simplification inherent to the limit of large values of T (Subsection 3.2). The existence of large *order parameters* in the deformed solution associated with the intrinsic system, permits the ordering of the terms in the Hamiltonian according to their magnitude. The leading terms reproduce the usual BCS equations between identical particles. The discussion of the *pn* excitations constitutes the central topic of the present contribution. The formalism not only allows for the construction of the elementary modes of excitation, carrying A, T and M as good quantum numbers, but also yields the matrix elements of the Hamiltonian between states differing in the value of T (Subsection 3.3).

3.1. The Hamiltonian

We choose a Hamiltonian that has been frequently used in the literature. It may be written as

$$\begin{aligned}
&H = H_{\rm sp} + H_{\rm pair} + H_\tau,\\
&H_{\rm sp} = \epsilon_{vj}\tau_{vj},\\
&H_{\rm pair} = -g_p S_p^+ S_p - g_n S_n^+ S_n - \frac{1}{2}\, g_\perp S_\perp^+ S_\perp,\\
&H_\tau = \kappa\left(\tau_0^2 - [\tau_1\tau_{-1}]_+\right),
\end{aligned} \tag{7}$$

where ϵ_{vj} and τ_{vj} are the single-particle energy and number operator of the protons or the neutrons in a j-shell. We denote by S_p^+, S_n^+ and $S_\perp^+$ the pairing operators creating a proton pair, a neutron pair and a proton–neutron pair coupled to isospin one and angular momentum zero

$$S_v^+ = \sum_{m>0} c_{vjm}^+ c_{vj\tilde{m}}^+, \qquad S_\perp^+ = \sum_{m>0} c_{pjm}^+ c_{nj\tilde{m}}^+ + c_{njm}^+ c_{pj\tilde{m}}^+, \tag{8}$$

where the symbol $\tilde{}$ indicates the time-reversal operation. The Hamiltonian allows for the differences between proton and neutron single-particle energies and pairing strengths, and for an arbitrary strength of the neutron–proton isoquadrupole pairing component. In this paper we do not attempt to discuss the derivation of this effective interaction from first principles, like Coulomb effects, neutron-proton mass differences, etc. The isospin interaction which manifests, for instance, in the Weizsäcker semiempirical mass formula, is constructed with the isospin operators τ_q. The case of isoscalar pairing is discussed in Appendix F.

It would be a trivial yet cumbersome procedure to adapt the present formalism to realistic interactions. In this work we value more the simplicity in the presentation associated with separable forces, which makes more transparent the geometrical aspects of the different transformation operations.

Although the Hamiltonian (7) is a scalar in gauge space, it is not so in isospace. In fact, it splits into isoscalar, isovector and isoquadrupole terms (c.f. Appendix B), namely

$$\begin{aligned}
&H = H_0 + H_1 + H_2,\\
&H_0 = \epsilon_{aj}\tau_{aj} - g_0\left(S_p^+ S_p + S_n^+ S_n + \frac{1}{2} S_\perp^+ S_\perp\right) + \chi\left(\tau_0^2 - [\tau_1\tau_{-1}]_+\right),\\
&H_1 = \epsilon_{0j}\tau_{0j} - g_1\left(S_p^+ S_p - S_n^+ S_n\right),\\
&H_2 = -g_2\left(S_p^+ S_p + S_n^+ S_n - S_\perp^+ S_\perp\right),
\end{aligned} \tag{9}$$

where

$$\begin{aligned}
&g_0 = \frac{1}{3}(g_p + g_n + g_\perp), \qquad g_1 = \frac{1}{2}(g_p - g_n), \qquad g_2 = \frac{1}{6}(g_p + g_n - 2g_\perp),\\
&\epsilon_{aj} = \epsilon_{pj} + \epsilon_{nj}, \qquad \epsilon_{0j} = \epsilon_{pj} - \epsilon_{nj},\\
&\tau_{aj} = \frac{1}{2}(\tau_{pj} + \tau_{nj}), \qquad \tau_{0j} = \frac{1}{2}(\tau_{pj} - \tau_{nj}).
\end{aligned} \tag{10}$$

Since the calculations are performed in the intrinsic system, any operator should be transformed to this frame. As usual [19], the transformation between laboratory (lab) and intrinsic (int) tensor operators is expressed by

$$\mathcal{O}_{\lambda\mu}^{\text{lab}} = D_{\mu\nu}^{\lambda} \mathcal{O}_{\lambda\nu}^{\text{intr}}, \tag{11}$$

where $D_{\mu\nu}^{\lambda}$ are the rotational matrices which are discussed in detail in the next subsection. The operators $\mathcal{O}_{\lambda\mu}$ are irreducible tensor operators carrying isospin λ and μ isospin-projection (see Appendix B). Therefore also the Hamiltonian should be transformed, since neither the single-particle nor the interaction terms are isoscalars in (9). We obtain for the transformed Hamiltonian,

$$H_1 = \epsilon_{0j} D^1_{0\sigma} \tau_{\sigma j} - g_1 \Big(D^1_{00}\big(S^+_p S_p - S^+_n S_n\big) - \frac{1}{\sqrt{2}} D^1_{01}\big(S^+_p S_\perp + S^+_\perp S_n\big) + \frac{1}{\sqrt{2}} D^1_{0\tilde{1}}\big(S^+_n S_\perp + S^+_\perp S_p\big)\Big) \tag{12}$$

$$H_2 = -g_2 \Big(D^2_{00}\big(S^+_p S_p + S^+_n S_n - S^+_\perp S_\perp\big) + \sqrt{\frac{3}{2}} D^2_{01}\big(S^+_\perp S_n - S^+_p S_\perp\big) + \sqrt{\frac{3}{2}} D^2_{0\tilde{1}}\big(S^+_\perp S_p - S^+_n S_\perp\big) + \sqrt{6}\big(D^2_{02} S^+_p S_n + D^2_{0\tilde{2}} S^+_n S_p\big)\Big), \tag{13}$$

where the irreducible isotensor operators are defined in Appendix B. The terms H_1 and H_2 of the Hamiltonian (13) are physical operators, because they commute with the gauge constraints (1). This is not the case for the components H_1 and H_2 in (9). In the usual case the transformation is a scalar and thus naturally commutes with the constraints.

Apparently, we have only succeeded in complicating the problem through the substitution of the Hamiltonian (9) by (13), which must be considered simultaneously with the constraints (1). In the following we show that this is not the case, but rather it constitutes the first step of a simplifying procedure.

3.2. The collective sector of the Hilbert space

As a needed detour in the construction of the Hamiltonian, we present hereafter the structure of the collective space. The collective Hilbert space appropriate for isospin conserving pairing interactions was originally introduced in Refs. [19,22]. A complete set of states for this sector, carrying the appropriate symmetries, is

$$\langle \phi, \phi_\alpha, \phi_\beta, \phi_\gamma | A T M K \rangle = \sqrt{\frac{2T+1}{16\pi^3}} \exp\left[\frac{iA}{2}\phi\right] D^T_{MK}(\phi_\alpha, \phi_\beta, \phi_\gamma). \tag{14}$$

Here ϕ is the collective angle in gauge space, and ϕ_α, ϕ_β and ϕ_γ represent the Euler angles in isospin space. The constant $\frac{1}{2}A$ is the eigenvalue of the pair of particles operator T_a, while M and K are the eigenvalues of the isospin projection along the laboratory and intrinsic frames of reference, respectively.

The problem may be solved in two limiting situations, according to whether the Coriolis type of interaction $\frac{1}{\mathcal{I}}(-1)^q T_q \tau_{\tilde{q}}$ is taken into account perturbatively or is wholly included in the intrinsic single-particle spectrum, as in the cranking model. The former solution is to be preferred if the magnitude of the isospin T is smaller than other parameters of the system, while the second solution is adopted for larger values of T. In the present paper we consider this second case.

The assumption of large values of T suggests the use of the Marshalek generalization of the Holstein–Primakoff representation [28]. Thus, the rotational isospin sector in (14) may be expressed in terms of the boson creation operators Υ^+, ξ^+ and ς^+

$$|T M K\rangle = \frac{(\Upsilon^+)^{2T}}{\sqrt{(2T)!}} \frac{(\xi^+)^m}{\sqrt{(m)!}} \frac{(\varsigma^+)^k}{\sqrt{(k)!}} |\rangle, \qquad m, k = 0, 1, 2, \ldots, \tag{15}$$

where the quantum numbers $m = \frac{1}{2}(T + M)$ and $k = \frac{1}{2}(T + K)$ substitute the isospin projections M and K, respectively. This representation is specially useful for values of m/T, $k/T \ll 1$, which we assume to be the case.

We may also write down the expression for the matrices $D^{\lambda}_{\mu\delta}$ acting on the states (15), namely

$$D^{\lambda}_{\mu,\delta} = \sum_{|s|\leqslant\lambda} \sqrt{\frac{2T+1}{2T+2s+1}} \sqrt{\frac{m!}{(m+s+\mu)!}} \sqrt{\frac{k!}{(k+s+\delta)!}} \times \langle \lambda, \mu; T, -T+m | T+s, -T+m+\mu \rangle \times \langle \lambda, \delta; T, -T+k | T+s, -T+m+\delta \rangle \Upsilon^{2s} \left(\xi^{\dagger}\right)^{s+\mu} \left(\varsigma^{\dagger}\right)^{s+\delta}. \tag{16}$$

The operators (16) are expanded in powers of $1/T$, as shown in the Appendix C for the cases of interest.

Within the same representation, the collective components of the isospin operators may be written in terms of the bosons ς^{+}, ς

$$T_0 = -T + \varsigma^{+}\varsigma, \qquad T_{\bar{1}} = \varsigma^{+}\sqrt{T - \frac{1}{2}\varsigma^{+}\varsigma} \approx -\sqrt{T}\,\varsigma^{+}$$
$$T_1 = -T^{+}_{\bar{1}} \approx \sqrt{T}\,\varsigma. \tag{17}$$

Therefore, the constraints (1) are written to leading order in $1/T$, as

$$\tau_a = \frac{A}{2}, \qquad \tau_0 = -T + \varsigma^{+}\varsigma, \qquad \tau_1 = -\sqrt{T}\,\varsigma, \qquad \tau_{\bar{1}} = \sqrt{T}\,\varsigma^{+}. \tag{18}$$

We also define the isospin raising operator β

$$\beta^2 |T\rangle = |T+1\rangle \leftrightarrow \beta^2 \frac{(\Upsilon^{+})^{2T}}{\sqrt{(2T)!}} |\rangle = \frac{(\Upsilon^{+})^{2T+1}}{\sqrt{(2T+1)!}} |\rangle. \tag{19}$$

3.3. The treatment of the Hamiltonian in the intrinsic frame

Up to now the Hamiltonian (13), together with the constraints (1), constitutes an exact reformulation of the initial problem. In particular, we have not introduced yet any violation of the gauge and isospin symmetries due to the formalism. From here on we simplify the treatment in the intrinsic system by fixing a convenient orientation of the body with respect to the intrinsic frame. Since there are four angular variables in the collective subspace (14), we may choose four conditions to fix such relative orientation. As usual we select[3]

$$\mathrm{Im}\langle S_p \rangle = \mathrm{Im}\langle S_n \rangle = \langle S_{\perp} \rangle = 0. \tag{20}$$

This selection of a gauge leads to the usual Bogoliubov–Valatin transformation between identical particles.[4] Note that the *pn*-pairing is only neglected in the intrinsic frame, but

[3] Other possible choices have been discussed [22].

[4] Although the quasiparticles are labeled by the quantum number $v = p, n$, the fact that we operate in an intrinsic system implies that they do not strictly correspond to either neutrons or protons. The expressions quasineutrons or quasiprotons should be more adequate to indicate their nature.

wholly incorporated in the laboratory frame through the collective rotations in isospin and gauge spaces.

The situation is identical to the one appearing in shape-deformed nuclei, for which the conditions $\langle Q_{\pm 1}\rangle = 0$, holding in the intrinsic system, does not mean that any part of the spherically symmetric quadrupole interaction is neglected in the laboratory system.

The two remaining expectation values $\langle S_p\rangle$ and $\langle S_n\rangle$ are real and they are considered to be the large quantities of the problem. This is consistent with the expansion implicit in the RPA, which assumes that $O(\langle S\rangle| > O(S^{(20)}) > O(S^{(11)})$. It is convenient to unify this expansion with the $1/T$ one by setting

$$O\big(\langle S\rangle\big) = T, \qquad O\big(S^{(20)}\big) = \sqrt{T}, \qquad O\big(S^{(11)}\big) = 1,$$
$$O(g_v) = O(g_\perp) = 1/T. \tag{21}$$

Combining the selection of the gauge (20) with the assumption (21), one may classify the different terms in the Hamiltonian according to their order of magnitude. One thus obtains the different contributions given in the Appendix D. By making intensive use of the simplified constraints (18), the summation of the leading order contributions yields the following terms

$$\begin{aligned}
&\langle H\rangle = \epsilon_{vj}\langle\tau_{vj}\rangle - g_v\langle S_v\rangle^2,\\
&H^{(20)} - \langle\lambda_v\rangle\tau_v^{(20)} = e_{vj}\tau_{vj}^{(20)} - g_v\langle S_v\rangle\big(S_v^{+(20)} + S_v^{(20)}\big),\\
&H^{(11)} - \langle\lambda_v\rangle\tau_v^{(11)} = e_{vj}\tau_{vj}^{(11)} - g_v\langle S_v\rangle\big(S_v^{+(11)} + S_v^{(11)}\big),\\
&H_\parallel = -g_v S_v^{+(20)} S_v^{(20)},\\
&H_\perp = -g_\perp\frac{1}{2}s^{+(20)}s^{(20)} + \omega_\xi\xi^+\xi - g_2\frac{3}{T}\langle S_p\rangle\langle S_n\rangle\big(\beta^4\xi^{+2} + \beta^{-4}\xi^2\big)\\
&\qquad -\big(\beta^2\xi^+\zeta^{+(20)} + \beta^{-2}\xi\zeta^{(20)}\big) - \frac{1}{2\mathcal{I}_\perp}\big[\tau_1^{(20)}, \tau_{\bar{1}}^{(20)}\big]_+.
\end{aligned} \tag{22}$$

According to (21) the expectation value of the Hamiltonian is of $O(T)$, the term with the superindex (20) is of $O(\sqrt{T})$ and the remaining terms are of order $O(1)$. The neglected terms in the Hamiltonian are of $O(\leqslant 1/\sqrt{T})$.

We have included the Lagrange multipliers in the single-particle energies of Eq. (22)

$$\begin{aligned}
&-\langle\lambda_v\rangle\tau_v^{(20+11)} = -\langle\lambda_a\rangle\tau_a^{(20+11)} - \langle\lambda_0\rangle\tau_0^{(20+11)},\\
&e_{vj} = \epsilon_{vj} - \langle\lambda_v\rangle.
\end{aligned} \tag{23}$$

The constants $\langle\lambda_v\rangle$ are chosen such that $\langle\tau_n\rangle = N$ and $\langle\tau_p\rangle = Z$, respectively. The single-particle energies e_{vj} are measured from $\langle\lambda_v\rangle$. However, note that within the BRST formalism, the introduction of the Lagrange multiplier does not constitute an additional improving approximation, but rather it is part of the exact formalism (c.f. Eq. (53)).

The operators β, ξ are defined in Subsection 3.2. The frequency ω_ξ and the moment of inertia $\mathcal{I}_\perp$ have the value

$$\omega_\xi = \frac{3g_2}{T}\left(\langle S_p\rangle^2 + \langle S_n\rangle^2\right) + \frac{g_1}{T}\left(\langle S_p\rangle^2 - \langle S_n\rangle^2\right) - \frac{1}{T}\epsilon_{0j}\langle\tau_{0j}\rangle,$$
$$\mathcal{I}_\perp = -\frac{T}{\langle\lambda_0\rangle}, \tag{24}$$

respectively, while the two quasiparticle operator $\zeta^{(20)}$ is defined in Eq. (84).

The Hamiltonian $H_\perp$ plus the (quadratic) RPA version of $H^{(11)} - \langle\lambda_v\rangle\tau_v^{(11)}$ commutes with the operators $\tau_{\pm1}^{(20)}$, but for the last term in Eq. (22).

According to the Hamiltonians (7) and (13) we should also include the isospin interaction H_τ. Neglecting again terms proportional to the constraints and using the approximation (17), we obtain

$$H_\tau = \kappa\left(T_0^2 - [T_1, T_{\bar{1}}]_+\right) = \kappa T(T+1). \tag{25}$$

In the following we interpret the various terms appearing in the Hamiltonian (22). We may already remark that all contributions conserve isospin, except for the terms including β^ν in $H_\perp$ ($\nu = \pm2, \pm4$). Thus, if we ignore for the time being these last terms, the basic single-particle and boson modes constructed from (22) carry the isospin T as a good quantum number. This is so in spite of the fact that not only the isoscalar, but also the isovector and isoquadrupole components enter in the construction of (22). This is explained because these last two components include isospin conserving contributions, notwithstanding their tensor character. It happens that such contributions are the leading order ones in the expansion of $D^\lambda_{0\delta}$ in powers of T^{-1} (see Appendices C and D).

3.3.1. The ground state energy

Following Appendix E we write

$$\langle H\rangle = \text{constant} + \frac{1}{2\mathcal{I}_v^{(1)}}\langle T_v\rangle^2, \tag{26}$$

where the moments of inertia $\mathcal{I}_v^{(1)}$ are defined as

$$\mathcal{I}_v^{(1)} = \frac{\langle T_v\rangle}{\langle\lambda_v\rangle}. \tag{27}$$

The expression (26) is used in the construction of the spectrum (37).

3.3.2. The mean field approximation

The second and third lines of (22) may be treated within the Bogoliubov–Valatin transformation between identical particles

$$\alpha^+_{vjm} = U_{vj}c^+_{vjm} - V_{vj}c_{vj\bar{m}}. \tag{28}$$

We require

$$H^{(20)} - \langle\lambda_v\rangle\tau_v^{(20)} = 0,$$
$$H^{(11)} - \langle\lambda_v\rangle\tau_v^{(11)} = E_{vj}\nu_{vj}. \tag{29}$$

Here ν_{vj} is the number of quasiparticles of type v in the j-shell (see Appendix A). We also obtain the quasiparticle energies and gaps

$$E_{vj} = \sqrt{e_{vj}^2 + \Delta_v^2}, \qquad \Delta_v = g_v \langle S_v \rangle, \tag{30}$$

and similarly the amplitudes U_{vj}, V_{vj}. The self-consistent conditions must be satisfied

$$\frac{4}{g_v} = \frac{2j+1}{E_{vj}}. \tag{31}$$

It is a remarkable property of the expansion in $1/T$ that the single quasiparticle terms are identical to those appearing in the naive treatment of the pairing interaction (7). These terms conserve isospin, in spite of the fact that they originate from all isotensor components of the Hamiltonian.

3.3.3. The quadratic Hamiltonian

The RPA solutions split into two branches, labeled as *parallel* ($H_{\parallel}^{\rm RPA}$) and *perpendicular* ($H_{\perp}^{\rm RPA}$). The parallel branch represents separate oscillations of quasiprotons and quasineutrons, each of them displaying a zero-frequency root together with the finite-frequency ones ($\omega_{v\iota} > 0$, $\iota = 2, 3, \ldots$). The treatment of this branch is the same as for the case of pairing between identical particles. One obtains to the RPA order

$$H_{\parallel}^{\rm RPA} = H^{(11)} - \langle \lambda_v \rangle \tau_v^{(11)} + (H_{\parallel})^{\rm RPA} = \omega_{v\iota} \left(\Gamma_{v\iota}^{+} \Gamma_{v\iota} + \frac{1}{2} \right) + \frac{1}{2\mathcal{I}_v} \big(\tau_v^{(20)}\big)^2. \tag{32}$$

The boson creation operators $\Gamma_{v\iota}^{+}$ are linear combinations of the operators γ_{vj}^{+}, γ_{vj} (Eq. (78)). The moments of inertia $\mathcal{I}_v$ for rotations in gauge space are calculated according to the Marshalek–Weneser prescription [29]. A regularization procedure for taking into account the inherent infrared problems is worked out in detail, as an example, in Chapter 7 of Ref. [16]. Therefore, in the present paper we confine the discussion to the inclusion of the second term in the r.h.s. of (32) into the spurious sector (see Eq. (56) and Appendix G).

We turn now our attention to the perpendicular branch of excitations.

$$H_{\perp}^{\rm RPA} = H^{(11)} - \langle \lambda_v \rangle \tau_v^{(11)} + (H_{\perp})^{\rm RPA}. \tag{33}$$

3.3.4. The real, intrinsic + collective, sectors

The independent quasiparticle Hamiltonian plus the interaction in the first line of $H_{\perp}$ (Eq. (22)) yield the quadratic intrinsic Hamiltonian to be diagonalized

$$\begin{aligned} H_{\rm int}^{\rm RPA} &= (E_{pj} + E_{nj}) \kappa_j^{+(20)} \kappa_j^{(20)} - \frac{g_{\perp}}{2} s^{+(20)} s^{(20)} \\ &= \omega_{\perp\iota} \left(\Gamma_{\perp\iota}^{+} \Gamma_{\perp\iota} + \frac{1}{2} \right), \quad \iota = 2, 3, \ldots . \end{aligned} \tag{34}$$

The operators $\kappa_j^{(20)}$ and $s^{(20)}$ are given in Eqs. (82). They are independent of the operators $\tau_{\pm 1}$. The linearization equation

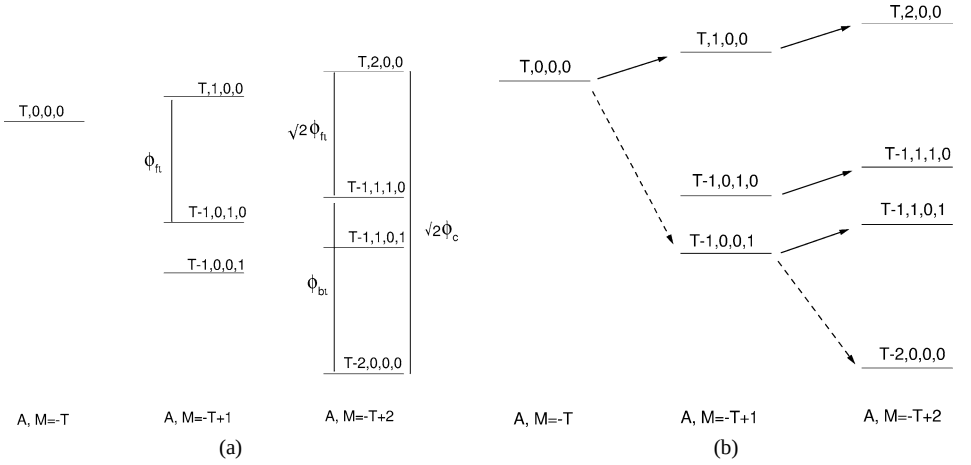

Fig. 1. Lowest energy states of a system with A nucleons. The set of quantum numbers $(A, T, M, n_\xi, n_{\perp\iota}, n_{\diamond\iota\mu})$ needed to specify a state are indicated for each state. (a) displays the Hamiltonian matrix elements connecting the different states, while (b) shows the Fermi single-beta decay transitions (solid arrows) and the Gamow–Teller transitions (dashed arrows).

$$\begin{aligned} &\left[H_\perp^{\mathrm{RPA}}, \Gamma_{\perp\iota}^+\right] = \omega_{\perp\iota}\Gamma_{\perp\iota}^+, \\ &\Gamma_{\perp\iota}^+ = \lambda_{\iota j}\gamma_{\perp j}^+ - \mu_{\iota j}\gamma_{\perp j}, \end{aligned} \tag{35}$$

determines the finite frequencies $\omega_{\perp\iota}$ and the amplitudes $\lambda_{\iota j}$, $\mu_{\iota j}$ in the orthonormal phonons $\Gamma_{\perp\iota}^+$. These last operators create the so-called antianalogue states in neighbor nuclei.[5]

In addition to these intrinsic excitations, the system displays the collective excitations corresponding to a change in the number of particles A, in the isospin T and in its projection M. According to the definition of the operator ξ^+ in Subsection 3.2, it increases the value of $M = m - T$ in one unit. Thus, the set of multiple I.A.S. is represented by the vibrational band $|m\rangle = (1/\sqrt{m!})(\xi^+)^m|0\rangle$ with frequency ω_ξ (see Eq. (24)). This frequency depends on the difference between proton and neutron single-particle energies ϵ_{0j} (not e_{0j}). As a result of the regularization procedure which is used in order to take into account the gauge constraints (1), the role of the unphysical operators $\tau_{\pm1}$ is taken by the collective operators ξ^+, ξ. Fig. 1 represents the set of vibrational and rotational excitations which are constructed on top of a state with the quantum numbers $(A, T, M = -T, m = 0, k = 0, n_{\perp\iota} = 0)$. All the states represented in this figure have the same value of A. The spectrum may be constructed by repeated applications of the intrinsic creation operators $\Gamma_{\perp\iota}^+$ and of the collective creation operators ξ^+. In addition, there may be excitations carrying different symmetries. One example is the set of the $I^\pi = 1^+$ states in odd–odd nuclei (see the Appendix F). These states are denoted by the indices $\diamond$, ι, μ ($\iota = 1, 2, 3, \ldots$; $\mu = 0, \pm1$ is the magnetic quantum number).

[5] The lack of sufficient intensity in the two-body transfer reactions populating antianalogue states has been pointed out in Ref. [30].

The spectrum is expanded by the basis

$$|A,T,M,n_{\perp\iota},n_{\diamond\iota\mu}\rangle = \frac{1}{\sqrt{2\pi}}\exp\left[\frac{iA}{2}\phi\right]|n_{\diamond\iota\mu}\rangle|n_{\perp\iota}\rangle|m\rangle$$
$$= \frac{1}{\sqrt{2\pi}}\exp\left[\frac{iA}{2}\phi\right]\frac{(\Gamma^{+}_{\diamond\iota\mu})^{n_{\diamond\iota\mu}}}{\sqrt{n_{\diamond\iota\mu}!}}\frac{(\Gamma^{+}_{\perp\iota})^{n_{\perp\iota}}}{\sqrt{n_{\perp\iota}!}}\frac{(\xi^{+})^{m}}{\sqrt{m!}}|0\rangle. \tag{36}$$

The energy of the states (36) is written

$$E(A,T,M,n_{\perp\iota},n_{\diamond\iota\mu})$$
$$= \frac{1}{2\mathcal{I}_{+}}\left(\frac{A}{2}\right)^{2} + \left(\kappa + \frac{1}{2\mathcal{I}_{+}}\right)T(T+1) + \frac{1}{2\mathcal{I}_{-}}A\left(T+\frac{1}{2}\right)$$
$$+ \omega_{\xi}\left(m+\frac{1}{2}\right) + \omega_{\perp\iota}\left(n_{\perp\iota}+\frac{1}{2}\right) + \omega_{\diamond\iota}\sum_{\mu}\left(n_{\diamond\iota\mu}+\frac{1}{2}\right), \tag{37}$$

where the moments of inertia are $\mathcal{I}_{\pm}^{-1} = (\mathcal{I}_{n}^{(1)})^{-1} \pm (\mathcal{I}_{p}^{(1)})^{-1}$. The derivation of the rotational pairing sector is given in the Appendix E. Therefore, there is a contribution from the pairing energy to the semiempirical mass term $\kappa T(T+1)$, which should decrease the effective value of κ. Note that the linear term T in the product $T(T+1)$ originates from the BRST treatment of the spurious sector (c.f. Eq. (63)).

3.3.5. The similitude with the self-consistent-cranking model (s.c.c.)

The model is equivalent to the s.c.c. model for rotations in ordinary space [31]. Since we have chosen the expectation values $\langle\lambda_{\pm1}\rangle = 0$, we are describing uniform rotations in gauge space and around the 0-axis of the isospin space both in the laboratory and intrinsic frames (they coincide). Note that the expectation values in the g.s. are

$$\langle T_p\rangle = Z, \qquad \langle T_n\rangle = N, \tag{38}$$

while in a representation conserving $R_{\pm1}$, i.e., a rotation of 180° around an axis in the xy plane, we should have the linear combinations $D^{T}_{MK} \pm D^{T}_{M\overline{K}}$. As a consequence, the expectation values of the operator T_0 should vanish, which is not consistent with Eqs. (38). The situation is analogous to the semiclassical limit for the motion of a particle in a symmetric double well, where approximations based on small oscillations around one minima yield good results. The $R_{\pm1}$ symmetry is of course restored by the exponentially small tunelling between two solutions.

The fluctuations of the direction of rotation in the laboratory frame, i.e., the wobbling motion, are represented by the ξ^{+}, ξ degree of freedom.

We also expect some sort of signature to exist, which should be associated with the remaining point symmetries under the operators $R_q = \exp[i\pi(\tau_q - T_q)]$. In fact, the Hamiltonian $H^{(20)+(11)} - \langle\lambda_v\rangle\tau_v^{(20)+(11)}$ determining the unperturbed basic set of states is invariant under the R_a and R_0 transformations or, equivalently, under separate R_v transformations. This expresses the well known conservation of the parity in the number of particles for the case of pairing between identical particles. If we assume an even number of protons and of neutrons in the g.s. band, application of the transformations $R_{p(n)}$ to this

band yields

$$R_{p(n)}|A,T,M,n_{\perp\iota}=0,n_{\diamond\iota\mu}=0\rangle=\exp\left[i\pi\left(T-\frac{1}{2}A\right)\right]|A,T,M,0,0\rangle, \tag{39}$$

and the selection rule for the g.s. band is

$$\begin{aligned}&1=(-1)^{T-\frac{1}{2}A},\\ &T=\frac{1}{2}A,\ \frac{1}{2}A-2,\ldots.\end{aligned} \tag{40}$$

The transformation of the boson excitation operators gives

$$\begin{aligned}&R_v\gamma^+_{vj}R^+_v=\gamma^+_{vj},\\ &R_v\gamma^+_{\perp j}R^+_v=-\gamma^+_{\perp j},\\ &R_v\gamma^+_{\diamond j_1j_2\mu}R^+_v=-\gamma^+_{\diamond j_1j_2\mu}.\end{aligned} \tag{41}$$

Therefore we obtain the selection rule

$$1=(-1)^{\sum_\iota(n_{\perp\iota}+n_{\diamond\iota\mu})+T-\frac{1}{2}A}, \tag{42}$$

which is verified by the spectrum shown in Fig. 1.

3.3.6. The isospin mixing terms

The isospin mixing terms are included in the second and third lines of $H_\perp$ (Eq. (22)). They conserve the number of particles A and the projection M in the laboratory frame, since the product operator $\beta^2\xi^+$ increases both the value of T and m by the same amount.

The product operator $\beta^4\xi^{+2}$ mixes the ground state of a nucleus having isospin $T-2$ and projection $M=2-T$ with the double I.A.S. with isospin T and the same projection. It is proportional to the isoquadrupole strength g_2. The corresponding matrix elements are labelled by ϕ_c in Fig. 1(a). They are due to the Hamiltonian

$$\begin{aligned}&H^{\text{mixing}}_A=-\phi_c\left(\beta^4\xi^{+2}+\beta^{-4}\xi^2\right),\\ &\phi_c=g_2\frac{3}{T}\langle S_p\rangle\langle S_n\rangle.\end{aligned} \tag{43}$$

According to (84), the operator $\zeta^{(20)}$ is the sum of two terms, $\zeta^{(20)}_f=\zeta_{fj}\gamma^+_{\perp j}$ and $\zeta^{(20)}_b=\zeta_{bj}\gamma_{\perp j}$, creating and destroying two quasiparticles, respectively. Therefore the term in the third line of $H_\perp$ splits into two contributions

$$\begin{aligned}H^{\text{mixing}}_B&=-\beta^2\xi^+\zeta^{+(20)}-\beta^{-2}\xi\zeta^{(20)}\\ &=-\left(\beta^2\xi^+\gamma_{\perp j}+\beta^{-2}\xi\gamma^+_{\perp j}\right)\zeta_{fj}-\left(\beta^2\xi^+\gamma^+_{\perp j}+\beta^{-2}\xi\gamma_{\perp j}\right)\zeta_{bj}\\ &=-\phi_{f\iota}\left(\beta^2\xi^+\Gamma_{\perp\iota}+\beta^{-2}\xi\Gamma^+_{\perp\iota}\right)-\phi_{b\iota}\left(\beta^2\xi^+\Gamma^+_{\perp\iota}+\beta^{-2}\xi\Gamma_{\perp\iota}\right),\\ \phi_{f\iota}&=\lambda_\iota j\zeta_{bj}+\mu_{\iota j}\zeta_{fj},\qquad \phi_{b\iota}=\lambda_{\iota j}\zeta_{fj}+\mu_{\iota j}\zeta_{bj}.\end{aligned} \tag{44}$$

The first term in the r.h.s. of the second line of Eq. (44) annihilates (creates) two quasiparticles while simultaneously increases (decreases) both the isospin and the number

of ξ-bosons. The second term in the same line increases (decreases) the number of quasiparticles, the isospin and the number of ξ-bosons. The existence of both types of terms is preserved if the operators $\zeta^{(20)}_{f,b}$ are expressed in terms of the final RPA boson operators $\Gamma^{+}_{\perp\iota}$, $\Gamma_{\perp\iota}$ ($\iota = 2, 3, \ldots$). Therefore, the operator $\beta^2\xi^{+}\Gamma_{\perp\iota}$ mixes the antianalogue states with the analogue state, in the neighbor odd–odd nucleus. The matrix elements associated with these terms are labelled by the quantities $\phi_{b\iota}$, and $\phi_{f\iota}$ in Fig. 1(a). The non-conserving isospin interaction H_B^{mixing} not only depends on the isoquadrupole strength, but also on the isodipole terms, since they both arise from the single-particle and the pairing contributions.

The naive RPA defines phonons which carry a mixture of isospin values, leading to unpredictable consequences. On the contrary, we have constructed a basic set of states carrying the isospin T as good quantum number, and the isospin mixing terms act within this basic set. Therefore, we have been able to disentangle the proper isospin mixing terms from the spurious ones through the application of the collective formalism.

3.3.7. The spurious intrinsic sector

While the operators $\gamma^{+(20)}_{\perp j}$ constitute a complete set of operators creating two quasiparticles coupled to zero angular momentum, the set of operators κ^{+}_{j} is not complete. According to its definition (82) it should be supplemented by a phonon related to the isospin operators $\tau_{\pm 1}$, which are orthogonal to the κ^{+}_{j}'s. Thus, it is natural to choose

$$\Gamma^{+}_{\perp 1} = \frac{1}{\sqrt{T}}\,\tau^{(20)}_{1}, \qquad \Gamma_{\perp 1} = -\frac{1}{\sqrt{T}}\,\tau^{(20)}_{\bar{1}}, \tag{45}$$

and therefore

$$\lambda_{1j} = -\sqrt{\frac{2j+1}{2T}}\,U_{pj}V_{nj}, \qquad \mu_{1j} = \sqrt{\frac{2j+1}{2T}}\,V_{pj}U_{nj}. \tag{46}$$

The non-vanishing commutation of the Hamiltonian with $\Gamma^{+}_{\perp 1}$ proceeds through the last term[6] in (22)

$$H^{\text{spurious}} = -\frac{1}{2\mathcal{I}_{\perp}}\left[\tau^{(20)}_{1}, \tau^{(20)}_{\bar{1}}\right]_{+} = \omega_{\perp 1}\left(\Gamma^{+}_{\perp 1}\Gamma_{\perp 1} + \frac{1}{2}\right),$$
$$\omega_{\perp 1} = \frac{T}{\mathcal{I}_{\perp}} = -\langle\lambda_0\rangle \tag{47}$$

(c.f. Eqs. (24)). Although the term in H^{spurious} plays a similar role as the RPA rotational term in (32), there is a difference given by the fact that the frequency $\omega_{\perp 1}$ in (47) is finite and it has a physical meaning. The inclusion of the H^{spurious} within the spurious sector is deferred to Section 4.

[6] The origin of this term may be traced back to the Lagrange multiplier term introduced in (23). It is straightforward to verify that (23) yields the commutation (47).

4. The BRST formalism

4.1. The BRST charge and Hamiltonian

So far the overcompleteness inherent to a description in terms of particle and collective degrees of freedom has been taken care through the assumption that the constrains (1) or, more precisely, their quadratic version (13), hold. If this is the case, the quadratic approximation to the Hamiltonian (13) is given by (22). The reader may be satisfied with the knowledge that physical states are annihilated by (1), in particular the vacuum state. He may perform many calculations as those presented in the Section 5. On the other hand, a deeper understanding of the problem, as well as more complicated calculations involving, for instance, higher orders of perturbation theory, requires the content of the present section.

A gauge theory has an underlying invariance under transformations generated by the charge $\mathcal{Q}$. This is a hermitian and nilpotent operator that is linear in the constrains and which includes information about the group of gauge transformations (1), through the presence of the structure constants c_{klj}

$$\begin{aligned} Q &= \bar{\pi}_v B_v - \eta_k(\tau_k - T_k) + \frac{i}{2} c_{klj}\eta_k\eta_l\pi_j \\ &= \bar{\pi}_v B_v - \eta_v(\tau_v - T_v) + \eta_1(\tau_{\bar{1}} - T_{\bar{1}}) + \eta_{\bar{1}}(\tau_1 - T_1) \\ &\quad + \frac{1}{2}\eta_{\bar{1}}\eta_1(\pi_n - \pi_p) + (\pi_{\bar{1}}\eta_1 - \pi_1\eta_{\bar{1}})(\eta_n - \eta_p). \end{aligned} \tag{48}$$

In addition to the constant term $\langle\lambda_v\rangle$, already introduced in (23), the Lagrange multipliers λ_v include a boson component λ'_v

$$\lambda_v = \langle\lambda_v\rangle + \lambda'_v. \tag{49}$$

The operators B_v in (48) are conjugate to the λ'_v. The charge $\mathcal{Q}$ also contains fermion ghost operators η_k, $\bar{\pi}_v$ with conjugate momenta π_k, $\bar{\eta}_v$,

$$\begin{aligned} &-i[\lambda'_v, B_w] = [\bar{\eta}_v, \bar{\pi}_w]_+ = [\eta_v, \pi_w]_+ = \delta_{vw}, \\ &[\eta_1, \pi_{\bar{1}}]_+ = [\eta_{\bar{1}}, \pi_1]_+ = -1. \end{aligned} \tag{50}$$

The constrains are automatically taken into account by operating within the subspace carrying zero charge. However, this subspace of states consists of physical states plus states $|\chi\rangle = Q|\text{unphysical}\rangle$, having zero-norm. Both physical operators and operators mapping physical states into zero-norm states, henceforth nil operators $\mathcal{O}_\chi$, commute with the charge $\mathcal{Q}$. Consequently, there are families of equivalent states and equivalent operators, namely

$$|\text{physical}\rangle \to |\text{physical}\rangle + |\chi\rangle, \qquad \mathcal{O}_{\text{physical}} \to \mathcal{O}_{\text{physical}} + \mathcal{O}_\chi, \tag{51}$$

which yield the same matrix elements as physical operators between physical states.

The strategy is to add to the Hamiltonian a convenient nil operator that simplifies the treatment of the spurious sector. In particular, the operator obtained by anticommuting an

arbitrary operator ρ—the "gauge fixing function"—with the charge $\mathcal{Q}$ is a nil operator. We may choose as gauge fixing function[7]

$$\rho = \lambda_v \pi_v + \omega_v^2 \bar{\eta}_v \left(\theta_v - \frac{1}{2\mathcal{I}_v} B_v \right), \tag{52}$$

where $\theta_v = \theta_v^{(20)}$ are the RPA angles conjugate to $\tau_v^{(20)}$. They are obtained, together with the moment of inertia $\mathcal{I}_v$, by applying the Marshalek–Weneser prescription [29]. The parameters ω_v are arbitrary and should disappear from any physical expression. The gauge fixing function (52) privileges the degrees of freedom associated with the quantum numbers v. The idea underlying this gauge is that, because of the finite value of the frequencies $\omega_{\perp 1}$, one may use straightforwardly the RPA solutions corresponding to the $\perp$-degrees of freedom, without worrying in this case about infrared divergencies. This choice facilitates the BRST treatment of the $\perp$-sector.

Using (52) we may construct the nil operator to be added to the original Hamiltonian H in order to obtain the (equivalent) BRST Hamiltonian.

$$\begin{aligned} H_{\mathrm{BRST}} &= H + [\rho, Q]_+ \\ &= H + i\pi_v \bar{\pi}_v - \lambda_v(\tau_v - T_v) + \omega_v^2 \left(B_v \theta_v^{(20)} - \frac{1}{2\mathcal{I}_v} B_v^2 \right) \\ &\quad + \omega_v^2 \eta_w \bar{\eta}_v \left[\theta_v^{(20)}, \tau_w \right] + \omega_v^2 \bar{\eta}_v \left(\eta_1 \left[\theta_v^{(20)}, \tau_{\bar{1}} \right] + \eta_{\bar{1}} \left[\theta_v^{(20)}, \tau_1 \right] \right) \\ &\quad - \lambda_0 (\pi_{\bar{1}} \eta_1 - \pi_1 \eta_{\bar{1}}). \end{aligned} \tag{53}$$

4.2. The quadratic BRST Hamiltonian

We consider first the Lagrange multiplier terms $-\lambda_v(\tau_v - T_v)$, where λ_v is given in Eq. (49). The differences $\langle \tau_v \rangle - \langle T_v \rangle$ vanish by construction and the terms $-\langle \lambda_v \rangle \tau_v^{(20)+(11)}$ have been already introduced in the single-particle terms of the Hamiltonian (22). Thus the two remaining quadratic contributions of the Lagrange multiplier term are (c.f. Eq. (17))

$$\langle \lambda_0 \rangle \varsigma^+ \varsigma - \lambda'_v \tau_v^{(20)}. \tag{54}$$

With the choice of θ_v as the RPA angles,

$$\begin{aligned} \left[\theta_v^{(20)}, \tau_w \right] &= i\delta_{vw} + \left[\theta_v^{(20)}, \tau_w \right]^{(r)}, \\ \left[\theta_v^{(20)}, \tau_{\pm 1} \right] &= \left[\theta_v^{(20)}, \tau_{\pm 1} \right]^{(r)}, \end{aligned} \tag{55}$$

where the contribution of the terms labeled by (r) is smaller than unity ($\leqslant O(T^{-1/2})$). Therefore the last but one line in the Hamiltonian (53) only contributes through the first term to the quadratic Hamiltonian, giving rise to the term $i\omega_v^2 \eta_v \bar{\eta}_v$.

The quadratic BRST Hamiltonian is distributed into the two branches, $\parallel$ and $\perp$. By adding the $\parallel$-terms to $H_{\parallel}^{\mathrm{RPA}}$ (Eq. (32)) we obtain

[7] There is a large latitude in the determination of the gauge fixing function. In previous treatments of the cranking model, we have used a more complicated ρ, including also the components $\theta_{\pm 1}$ [32]. The gauge fixing function used in this paper was suggested by J.P. Garrahan [33]. To our knowledge, it has not been applied before.

$$
\begin{aligned}
&H^{\mathrm{RPA}}_{\parallel,\mathrm{BRST}} \\
&\quad = \omega_{v\iota}\left(\Gamma^{+}_{v\iota}\Gamma_{v\iota} + \frac{1}{2}\right) + \frac{1}{2\mathcal{I}_v}\left(\tau^{(20)}_v\right)^2 \\
&\qquad - \lambda'_v\tau^{(20)}_v + \omega^2_v\left(B_v\theta^{(20)}_v - \frac{1}{2\mathcal{I}_v}B^2_v\right) + i\left(\pi_v\bar{\pi}_v + \omega^2_v\eta_v\bar{\eta}_v\right) \\
&\quad = \omega_{v\iota}\left(\Gamma^{+}_{v\iota}\Gamma_{v\iota} + \frac{1}{2}\right) + \omega_v\left(\Gamma^{+}_{v1}\Gamma_{v1} - \Gamma^{+}_{v0}\Gamma_{v0} + \bar{a}_v a_v + \bar{b}_v b_v\right), \quad \iota = 2, 3, \ldots . \qquad (56)
\end{aligned}
$$

Thus this Hamiltonian includes a real sector, corresponding to the finite-frequency RPA bosons, and a supersymmetric spurious sector. The zero-frequency term proportional terms $(\tau^{(20)}_v)^2$ are incorporated into this spurious sector, which is diagonalized in Appendix G. The following commutation relations are satisfied[8]

$$
\left[\Gamma_{v1}, \Gamma^{+}_{v1}\right] = \left[\Gamma^{+}_{v0}, \Gamma_{v0}\right] = [\bar{a}_v, a_v]_+ = [\bar{b}_v, b_v]_+ = 1. \qquad (57)
$$

The vacuum state satisfies the conditions

$$
\Gamma_{v\iota}|0\rangle = \Gamma_{v1}|0\rangle = \Gamma_{v0}|0\rangle = a_v|0\rangle = b_v|0\rangle = 0. \qquad (58)
$$

It is a physical state, unlike the excited states of the spurious sector.

We consider now the $\perp$-branch. It includes the contribution (47).

$$
\begin{aligned}
H^{\mathrm{RPA}}_{\perp,\mathrm{BRST}} \;=\; & \omega_\xi\left(\xi^{+}\xi + \frac{1}{2}\right) + \omega_{\perp\iota}\left(\Gamma^{+}_{\perp\iota}\Gamma_{\perp\iota} + \frac{1}{2}\right) \\
& - \frac{1}{2\mathcal{I}_\perp}\left[\tau^{(20)}_1, \tau^{(20)}_{\bar{1}}\right]_+ + \langle\lambda_0\rangle\left(\varsigma^{+}\varsigma + \pi_1\eta_{\bar{1}} - \pi_{\bar{1}}\eta_1\right). \qquad (59)
\end{aligned}
$$

Making use of the transformation

$$
\begin{aligned}
&\pi_1 = i\bar{a}_\perp, \qquad \pi_{\bar{1}} = b_\perp, \qquad \eta_1 = -\bar{b}_\perp, \qquad \eta_{\bar{1}} = i a_\perp, \\
&[\bar{a}_\perp, a_\perp]_+ = [\bar{b}_\perp, b_\perp]_+ = 1, \qquad (60)
\end{aligned}
$$

we obtain

$$
\begin{aligned}
H^{\mathrm{RPA}}_{\perp,\mathrm{BRST}} \;=\; & \omega_\xi\left(\xi^{+}\xi + \frac{1}{2}\right) + \omega_{\perp\iota}\left(\Gamma^{+}_{\perp\iota}\Gamma_{\perp\iota} - \frac{1}{2}\right) \\
& + \omega_{\perp 1}\left(\Gamma^{+}_{\perp 1}\Gamma_{\perp 1} - \varsigma^{+}\varsigma + \bar{a}_\perp a_\perp + \bar{b}_\perp b_\perp - \frac{1}{2}\right). \qquad (61)
\end{aligned}
$$

The quadratic term $-\omega_{\perp 1}\varsigma^{+}\varsigma$ amounts to a negative energy boson. This leads to the (at least cumbersome) situation of a degenerate vacuum state. The problem may be eliminated through the transformation

$$
\begin{aligned}
&\Gamma^{+}_{\perp 0} = -\varsigma, \\
&\left[\Gamma_{\perp 0}, \Gamma^{+}_{\perp 0}\right] = -1, \qquad (62)
\end{aligned}
$$

[8] This is the same case as in electromagnetism, where an indefinite metric associated with the Lagrange multiplier is used.

at the expense of working again with the indefinite metric (62) (cf. Eq. (57)). The final quadratic Hamiltonian has the form

$$H^{\mathrm{RPA}}_{\perp,\mathrm{BRST}} = \omega_\xi\left(\xi^+\xi+\frac{1}{2}\right)+\omega_{\perp\iota}\left(\Gamma^+_{\perp\iota}\Gamma_{\perp\iota}+\frac{1}{2}\right) + \omega_{\perp 1}\left(\Gamma^+_{\perp 1}\Gamma_{\perp 1}-\Gamma^+_{\perp 0}\Gamma_{\perp 0}+\bar{a}_\perp a_\perp+\bar{b}_\perp b_\perp+\frac{1}{2}\right). \tag{63}$$

The vacuum state has been redefined through the transformations (60) and (62). It is annihilated by the operators

$$\Gamma_{\perp\iota}|0\rangle=\xi|0\rangle=\Gamma_{\perp 1}|0\rangle=\Gamma_{\perp 0}|0\rangle=a_\perp|0\rangle=b_\perp|0\rangle=0. \tag{64}$$

Using the previous transformations, the leading (quadratic) terms in the charge associated with the $\perp$-degrees of freedom are given by

$$Q^{(2)}_\perp = -i\sqrt{\mathcal{I}_\perp}\,a_\perp\left(\Gamma^+_{\perp 1}+\Gamma^+_{\perp 0}\right)+\sqrt{\mathcal{I}_\perp}\,\bar{b}_\perp(\Gamma_{\perp 1}+\Gamma_{\perp 0}), \tag{65}$$

which clearly annihilates the vacuum state, according to (64).

The spurious spectrum is constructed by repeated applications of the spurious supersymmetric quartet of creation operators $\Gamma^+_{\perp 1}$, $\Gamma^+_{\perp 0}$, $\bar{a}_\perp$, $\bar{b}_\perp$

$$\begin{aligned}|\mathrm{spurious}\rangle &= |n_{\perp 1}\rangle|n_{\perp 0}\rangle|n_{\perp a}\rangle|n_{\perp b}\rangle \\ &= \prod_{d=0,1}\frac{(\Gamma^+_{\perp d})^{n_{\perp d}}}{\sqrt{n_{\perp d}!}}\prod_{c=a,b}(\bar{c}_\perp)^{n_{\perp c}}|0\rangle,\end{aligned} \tag{66}$$

with $n_{\perp d}=0,1,2,\ldots$, and $n_{\perp c}=0,1$. It may be shown that all the states (66) are either unphysical or nil states, but the vacuum state (64) (see Ref. [16]). Therefore, the word *spurious* has a definite meaning here.

We note the similitude between the spurious sector of the Hamiltonian (63) and those associated with either protons or neutrons in the $\|$-branch (Eq. (56)). There are also some differences, consisting on the fact that the frequencies ω_v are arbitrary (and thus should not appear in any physical result), while the frequency $\omega_{\perp 1}$ has physical meaning, and on the existence of a vacuum energy $\frac{1}{2}\omega_{\perp 1}$ in (63). In fact, this energy is incorporated to the rotational energy in Eq. (37). The treatment beyond the RPA order is discussed in Appendix H.

5. Double-beta decay transitions

5.1. Fermi double-beta decay transitions

The Fermi operator is written

$$\beta^{(F-)}=\sqrt{2}\,\tau_1. \tag{67}$$

As we proceeded in the case of the Hamiltonian, we must transform the operator $\beta^{(F-)}$ to the intrinsic frame

$$\begin{aligned}\beta^{(F-)} &\to \sqrt{2}\left(D^1_{11}\tau_1 + D^1_{10}\tau_0 + D^1_{1(-1)}\tau_{-1}\right)\\ &= \sqrt{2}\,D^1_{10}\langle\tau_0\rangle + \text{nil operator} + O\left(T^{-1/2}\right)\\ &= -\sqrt{2T}\,\xi^+ + \text{nil operator} + O\left(T^{-1/2}\right).\end{aligned} \tag{68}$$

This is another instance where, as a result of the renormalization procedure, the badly-behaved operator τ_1 is replaced by the well-behaved collective operator ξ^+ creating the sequence of analogue states. The allowed Fermi transitions, for single-beta decay virtual transitions, are represented in Fig. 1(b).

The Fermi double-beta decay requires the existence in the Hamiltonian of terms that do not preserve the isobaric symmetry. Thus, the interactions discussed in Subsection 3.3.6 are needed. Table 1 displays the matrix elements of the mixing Hamiltonian (44) between one-phonon states in the intermediate odd–odd nucleus with the isospin projection $M = -T + 1$, consistently with the selection rule (42). Table 2 shows the corresponding matrix elements between zero and two-phonon states in the $M = -T + 2$ (final) system. They are extracted from Eqs. (43) and (44). The unperturbed energies are obtained from expression (37). The quantum numbers shown in the tables are $(T, n_\xi, n_{\perp_\iota})$.

After the diagonalization, the states in the intermediate system are labelled by ι', with energies $\omega_{\iota'}$. Subsequently, we obtain the beta-decay amplitudes $M^{(F)}_{T\,\text{g.s.};\iota'}$ $(|T\,\text{g.s.}\rangle \to |\iota'\rangle)$ and $M^{(F)}_{\iota';(T-2)\,\text{g.s.}}$ $(|\iota'\rangle \to |(T-2)\,\text{g.s.}\rangle)$ for the transitions involved in the double-beta decay through the application of the operator (68), satisfying the selection rule $\Delta n_\xi = \pm 1$ between unperturbed states. The matrix element of the allowed double-Fermi decay, in the two-neutrino channel, is written

$$M^{(F)}_{2\nu} = \frac{M^{(F)}_{T\,\text{g.s.};\iota'} M^{(F)}_{\iota';(T-2)\,\text{g.s.}}}{\Delta + \omega_{\iota'}}, \tag{69}$$

where Δ is the energy released during the decay [5].

Table 1
The matrix elements of the Hamiltonian corresponding to the states carrying $M = -T + 1$

	$T-1,0,1$	$T,1,0$
$T-1,0,1$	$\omega_{\perp_\iota}$	$-\phi_{f_\iota}$
$T,1,0$	$-\phi_{f_\iota}$	$\omega_\xi + 2\kappa T$

Table 2
The matrix elements of the Hamiltonian corresponding to the states carrying $M = -T + 2$

	$T-2,0,0$	$T-1,1,1$	$T,2,0$
$T-2,0,0$	0	$-\phi_{b\iota}$	$-\sqrt{2}\phi_c$
$T-1,1,1$	$-\phi_{b\iota}$	$\omega_\xi + \omega_{\perp_\iota} + 2\kappa(T-1)$	$-\sqrt{2}\phi_{f_\iota}$
$T,2,0$	$-\sqrt{2}\phi_c$	$-\sqrt{2}\phi_{f_\iota}$	$2\omega_\xi + \kappa(4T-2)$

5.2. Schematic models

We center the discussion of this section on the results presented in Ref. [13], where the case of a single j-shell has been assumed. Since in this model antianalogue states are not present, the interaction H_B^{mixing} does not enter in the calculation. The values given in Table 3 are the parameters used in the calculations of Ref. [13], together with the relation $g = g_n = g_p$.

The Fig. 1 of Ref. [13] displays the energy ω_ξ of the I.A.S. (Eq. (24)), as a function of g_2/g. It is seen that the point where ω_ξ vanishes and the point where the isospin symmetry is restored are different. This exact result cannot be reproduced by other approximations. The amplitude M_1 of the Fermi transition to the intermediate I.A.S. $(T, 1, 0)$ and the amplitude M_2 between the I.A.S. and the final state $(T-2, 0, 0)$ are displayed in the lower boxes of the Fig. 1 of Ref. [13]. As it has been shown there, the matrix element M_2 is proportional to the admixture of the double I.A.S. $(T, 2, 0)$ in the final state. Such admixtures have been obtained in perturbation theory by using the matrix elements given in Table 2. Fig. 2 of Ref. [13], shows Fermi double-beta decay matrix elements. In addition to the exact and collective values of these matrix elements, we have included the results

Table 3
The value of the quantum numbers j, A, T and of the parameters ϵ_0, g used in the schematic calculations. The value of the isorotational parameter κ is fixed at $\kappa = 0$ for all cases

j	A	T	ϵ_0	g
9/2	10	3	0.8 MeV	0.4 MeV
19/2	20	4	0.63 MeV	0.2 MeV

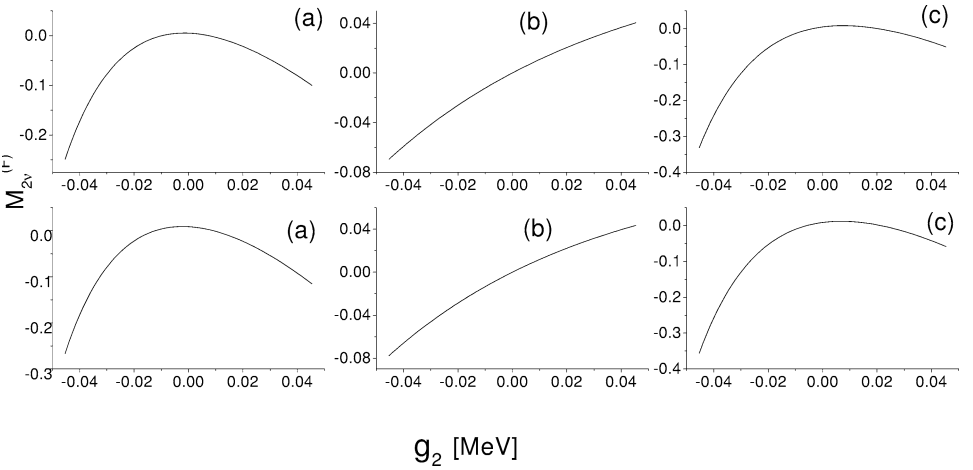

Fig. 2. Matrix elements for allowed double Fermi transitions, as a function of the coupling constant g_2. The upper three boxes correspond to $g_1 = 0$, while the lower ones to $g_1 = 0.026$ MeV. Insets (a) are obtained with $\phi_c = 0$; (b) with $\phi_{bl} = \phi_{fl} = 0$ and (c) with all matrix elements different from zero. The matrix elements are given in units of MeV^{-1}.

obtained by means of other approximations. As expected, *provided T is conserved*, the exact results show the suppression of the matrix elements around the point $g_2 = 0$. The result is reproduced by the naive RPA (QRPA) and by the collective approach, *but not* by the so-called renormalized RPA (RQRPA). On the other side, the naive RPA cannot be extended beyond the unstable region of small g_2 values. Ikeda's sum rule is also reproduced by the collective approach and by the QRPA, but not by the RQRPA.

5.3. Realistic calculations

We have performed a realistic calculation for the two-neutrino double-beta-decay mode in ^{76}Ge. Single-particle energies are obtained from the parametrization of the Wood–Saxon potential recommended in [19], including the proton electrostatic field. The valence energy levels are listed in Appendix I. The values Δ_v, $\langle\lambda_v\rangle$ are also given in Appendix I, for each value of the proton or neutron strength parameter g_v that is used.

The calculation has been performed for two values of g_1, i.e., $g_1 = 0$ MeV and $g_1 = 0.026$ MeV. In correspondence to the first value we have taken $g_n = g_p = 0.289$ MeV; while for the second one we have taken $g_n = 0.289$ MeV and $g_p = 0.342$ MeV, so that the proton gap becomes slightly larger than the neutron one. The isorotational parameter has the value $\kappa = 1$ MeV. The Fermi amplitudes are represented as a function of the isoquadrupole strength g_2. The results of the calculations are shown in Fig. 2. We note that:

(i) both interactions H_A^{mixing} and H_B^{mixing} yield contributions which are of similar orders of magnitude, with the same sign if $g_2 < 0$ and with the opposite sign if $g_2 > 0$;
(ii) the interaction H_A^{mixing} is mainly responsible for the amplitude through the intermediate I.A.S. state, while the one labelled by H_B^{mixing} yields the amplitudes that proceed via the intermediate antianalogue states. This can be easily seen by diagonalizing the matrices of Tables 1 and 2 in perturbation theory (c.f. Appendix J);
(iii) although the contribution via the interaction H_B^{mixing} predominates, its effects are only due to the splitting between intermediate states, since they would be cancelled for intermediate degenerate states (c.f. Appendix J). As a consequence of this cancellation the predicted values tend to be suppressed;
(iv) estimates of the magnitude of g_2 are given in Appendix K. They range in the region $0 \leqslant g_2 \leqslant 0.007$ MeV. Within this region the curves (c) of Fig. 2 display an additional cancellation because the contributions (a) and (b) add incoherently (c.f. (i)). As a consequence, the results are stable and too small to be of significance in the $2\nu\beta\beta$ processes;
(v) dipole effects are also negligible for realistic differences between neutron and proton Hamiltonians.

5.4. Gamow–Teller double-beta decay transitions

In the case of allowed Gamow–Teller transitions, the operator is defined as

$$\beta_q^{(\mathrm{GT}-)} = \sigma_{1q} \equiv \frac{1}{\sqrt{3}} \sum_l \langle j_1 \| \sigma \| j_2 \rangle \left[c^+_{pj_1} c_{nj_2} \right]^1_q, \tag{70}$$

where $\langle j_1 \| \sigma \| j_2 \rangle$ is the reduced matrix element of the spin operator. The transformation to the intrinsic system, leads to

$$\begin{aligned}\beta_q^{(\mathrm{GT}-)} &\to D^1_{11}\sigma_{1q} + D^1_{10}\sigma_{0q} + D^1_{1(-1)}\sigma_{(-1)q} \\ &= D^1_{11}\sigma_{1q} + O\big(T^{-1/2}\big) \\ &= \beta^{-2}\sigma_{1q} + O\big(T^{-1/2}\big). \end{aligned} \tag{71}$$

Each time that a phonon is created or destroyed through the operator $\sigma^{(20)}_{1q}$, the operator β^{-2} decreases the isospin by one unit. This is consistent with the selection rules of Eq. (42). Moreover, we may elaborate the $^{(20)}$ terms of the Gamow–Teller operator

$$\begin{aligned}\sigma^{(20)}_{1q} &= -\frac{\langle j_1 \| \sigma \| j_2 \rangle}{\sqrt{3}} \left(U_{pj_1} V_{nj_2} \gamma^+_{\diamond j_1 j_2 q} + (-1)^q V_{pj_1} U_{nj2} \gamma_{\diamond j_1 j_2 (-q)} \right) \\ &= \sigma_{f\iota} \Gamma^+_{\diamond \iota q} + (-1)^q \sigma_{b\iota} \Gamma_{\diamond \iota(-q)}, \end{aligned} \tag{72}$$

where

$$\begin{aligned}\sigma_{f\iota} &= M^{\diamond}_{T\,\mathrm{g.s.};(T-1)\iota q} = -\frac{\langle j_1 \| \sigma \| j_2 \rangle}{\sqrt{3}} \left(U_{pj_1} V_{nj_2} \lambda_{\iota j_1 j_2} + V_{pj_1} U_{nj_2} \mu_{\iota j_1 j_2} \right), \\ \sigma_{b\iota} &= M^{\diamond}_{(T-1)\iota q;(T-2)\,\mathrm{g.s.}} = -\frac{\langle j_1 \| \sigma \| j_2 \rangle}{\sqrt{3}} \left(V_{pj_1} U_{nj_2} \lambda_{\iota j_1 j_2} + U_{pj_1} V_{nj_2} \mu_{\iota j_1 j_2} \right). \end{aligned} \tag{73}$$

The amplitudes $\lambda_{\iota j_1 j_2}$ and $\mu_{\iota j_1 j_2}$ are obtained within the RPA, as discussed in Appendix F.

Therefore, the double Gamow–Teller matrix element contributing to the two-neutrino mode is written

$$M^{(\mathrm{GT})}_{2\nu} = \frac{(-1)^q M^{\diamond}_{T\,\mathrm{g.s.};(T-1)\iota q} M^{\diamond}_{(T-1)\iota(-q);(T-2)\,\mathrm{g.s.}}}{\Delta + \omega_{\diamond\iota}} = \frac{3\sigma_{f\iota}\sigma_{b\iota}}{\Delta + \omega_{\diamond\iota}}. \tag{74}$$

For the sake of comparison, we show in Fig. 3 the values of the matrix element (74) as a function of the isoscalar pairing interaction g_d (see Appendix F). The strength of the

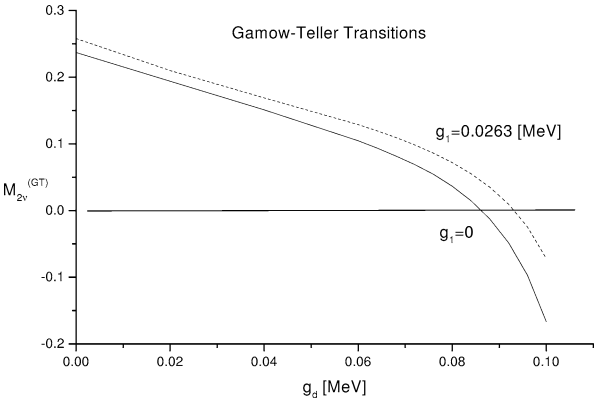

Fig. 3. Matrix elements for allowed double Gamow–Teller g.s. to g.s. transitions in ^{76}Ge, as a function of $g_\diamond$ (see the text) and for two values of g_1. The matrix elements are given in units of MeV^{-1}.

isospin-spin interaction was fixed at $\chi = 0.030$ MeV for the case $g_d = 0$ and changed as g_d increases so that the energy weighted sum rule

$$\left[S^+_{\diamond q}, [H, S_{\diamond q}]\right] = \omega_{\diamond\iota}\left(\sigma^2_{f\iota} + \sigma^2_{bJ}\right) \tag{75}$$

stays constant.

Since the results are an order of magnitude larger than those of the Fermi contributions, the present comparison confirms the dominance of the allowed Gamow–Teller transitions in the amplitude for the two-neutrino double-beta decay channel.

In this discussion of the matrix elements of the two-neutrino double-beta decay process we have disregarded the effects associated with the lack of overlap between the initial and final states.

6. Conclusions

We have presented in detail a suitable formalism for the description of proton–neutron excitations in superfluid nuclei in which, at first sight, pairing effects are included only between identical particles. However, this violation of the symmetry is only performed in the intrinsic frame. We have shown that the introduction of the overcomplete set of intrinsic and collective variables constitutes an adequate tool to treat such excitations, provided the appropriate constraints are simultaneously taken into account.

Briefly, one transforms the Hamiltonian to the moving frame and simplifies the subsequent expression by adding nil contributions. At the quadratic approximation, one obtains two set of real excitations: (a) the parallel excitations, involving identical quasiparticles, which have been previously treated within the BRST framework; and (b) perpendicular excitations, involving proton–neutron configurations, which are the central topic of the present contribution. The resulting states carry the number of particles A, the isospin T and its laboratory projection M, as good quantum numbers. They are also labelled by the number of different well-behaved physical bosons associated with the I.A.S. band, with the antianalogue states and with states carrying different symmetries. Moreover, matrix elements of the Hamiltonian between states differing in one and two units of T are explicitly obtained. Thus, the formalism is able to disentangle the physical effects arising from the lack of symmetry of the original Hamiltonian from those unphysical effects originated from the application of the BCS formalism for identical particles.

The spurious effects appear concentrated in the Hamiltonian on a term depending on the isospin components $\tau_{\pm 1}$. We may ignore this term if we are satisfied with calculations to leading order, for instance, with those presented in this paper concerning allowed double-Fermi and double-Gamow–Teller beta-decay transitions. Nevertheless, it is also possible to transfer such an isospin dependent term to the spurious sector via a rigorous procedure based on the BRST invariance. The BRST spurious sector may be expressed as a supersymmetric system of two bosons and two fermions, which is very similar to the one appearing in the BRST treatment of pairing interactions between identical particles. Higher order corrections would demand the inclusion of all these spurious sectors in the mathematical manipulations.

Within the physical sector, the badly-behaved operators $\tau_{\pm 1}^{(20)}$ become systematically replaced by the well-behaved operators ξ^+, ξ, creating and destroying the states within the I.A.S. band. This is apparent in the expression for the Fermi β-decay operator, after being transformed to the intrinsic frame.

The results concerning the value of the I.A.S. frequency ω_ξ, the β-decay matrix elements and the $2\nu\beta\beta$ amplitudes have been checked against the predictions of an exact calculation. These last ones are very well reproduced. This should not be a surprise, since our calculation is exact, within a perturbation framework. This result is at variance with those of other, often arbitrary, modifications of the RPA, which have appeared in the literature.

Realistic calculations have been made, displaying also a dependence with the isoquadrupole strength g_2. The empirical value of this parameter yields an upper limit for the contribution of the Fermi transitions to the two-neutrino double-beta decay processes. Moreover, the existence of at least two cancellations further decrease the predicted amplitude. Thus, the final results in ^{76}Ge are (as expected) too small to be of significance, both relative to the empirical value and to the Gamow–Teller contribution.

Acknowledgements

This work was supported in part by the CONICET through the Carrera del Investigador Científico; by the Fundacion Antorchas and by the University Favaloro (proyecto 010/99). Contribution of N.N. Scoccola to some of the above presented results is gratefully acknowledged.

Appendix A. Notation and definitions

The operators S_{pj}^+, S_{nj}^+ and $S_{\perp j}^+$ create a proton pair, a neutron pair and a proton–neutron pair. In all cases the pairs are created in time-reversed states $(m, \tilde{m})$

$$S_{vj}^+ = \sum_{m>0} c_{vjm}^+ c_{vj\tilde{m}}^+, \qquad S_{\perp j}^+ = \sum_{m>0} \left(c_{pjm}^+ c_{nj\tilde{m}}^+ + c_{njm}^+ c_{pj\tilde{m}}^+\right). \tag{76}$$

Moreover,

$$\tau_k = \sum_j \tau_{kj}, \qquad S_\rho^+ = \sum_j S_{\rho j}^+, \tag{77}$$

with $k = a, q$; $q = 0, \pm 1$, and $\rho = v, \perp$.

We use the quasiparticle operators

$$\begin{aligned}
&\nu_{vj} = \alpha_{vjm}^+ \alpha_{vjm}, \qquad \nu_{vwj} = \alpha_{wjm}^+ \alpha_{vjm}, \\
&\gamma_{vj}^+ = \sqrt{\frac{2}{2j+1}} \sum_{m>0} \alpha_{vjm}^+ \alpha_{vj\tilde{m}}^+, \\
&\gamma_{\perp j}^+ = \frac{1}{\sqrt{2j+1}} \sum_{m>0} \left(\alpha_{njm}^+ \alpha_{pj\tilde{m}}^+ + \alpha_{pjm}^+ \alpha_{nj\tilde{m}}^+\right).
\end{aligned} \tag{78}$$

The operators (3), (4), (76) may be written in terms of the operators (78)

$$\begin{aligned}
&\tau_{vj}^{(00)} = V_{vj}^2(2j+1), && \tau_1^{(00)} = 0,\\
&S_{vj}^{(00)} = \left(j+\frac{1}{2}\right)U_{vj}V_{vj}, && S_{\perp j}^{(00)} = 0,\\
&\tau_{vj}^{(11)} = \left(U_{vj}^2 - V_{vj}^2\right)\nu_{vj},\\
&\tau_{1j}^{(11)} = -\frac{1}{\sqrt{2}}\left(U_{nj}U_{pj}\nu_{pnj} - V_{nj}V_{pj}\nu_{pnj}\right),\\
&S_{vj}^{+(11)} = -U_{vj}V_{vj}\nu_{vj},\\
&S_{\perp j}^{+(11)} = -U_{pj}V_{nj}\nu_{pnj} - U_{nj}V_{pj}\nu_{pnj},
\end{aligned} \tag{79}$$

$$\begin{aligned}
&\tau_{vj}^{(20)} = 2\sqrt{j+\frac{1}{2}}\,U_{vj}V_{vj}\left(\gamma_{vj}^{+} + \gamma_{vj}\right),\\
&\tau_{1j}^{(20)} = -\sqrt{j+\frac{1}{2}}\left(U_{pj}V_{nj}\gamma_{\perp j}^{+} + V_{pj}U_{nj}\gamma_{\perp j}\right),\\
&S_{vj}^{+(20)} = \sqrt{j+\frac{1}{2}}\left(U_{vj}^2\gamma_{vj}^{+} - V_{vj}^2\gamma_{vj}\right),\\
&S_{\perp j}^{+(20)} = \sqrt{2j+1}\left(U_{nj}U_{pj}\gamma_{\perp j}^{+} - V_{nj}V_{pj}\gamma_{\perp j}\right).
\end{aligned} \tag{80}$$

$$\tag{81}$$

It is convenient to extract the terms which are proportional to $\tau_{\pm1}^{(20)}$, from the operators $\gamma_{\perp j}^{+}$, $\tau_{\pm1j}^{(20)}$ and $S_{\perp j}^{+(20)}$. Using (6) we obtain

$$\begin{aligned}
\kappa_j^{+(20)} &= \gamma_{\perp j}^{+} + \frac{\sqrt{j+1/2}}{T}\left(U_{pj}V_{nj}\tau_1^{(20)} + V_{pj}U_{nj}\tau_{\bar{1}}^{(20)}\right)\\
&= \gamma_{\perp j}^{+} - \frac{\sqrt{(j+1/2)(j'+1/2)}}{T}\Big(\left(U_{pj}V_{nj}U_{pj'}V_{nj'} - V_{pj}U_{nj}V_{pj'}U_{nj'}\right)\gamma_{\perp j'}^{+}\\
&\quad + \left(U_{pj}V_{nj}V_{pj'}U_{nj'} - V_{pj}U_{nj}U_{pj'}V_{nj'}\right)\gamma_{\perp j}\Big),\\
t_{1j}^{(20)} &= \tau_{1j}^{(20)} + \frac{\langle\tau_{0j}\rangle}{T}\,\tau_1^{(20)}\\
&= -\left(\delta_{j\,j'} + \frac{\langle\tau_{0j}\rangle}{T}\right)\left(U_{pj'}V_{nj'}\gamma_{\perp j'}^{+} + V_{pj'}U_{nj'}\gamma_{\perp j'}\right),\\
s_j^{+(20)} &= S_{\perp j}^{+(20)} + \frac{\sqrt{2}}{T}\left(\langle S_{nj}\rangle\tau_1^{(20)} + \langle S_{pj}\rangle\tau_{\bar{1}}^{(20)}\right)\\
&= \sqrt{2j'+1}\left(U_{nj}U_{pj}\delta_{j\,j'} - \frac{\langle S_{nj}\rangle}{T}\,U_{pj'}V_{nj'} + \frac{\langle S_{pj}\rangle}{T}\,V_{pj'}U_{nj'}\right)\gamma_{\perp j'}^{+}\\
&\quad - \sqrt{2j'+1}\left(V_{nj}V_{pj}\delta_{j\,j'} + \frac{\langle S_{nj}\rangle}{T}\,V_{pj'}U_{nj'} - \frac{\langle S_{pj}\rangle}{T}\,U_{pj'}V_{nj'}\right)\gamma_{\perp j'},
\end{aligned} \tag{82}$$

where

$$\left[\kappa_j^{+},\tau_{\pm1}\right]^{(00)} = \left[t_{1j},\tau_{\pm1}\right]^{(00)} = \left[s_j^{+},\tau_{\pm1}\right]^{(00)} = 0. \tag{83}$$

We define the operators $h^{(20)}$, $s^{+(20)}$ and $\zeta^{(20)}$

$$h^{(20)} = h_f^{(20)} + h_b^{(20)}, \qquad s^{(20)} = s_f^{(20)} + s_b^{(20)},$$
$$\zeta^{(20)} = \zeta_f^{(20)} + \zeta_b^{(20)}, \tag{84}$$

where

$$h_f^{(20)} = -\left(\epsilon_{0j} + \frac{1}{T}\,\epsilon_{0j'}\langle\tau_{0j'}\rangle\right)\sqrt{2j+1}\,U_{pj}V_{nj}\gamma_j^+,$$
$$h_b^{(20)} = -\left(\epsilon_{0j} + \frac{1}{T}\,\epsilon_{0j'}\langle\tau_{0j'}\rangle\right)\sqrt{2j+1}\,V_{pj}U_{nj}\gamma_j,$$
$$s_f^{+(20)} = \sqrt{2j+1}\left(-V_{pj}V_{nj} - \frac{\langle S_n\rangle}{T}\,V_{pj}U_{nj} + \frac{\langle S_p\rangle}{T}\,U_{pj}V_{nj}\right)\gamma_j,$$
$$s_b^{+(20)} = \sqrt{2j+1}\left(U_{pj}U_{nj} - \frac{\langle S_n\rangle}{T}\,U_{pj}V_{nj} + \frac{\langle S_p\rangle}{T}\,V_{pj}U_{nj}\right)\gamma_j^+,$$
$$\zeta_f^{(20)} = \zeta_{fj}\gamma_{\perp j}^+$$
$$= h_f^{(20)} + (g_1+3g_2)\langle S_p\rangle s_b^{+(20)} + (g_1-3g_2)\langle S_n\rangle s_f^{(20)},$$
$$\zeta_b^{(20)} = \zeta_{bj}\gamma_{\perp j}$$
$$= h_b^{(20)} + (g_1+3g_2)\langle S_p\rangle s_f^{+(20)} + (g_1-3g_2)\langle S_n\rangle s_b^{(20)}. \tag{85}$$

Appendix B. Irreducible isotensor operators

In this appendix we give the irreducible isotensor operators $\mathcal{O}_{\lambda\mu}$ constructed from the pairing operators S_p^+, S_n^+ and $S_\perp^+$, and their hermitian conjugate expressions.

The irreducible operators S_p^+, S_n^+ and $\frac{1}{2}S_\perp^+$ carry isospin $\lambda = 1$ and projections $\mu = 1$, -1 and 0. Therefore, the product of two of them is a linear combination of terms with isospin $\lambda = 0, 1$ and 2. By taking a linear combination of the products indicated in Table 4 one may easily verified that only the indicated value of λ survives.

Table 4
Irreducible isotensor operators

λ,μ	$\mathcal{O}_{\lambda\mu}$	λ,μ	$\mathcal{O}_{\lambda\mu}$
0, 0	$S_p^+S_p + S_n^+S_n + \frac{1}{2}S_\perp^+S_\perp$	2, 2	$\sqrt{6}\,S^+pS_n$
		2, 1	$\sqrt{3/2}\,(S_\perp^+S_n - S_p^+S_\perp)$
1, 1	$-1/\sqrt{2}\,(S_p^+S_\perp + S_\perp^+S_n)$	2, 0	$S_p^+S_p + S_n^+S_n - S_\perp^+S_\perp$
1, 0	$S_p^+S_p - S_n^+S_n$	2, −1	$\sqrt{3/2}\,(S_\perp^+S_p - S_n^+S_\perp)$
1, −1	$1/\sqrt{2}\,(S_n^+S_\perp + S_\perp^+S_p)$	2, −2	$\sqrt{6}\,S_n^+S_p$

Appendix C. The collective operators

In this appendix we briefly present a derivation of the relations used to express the collective operators in terms of creation and annihilation operators. In general

$$\begin{aligned} D^{\lambda}_{\mu,\delta} D^{T}_{-T+m,-T+k} &= \sum_{|s|\leqslant\lambda} \langle \lambda,\mu;T,-T+m|T+s,-T+m+\mu\rangle \\ &\quad\times \langle \lambda,\delta;T,-T+k|T+s,-T+k+\delta\rangle D^{T+s}_{-T+m+\mu,-T+k+\delta}. \end{aligned} \tag{86}$$

Using the Holstein–Primakoff representation of the rotational wave functions as extended by Marshalek, we can express the SU(2)-Wigner functions as

$$\begin{aligned} D^{T+s}_{-T+m+\mu,-T+k+\delta} &= \sqrt{\frac{2T+1}{2T+2s+1}}\sqrt{\frac{m!}{(m+s+\mu)!}}\sqrt{\frac{k!}{(k+s+\delta)!}} \\ &\quad\times \beta^{2s}\left(\xi^{+}\right)^{s+\mu}\left(\varsigma^{+}\right)^{s+\delta} D^{T}_{-T+m,-T+k}. \end{aligned} \tag{87}$$

Therefore, we can identify

$$\begin{aligned} D^{\lambda}_{\mu,\delta} &= \sum_{|s|\leqslant\lambda} \sqrt{\frac{2T+1}{2T+2s+1}}\sqrt{\frac{m!}{(m+s+\mu)!}}\sqrt{\frac{k!}{(k+s+\delta)!}} \\ &\quad\times \langle \lambda,\mu;T,-T+m|T+s,-T+m+\mu\rangle \\ &\quad\times \langle \lambda,\delta;T,-T+k|T+s,-T+k+\delta\rangle \beta^{2s}\left(\xi^{\dagger}\right)^{s+\mu}\left(\varsigma^{\dagger}\right)^{s+\delta}. \end{aligned} \tag{88}$$

Expanding this relation in powers of $1/T$ for the cases of interest, we obtain:

(a) Hamiltonian dipole operators

$$\begin{aligned} D^{(1)}_{0,1} &= \left(\varsigma^{\dagger} - \beta^{-2}\xi\right)T^{-1/2} + \mathcal{O}\left(T^{-3/2}\right), \\ D^{(1)}_{0,0} &= 1 - \left(1 + \varsigma^{\dagger}\varsigma + \xi^{\dagger}\xi - \beta^{-2}\varsigma\xi - \beta^{2}\varsigma^{\dagger}\xi^{\dagger}\right)T^{-1} + \mathcal{O}\left(T^{-2}\right), \\ D^{(1)}_{0,\bar{1}} &= -\left(\varsigma - \beta^{2}\xi^{+}\right)T^{-1/2} + \mathcal{O}\left(T^{-3/2}\right). \end{aligned} \tag{89}$$

(b) Hamiltonian quadrupole operators

$$\begin{aligned} D^{(2)}_{0,2} &= \sqrt{\frac{3}{2}}\left(\left(\varsigma^{\dagger}\right)^{2} - 2\beta^{-2}\varsigma^{\dagger}\xi + \beta^{-4}\xi^{2}\right)T^{-1} + \mathcal{O}\left(T^{-2}\right), \\ D^{(2)}_{0,1} &= \sqrt{3}\left(\varsigma^{\dagger} - \beta^{-2}\xi\right)T^{-1/2} + \mathcal{O}\left(T^{-3/2}\right), \\ D^{(2)}_{0,0} &= 1 - 3\left(1 + \varsigma^{\dagger}\varsigma + \xi^{\dagger}\xi - \beta^{-2}\varsigma\xi - \beta^{2}\varsigma^{\dagger}\xi^{\dagger}\right)T^{-1} + \mathcal{O}\left(T^{-2}\right), \\ D^{(2)}_{0,\bar{1}} &= -\sqrt{3}\left(\varsigma - \beta^{2}\xi^{\dagger}\right)T^{-1/2} + \mathcal{O}\left(T^{-3/2}\right), \\ D^{(2)}_{0,\bar{2}} &= \sqrt{\frac{3}{2}}\left(\varsigma^{2} - 2\beta^{2}\varsigma\xi^{\dagger} + \beta^{4}\left(\xi^{\dagger}\right)^{2}\right)T^{-1} + \mathcal{O}\left(T^{-2}\right). \end{aligned} \tag{90}$$

(c) Transition dipole operators

$$
\begin{aligned}
D^{(1)}_{1,1} &= \beta^{-2} - \left(\beta^{-2}\left(1 + \varsigma^{\dagger}\varsigma + \xi^{\dagger}\xi\right) - \varsigma^{\dagger}\xi^{\dagger}\right)T^{-1} + \mathcal{O}\left(T^{-2}\right),\\
D^{(1)}_{1,0} &= \left(\xi^{\dagger} - \beta^{-2}\varsigma\right)T^{-1/2} + \mathcal{O}\left(T^{-3/2}\right),\\
D^{(1)}_{1,\tilde{1}} &= \frac{1}{2}\left(\beta^{-2}\varsigma^{2} - 2\varsigma\xi^{\dagger} + \beta^{2}\left(\xi^{\dagger}\right)^{2}\right)T^{-1} + \mathcal{O}\left(T^{-2}\right).
\end{aligned}
\tag{91}
$$

Appendix D. The expansion of the different contributions to the Hamiltonian (13)

The terms are ordered according to the combined powers of $1/\langle S_v\rangle$ and $1/T$. The largest ones are included as expectation values ($O(T)$); the following ones carry the superindex (20) and are proportional to one-body operators creating and destroying two quasiparticles ($O(\sqrt{T})$); the remaining terms include the (11) component of one-body operators and/or two interacting quasibosons[9] ($O(T^0)$).

The first subindex denotes the origin of the term, i.e., whether it arises from the single-particle or the pairing Hamiltonian. The second subindex denotes the isotensor order. Whenever a third subindex is present, it indicates the number of operators $\tau_{\pm 1}$ appearing in it.

Each term is correct up to a (null) contribution which is proportional to the operators $(\varsigma + \tau^{(20)}_1/\sqrt{T})$ or $(\varsigma^{+} - \tau^{(20)}_{\tilde{1}}/\sqrt{T})$ acting on the vacuum state $|0\rangle$.

To start with, we consider the single-particle isoscalar term

$$
\begin{aligned}
&H_{\mathrm{sp},0} = e_{aj}\tau_{aj},\\
&\langle H_{\mathrm{sp},0}\rangle = e_{aj}\langle\tau_{aj}\rangle,\\
&H^{(20)}_{\mathrm{sp},0} = e_{aj}\tau^{(20)}_{aj},\\
&H_{\mathrm{sp},0,0} = e_{aj}\tau^{(11)}_{aj}.
\end{aligned}
\tag{92}
$$

This term includes contributions from the single-particle Hamiltonian (7) and from the Lagrange multiplier term (23).

The following are the isovector terms of the single-particle Hamiltonian

$$
\begin{aligned}
&H_{\mathrm{sp},1} = \epsilon_{0j}\tau_{qj}D^{1}_{0q} - \langle\lambda_0\rangle\tau_0 = e_{0j}\tau_{qj}D^{1}_{0q} + \langle\lambda_0\rangle\left(\tau_q D^{1}_{0q} - \tau_0\right),\\
&\langle H_{\mathrm{sp},1}\rangle = e_{0j}\langle\tau_{0j}\rangle,\\
&H^{(20)}_{\mathrm{sp},1} = e_{0j}\tau^{(20)}_{0j},\\
&H_{\mathrm{sp},1,0} = e_{0j}\tau^{(11)}_{0j} - \frac{\epsilon_{0j}\langle\tau_{0j}\rangle}{T}\xi^{+}\xi - \frac{1}{\sqrt{2T}}\left(\beta^{2}\xi^{+}h^{+(20)} + \beta^{-2}\xi h^{(20)}\right),\\
&H_{\mathrm{sp},1,1} = \frac{1}{T}\left(\tau^{(20)}_{\tilde{1}}t^{(20)} - \tau^{(20)}_{1}t^{+(20)}\right),\\
&H_{\mathrm{sp},1,2} = -\frac{\epsilon_{0j}\langle\tau_{0j}\rangle}{2T^{2}}\left[\tau^{(20)}_{1}, \tau^{(20)}_{\tilde{1}}\right]_{+}.
\end{aligned}
\tag{93}
$$

[9] The procedure also yields constant terms of ($O(T^0)$), which are not included, since there are other contributions of the same order.

As said before, these terms include the contribution from the single-particle Hamiltonian (7) and from the Lagrange multiplier term (23). The operator $h^{(20)}$ is defined in (84).

The pairing Hamiltonian gives the isoscalar terms

$$
\begin{aligned}
H_{\text{pair},0} &= -g_0\biggl(S_p^+ S_p + S_n^+ S_n + \frac{1}{2}\,S_\perp^+ S_\perp\biggr),\\
\langle H_{\text{pair},0}\rangle &= -g_0\bigl(\langle S_p\rangle^2 + \langle S_n\rangle^2\bigr),\\
H_{\text{pair},0}^{(20)} &= -g_0\bigl(\langle S_p\rangle\bigl(S_p^{+(20)} + S_p^{(20)}\bigr) + \langle S_n\rangle\bigl(S_n^{+(20)} + S_n^{(20)}\bigr)\bigr),\\
H_{\text{pair},0,0} &= -g_0\biggl(\langle S_p\rangle\bigl(S_p^{+(11)} + S_p^{(11)}\bigr) + \langle S_n\rangle\bigl(S_n^{+(11)} + S_n^{(11)}\bigr)\\
&\qquad + S_p^{+(20)} S_p^{(20)} + S_n^{+(20)} S_n^{(20)} + \frac{1}{2}\,s^{+(20)} s^{(20)}\biggr),\\
H_{\text{pair},0,1} &= -g_0\,\frac{1}{T\sqrt{2}}\biggl(\langle S_n\rangle\bigl(s^{+(20)}\tau_{\bar{1}}^{(20)} - s^{(20)}\tau_1^{(20)}\bigr)\\
&\qquad + \langle S_p\rangle\bigl(s^{+(20)}\tau_1^{(20)} - s^{(20)}\tau_{\bar{1}}^{(20)}\bigr)\biggr),\\
H_{\text{pair},0,2} &= g_0\,\frac{1}{2T^2}\bigl(\bigl(\langle S_p\rangle^2 + \langle S_n\rangle^2\bigr)\bigl[\tau_{\bar{1}}^{(20)}, \tau_1^{(20)}\bigr]_+ 2\langle S_n\rangle\langle S_p\rangle\bigl(\bigl(\tau_1^{(20)}\bigr)^2 + \bigl(\tau_{\bar{1}}^{(20)}\bigr)^2\bigr)\bigr).
\end{aligned}
\tag{94}
$$

The isovector pairing terms are

$$
\begin{aligned}
H_{\text{pair},1} &= -g_1\biggl(D_{00}^1\bigl(S_p^+ S_p - S_n^+ S_n\bigr) - \frac{1}{\sqrt{2}}\,D_{01}^1\bigl(S_p^+ S_\perp + S_\perp^+ S_n\bigr)\\
&\qquad + \frac{1}{\sqrt{2}}\,D_{0\bar{1}}^1\bigl(S_n^+ S_\perp + S_\perp^+ S_p\bigr)\biggr),\\
\langle H_{\text{pair},1}\rangle &= -g_1\bigl(\langle S_p\rangle^2 - \langle S_n\rangle^2\bigr),\\
H_{\text{pair},1}^{(20)} &= -g_1\bigl(\langle S_p\rangle\bigl(S_p^{+(20)} + S_p^{(20)}\bigr) - \langle S_n\rangle\bigl(S_n^{+(20)} + S_n^{(20)}\bigr)\bigr),\\
H_{\text{pair},1,0} &= -g_1\biggl(\langle S_p\rangle\bigl(S_p^{+(11)} + S_p^{(11)}\bigr) - \langle S_n\rangle\bigl(S_n^{+(11)} + S_n^{(11)}\bigr) - \frac{1}{T}\,\bigl(\langle S_p\rangle^2 - \langle S_n\rangle^2\bigr)\\
&\qquad + S_p^{+(20)} S_p^{(20)} - S_n^{+(20)} S_n^{(20)} + \frac{1}{\sqrt{2T}}\langle S_p\rangle\bigl(\beta^{-2}\xi s^{(20)} + \beta^2\xi^+ s^{+(20)}\bigr)\\
&\qquad + \frac{1}{\sqrt{2T}}\,\langle S_n\rangle\bigl(\beta^{-2}\xi s^{+(20)} + \beta^2\xi^+ s^{(20)}\bigr)\biggr),\\
H_{\text{pair},1,1} &= g_1\,\frac{1}{T\sqrt{2}}\,\bigl(\langle S_n\rangle\bigl(s^{+(20)}\tau_{\bar{1}}^{(20)} - s^{(20)}\tau_1^{(20)}\bigr)\langle S_p\rangle\bigl(s^{(20)}\tau_{\bar{1}}^{(20)} - s^{+(20)}\tau_1^{(20)}\bigr)\bigr),\\
H_{\text{pair},1,2} &= g_1\,\frac{1}{2T^2}\,\bigl(\langle S_p\rangle^2 - \langle S_n\rangle^2\bigr)\bigl[\tau_1^{(20)}, \tau_{\bar{1}}^{(20)}\bigr]_+.
\end{aligned}
\tag{95}
$$

Finally, the isoquadrupole pairing interaction yields

$$
\begin{aligned}
H_{\text{pair},2} &= -g_2\Big(D^2_{00}\big(S^+_p S_p + S^+_n S_n - S^+_\perp S_\perp\big) + \sqrt{6}\,\big(D^2_{02} S^+_p S_n + D^2_{0\bar{2}} S^+_n S_p\big) \\
&\qquad + \sqrt{\frac{3}{2}}\, D^2_{01}\big(S^+_\perp S_n - S^+_p S_\perp\big) + \sqrt{\frac{3}{2}}\, D^2_{0\bar{1}}\big(S^+_\perp S_p - S^+_n S_\perp\big)\Big), \\
\langle H_{\text{pair},2}\rangle &= -g_2\big(\langle S_p\rangle^2 + \langle S_n\rangle^2\big), \\
H_{\text{pair},2} &= -g_2\big(\langle S_p\rangle\big(S^{+(20)}_p + S^{(20)}_p\big) + \langle S_n\rangle\big(S^{+(20)}_n + S^{(20)}_n\big)\big), \\
H_{\text{pair},2,0} &= -g_2\Big(\langle S_p\rangle\big(S^{+(11)}_p + S^{(11)}_p\big) + \langle S_n\rangle\big(S^{+(11)}_n + S^{(11)}_n\big) \\
&\qquad + S^{+(20)}_p S^{(20)}_p + S^{+(20)}_n S^{(20)}_n - s^{+(20)} s^{(20)} - \frac{3}{T}\big(\langle S_p\rangle^2 + \langle S_n\rangle^2\big)\xi^+\xi \\
&\qquad + \frac{3}{T}\langle S_p\rangle\langle S_n\rangle\big(\beta^{-4}\xi^2 + \beta^4\xi^{+2}\big) + \frac{3}{\sqrt{2T}}\langle S_p\rangle\big(\beta^{-2}\xi s^{(20)} + \beta^2\xi^+ s^{+(20)}\big) \\
&\qquad - \frac{3}{\sqrt{2T}}\langle S_n\rangle\big(\beta^{-2}\xi s^{+(20)} + \beta^2\xi^+ s^{(20)}\big)\Big), \\
H_{\text{pair},2,1} &= -g_2\frac{1}{T\sqrt{2}}\big(\langle S_n\rangle\big(s^{+(20)}\tau^{(20)}_{\bar{1}} - s^{(20)}\tau^{(20)}_{\bar{1}}\big) + \langle S_p\rangle\big(s^{+(20)}\tau^{(20)}_1 + s^{(20)}\tau^{(20)}_{\bar{1}}\big)\big), \\
H_{\text{pair},2,2} &= g_2\frac{1}{T^2}\Big(\langle S_n\rangle\langle S_p\rangle\big(\big(\tau^{(20)}_1\big)^2 + \big(\tau^{(20)}_{\bar{1}}\big)^2\big) + \frac{1}{2}\big(\langle S_p\rangle^2 + \langle S_n\rangle^2\big)\big[\tau^{(20)}_1, \tau^{(20)}_{\bar{1}}\big]_+\Big).
\end{aligned}
\tag{96}
$$

As for the quadratic terms in the case of pairing between identical particles, we include a contribution arising from the independent quasiparticle term, which may be written as $H'^{(11)} = E_{vj}\nu_{vj} \approx (E_{nj} + E_{pj})\gamma^+_j\gamma_j$, which yields

$$
\begin{aligned}
H_{\text{qp},\perp,0} &= (E_{pj} + E_{nj})\kappa^+_j\kappa_j, \\
H_{\text{qp},\perp,1} &= \frac{\sqrt{j+1/2}}{T}\,(E_{pj} + E_{nj})\Big(U_{pj}V_{nj}\big(\kappa^+_j\tau^{(20)}_{\bar{1}} - \kappa_j\tau^{(20)}_1\big) \\
&\qquad + V_{pj}U_{nj}\big(\kappa^+_j\tau^{(20)}_1 - \kappa_j\tau^{(20)}_{\bar{1}}\big)\Big) \\
&= -\frac{1}{T}\big(t^{(20)}\tau^{(20)}_{\bar{1}} - t^{+(20)}\tau^{(20)}_1\big) \\
&\quad + (g_0 + g_2)\frac{1}{T\sqrt{2}}\Big(\langle S_p\rangle\big(s^{+(20)}\tau^{(20)}_1 - s^{(20)}\tau^{(20)}_{\bar{1}}\big) \\
&\qquad + \langle S_n\rangle\big(s^{+(20)}\tau^{(20)}_{\bar{1}} - s^{(20)}\tau^{(20)}_1\big)\Big) \\
&\quad + g_1\frac{1}{T\sqrt{2}}\Big(\langle S_p\rangle\big(s^{+(20)}\tau^{(20)}_1 - s^{(20)}\tau^{(20)}_{\bar{1}}\big) \\
&\qquad + \langle S_n\rangle\big(s^{(20)}\tau^{(20)}_1 - s^{+(20)}\tau^{(20)}_{\bar{1}}\big)\Big),
\end{aligned}
$$

$$H_{\mathrm{qp},\perp,2} = -\frac{2j+1}{4T^2}(E_{pj}+E_{nj})\Big(2U_{pj}V_{pj}U_{nj}V_{pj}\big(\big(\tau_1^{(20)}\big)^2+\big(\tau_{\bar{1}}^{(20)}\big)^2\big) + \big(U_{pj}^2V_{nj}^2+U_{nj}^2V_{pj}^2\big)\big[\tau_1^{(20)},\tau_{\bar{1}}^{(20)}\big]_+\Big)$$
$$= \frac{1}{2T^2}\big(e_{0j}\langle\tau_{0j}\rangle-(g_0+g_2)\big(\langle S_p\rangle^2+\langle S_n\rangle^2\big) - g_1\big(\langle S_p\rangle^2-\langle S_n\rangle^2\big)\big)\big[\tau_1^{(20)},\tau_{\bar{1}}^{(20)}\big]_+ - \frac{1}{T^2}(g_0+g_2)\langle S_p\rangle\langle S_n\rangle\big(\big(\tau_1^{(20)}\big)^2+\big(\tau_{\bar{1}}^{(20)}\big)^2\big). \quad (97)$$

The final form of $H^{(2)}_{\mathrm{qp},\perp,1}$ and $H^{(2)}_{\mathrm{qp},\perp,2}$ are derived through conventional algebraic manipulations which involve the self-consistency conditions.

Appendix E. The rotational pairing energy

The self-consistent conditions (31) are obtainable from the minimization of the Hamiltonians

$$\delta\big\langle\big(H'-\langle\lambda_v\rangle\tau_v\big)\big\rangle = 0. \quad (98)$$

The expectation values of the Lagrange multipliers $\langle\lambda_v\rangle$ are such that

$$\langle\tau_p\rangle = Z, \qquad \langle\tau_n\rangle = N. \quad (99)$$

Thus, the invariance (98) yields

$$\langle\lambda_p\rangle = \frac{\partial}{\partial Z}\big\langle H'\big\rangle = \frac{Z}{\mathcal{I}_p^{(1)}}, \qquad \langle\lambda_n\rangle = \frac{\partial}{\partial N}\big\langle H'\big\rangle = \frac{N}{\mathcal{I}_n^{(1)}}, \quad (100)$$

defining the moments of inertia $\mathcal{I}_v^{(1)}$. Therefore, the expectation value of the Hamiltonian includes the rotational energies

$$\frac{1}{2}Z\langle\lambda_p\rangle+\frac{1}{2}N\langle\lambda_n\rangle = \frac{1}{2\mathcal{I}_p^{(1)}}Z^2+\frac{1}{2\mathcal{I}_n^{(1)}}N^2. \quad (101)$$

In addition, the BRST Hamiltonian (63) provides the term

$$\frac{1}{2}\omega_{\perp 1} = -\frac{1}{2}\langle\lambda_p\rangle+\frac{1}{2}\langle\lambda_n\rangle = -\frac{1}{2\mathcal{I}_p^{(1)}}Z+\frac{1}{2\mathcal{I}_n^{(1)}}N. \quad (102)$$

Adding (101) and (102) yields the rotational energy given in (37).

Appendix F. The RPA associated with the isoscalar pairing Hamiltonian

We here discuss some possible manifestations of an isoscalar pairing interaction within our formalism, although so far there has not appeared any positive evidence regarding the strength $g_\diamond$ of this force in medium and heavy mass nuclei (see Refs. [26,27]). In

addition, we also introduce a repulsive spin-isospin force, in order to keep the Gamow–Teller resonance at its empirical energy,

$$H_\diamond = -g_d S^+_{\diamond q} S_{\diamond q},$$
$$H_{\sigma\tau} = \chi \sigma^+_q \sigma_q, \tag{103}$$

where

$$S^+_{\diamond q} = \frac{1}{\sqrt{3}} \sum_l \langle j_1 \| \sigma \| j_2 \rangle \left[c^+_{pj_1} c^+_{nj_2} \right]^1_q,$$
$$\sigma^+_q = \frac{1}{\sqrt{3}} \sum_l \langle j_1 \| \sigma \| j_2 \rangle \left[c^+_{pj_1} c_{nj_2} \right]^1_q. \tag{104}$$

Within the RPA, the two-particle coupled creation operators and the particle–hole coupled operators yield

$$\left[c^+_{pj_1} c^+_{nj_2} \right]^{1(\mathrm{RPA})}_q = U_{pj_1} U_{nj_2} \gamma^+_{\diamond j_1 j_2 q} + (-1)^q V_{pj_1} V_{nj_2} \gamma_{\diamond j_1 j_2 (-q)},$$
$$\left[c^+_{pj_1} c_{nj_2} \right]^{1(\mathrm{RPA})}_q = -U_{pj_1} V_{nj_2} \gamma^+_{\diamond j_1 j_2 q} + (-1)^q V_{pj_1} U_{nj_2} \gamma_{\diamond j_1 j_2 (-q)}, \tag{105}$$

the coupled two-quasiparticle operators being defined as

$$\gamma^+_{\diamond j_1 j_2 q} = \left[\alpha^+_{pj_1} \alpha^+_{nj_2} \right]^1_q,$$
$$\left[\gamma_{\diamond j_1 j_2 q}, \gamma^+_{\diamond j'_1 j'_2 q'} \right] = \delta_{j_1 j'_1} \delta_{j_2, j'_2} \delta_{q q'}. \tag{106}$$

Since both $H_\diamond$ and $H_{\sigma\tau}$ are isoscalar operators that conserve the number of particles, their transformation to the intrinsic system is trivial. The RPA treatment of this Hamiltonian yields the uncoupled bosons $\Gamma^+_{\diamond \iota q}$ with frequencies $\omega_{\diamond \iota}$ and amplitudes $\lambda_{\iota j_1 j_2}$, $\mu_{\iota j_1 j_2}$. There is no problem here associated with spurious components, because these phonons carry a different symmetry from those originating the condensate. Since these phonons also have a single proton and a single neutron, the selection rule associated with them is the same as for the $\perp$-phonons discussed earlier (c.f. Eqs. (42))

$$\Gamma^+_{\diamond \iota q} = \sum_{j_1 j_2} \left(\lambda_{\iota j_1 j_2} \gamma^+_{\diamond j_1 j_2 q} - (-1)^q \mu_{\iota j_1 j_2} \gamma_{\diamond j_1 j_2 (-q)} \right),$$
$$\omega_{\diamond \iota} \Gamma^+_{\diamond \iota q} \Gamma_{\diamond \iota q} = H^{(11)} - \langle \lambda_v \rangle \tau^{(11)}_v + H^{\mathrm{RPA}}_\diamond + H^{\mathrm{RPA}}_{\sigma\tau}. \tag{107}$$

Appendix G. The diagonalization of the spurious sector in the $\|$-quadratic Hamiltonian

One applies the following transformations

$$\tau^{(20)}_v = \sqrt{\frac{\mathcal{I}_v \omega_v}{2}} \left(\Gamma^+_{v1} + \Gamma_{v1} + \Gamma^+_{v0} + \Gamma_{v0} \right),$$
$$\theta^{(20)}_v = -i \frac{1}{\sqrt{2\mathcal{I}_v \omega_v}} \left(\Gamma^+_{v1} - \Gamma_{v1} \right),$$

$$
\begin{aligned}
B_v &= -i\sqrt{\frac{\mathcal{I}_v}{2\omega_v}}\left(\Gamma^+_{v1} - \Gamma_{v1} + \Gamma^+_{v0} - \Gamma_{v0}\right),\\
\lambda'_v &= \sqrt{\frac{\omega_v}{2\mathcal{I}_v}}\left(\Gamma^+_{v0} + \Gamma_{v0}\right), \qquad (108)\\
\eta_v &= \frac{1}{\sqrt{2\omega_v}}\,(ia_v + \bar{b}_v),\\
\pi_v &= \sqrt{\frac{\omega_v}{2}}\,(b_v - i\bar{a}_v),\\
\bar{\eta}_v &= \frac{1}{\sqrt{2\omega_v}}\,(-ib_v + \bar{a}_v),\\
\bar{\pi}_v &= \sqrt{\frac{\omega_v}{2}}\,(a_v + i\bar{b}_v) \qquad (109)
\end{aligned}
$$

with the commutation relations given in (57). Note also that $a^+_v \neq \bar{a}_v$ and $b^+_v \neq \bar{b}_v$.

Appendix H. Beyond the RPA

Up to now we have treated the terms creating or destroying two quasiparticles in a somewhat cavalier way, since a term $O^{(20)}$ behaves like a pure boson term only to the RPA order. Beyond such order we should distinguish two contributions

$$O^{(20)} = O' + O^{(r)}, \qquad (110)$$

where O' is a pure single-boson term and the remaining part $O^{(r)}$ is at least $T^{-1/2}$ times smaller. As an example, the l.h.s. of the two first lines in Eqs. (108) should read τ'_v and θ'_v, respectively, $([\theta'_v, \tau'_v]) = i$. The explicit inclusion of both terms (110) in the BRST Hamiltonian (53) yields the residual Hamiltonian. For instance, from the BRST $[\rho, Q]_+$ contribution we obtain the terms of $\mathrm{O}(\leqslant T^{-1/2})$

$$
\begin{aligned}
[\rho, Q]_+ \;\rightarrow\; & -\lambda'_v\,\tau^{(11+r)}_w + \omega_v^2 B_v \theta^{(r)}_v + \lambda'_0\left(\xi^+\xi + \pi_{\bar{1}}\eta_{\bar{1}} - \pi_{\bar{1}}\eta_1\right)\\
& - \omega_v^2\bar{\eta}_v\left(\eta_w\left[\theta^{(r)}_v, \tau_w\right] + \eta_w l[\theta'_v, \tau^{(11+r)}_w\right] - \eta_1\left[\theta^{(20)}_v, \tau_{\bar{1}}\right]\\
& - \eta_{\bar{1}}\left[\theta^{(20)}_v, \tau_1\right]\Big). \qquad (111)
\end{aligned}
$$

Here, the boson operators λ'_v, λ'_0, B_v, ς must be replaced by their expression in terms of the final phonons (Eqs. (108) and (62)). Similarly the ghost operators are to be substituted according to Eqs. (109) and (60). The residual Hamiltonian should be treated with one of the well-known procedures to go beyond the RPA, such as the NFT [34] or the boson expansions [35]. Although care must be taken concerning the indefinite metric of the bosons Γ_{v0}, $\Gamma_{\perp 0}$, this is by no means a difficult complication. In fact, the two loop correction to the g.s. and one-phonon state has been worked in detail, for the case of pairing between identical particles, in Chapter 7 of Ref. [16]. In the present case of isospin pairing, it is straightforward, albeit cumbersome, to expand H while simultaneously being consistent with the order of magnitude of the terms that are retained. This program includes

the use of Eq. (16) and the expansion of the functions D^1 and D^2 to the appropriate order, following the procedure illustrated in Appendix C.

Appendix I. Parameters used in the realistic calculation

Table 5
The single-particle levels used in the calculation

n	l	j	ϵ_n [MeV]	ϵ_p [MeV]
1	1	3/2	−11.52	−6.79
0	3	5/2	−10.72	−9.01
0	3	7/2	−15.52	−8.21
2	0	1/2	−2.17	−13.97
1	2	3/2	−0.80	1.61
1	2	5/2	−3.30	3.07
0	4	7/2	−0.18	−0.27
0	4	9/2	−7.03	3.05
0	5	11/2	1.65	−5.31
0	5	9/2	5.00	7.00

Table 6
The strength of the interaction, the gap and the expectation value of the Lagrange multiplier for the relevant numbers of particles. The values are given in units of MeV

	$N=24$	$N=22$	$Z=12$	$Z=14$	$Z=12$	$Z=14$
g_v	$22/A$	$22/A$	$22/A$	$22/A$	$26/A$	$26/A$
Δ_v	2.18	2.07	1.61	1.82	2.24	2.52
$\langle\lambda_v\rangle$	−7.39	−8.08	−9.10	−8.53	−9.41	−8.75

Appendix J. Perturbation treatment of the Fermi amplitude

In this appendix we diagonalize the matrices of Tables 1 and 2 in perturbation theory. The change in the unperturbed wave functions corresponding to the states (36) is

$$\begin{aligned}
&\delta|T, n_\xi = 1, 0\rangle = -f_\iota|(T-1), 0, n_{\perp\iota} = 1\rangle,\\
&\delta|T-1, 0, n_{\perp\iota} = 1\rangle = f_\iota|T, n_\xi = 1, 0\rangle,\\
&\delta|T-2, \rangle = b_\iota|T-1, n_\xi = 1, n_{\perp\iota} = 1\rangle + c|T, n_\xi = 2, 0\rangle,
\end{aligned} \tag{112}$$

where the coefficients are given by

$$\begin{aligned}
f_\iota &= -\frac{\phi_{f\iota}}{\omega_{\perp\iota} - 2\kappa T - \omega_\xi},\\
b_\iota &= \frac{\phi_{b\iota}}{2\kappa(T-1) + \omega_\xi + \omega_{\perp\iota}},\\
c &= \frac{\phi_c}{\sqrt{2}\,T(\kappa(2T-1) + \omega_\xi)}.
\end{aligned} \tag{113}$$

Therefore the relevant transition matrix elements become

$$M^{(F)}_{T\text{g.s.;I.A.S.}} = \langle T,1,0|\beta^{(F-)}|T,0,0\rangle = -\sqrt{2T},$$
$$M^{(F)}_{T\text{g.s.;}(T-1),0,n_{\perp_\iota}} = \langle T-1,0,n_{\perp_\iota}=1|\beta^{(F-)}|T,0,0\rangle = -\sqrt{2T}\,f_\iota,$$
$$M^{(F)}_{\text{I.A.S.;}T-2,0,0} = \langle T-2,0,0|\beta^{(F-)}|\,\text{I.A.S.}\rangle = -\sqrt{2T}\,(c-f_\iota b_\iota),$$
$$M^{(F)}_{T-1,0,n_{\perp_\iota}=1;T-2,0,0} = \langle T-2,0,0|\beta^{(F-)}|T-1,0,1\rangle = -\sqrt{2T}\,b_\iota. \tag{114}$$

The double decay matrix element is written

$$M^{(F)}_{2\nu} = \frac{M^{(F)}_{T\text{g.s.;I.A.S.}} M^{(F)}_{\text{I.A.S.;}T-2,0,0}}{\Delta + E(\text{I.A.S.})} + \frac{M^{(F)}_{T\text{g.s.;}T-1,0,n_{\perp_\iota}} M^{(F)}_{T-1,0,n_{\perp_\iota};T-2,0,0}}{\Delta+\omega_{\perp_\iota}}. \tag{115}$$

Appendix K. The empirical value of the strength g_2

The present determination of the empirical value is based on the Isobaric Multiplet Mass Equation (IMME) [36], which expresses the energy of a given isomultiplet as

$$E(T,M) = a + bM + cM^2. \tag{116}$$

We must obtain the expectation value of the isoquadrupole Hamiltonian

$$H_2 = -g_2\big(S_p^2+S_n^2\big)D_{00}^2 \approx -g_2\left(\frac{\Delta}{g_0}\right)^2 D_{00}^2 \approx g_2\,\frac{A}{2}\,D_{00}^2, \tag{117}$$

where we have used the values $\Delta = 11\ \text{MeV}/\sqrt{A}$; $g_0 = 22\ \text{MeV}/A$, as in ([19]). Therefore, for $T=1$ states,

$$g_2 = \frac{3.3}{A}\,c_{\text{pairing}}. \tag{118}$$

The pairing contribution can be determined by subtracting the Coulomb contribution,

$$E_{\text{Coulomb}} = \frac{3}{5}\,\frac{Z^2e^2}{R_c} \to 0.70\,M^2\ \text{MeV}/A^{1/3}, \tag{119}$$

from the empirical value of cM^2 in 0^+ states, and ascribing the residual value of cM^2 to pairing effects. This would yield an upper limit for c_{pairing} (column 4 of Table 6). Alternatively, we note that the empirical value of the coefficient c is systematically larger for the $I^\pi = 0^+$ g.s., than for the $I^\pi \neq 0^+$ excited states in the same nucleus. Therefore we may attribute to pairing effects the difference in the value of c between the g.s. and the excited states. For each nucleus, the average value of the last one is listed in column 5 of Table 7 and the values of c_{pairing} thus obtained are given in the last column.

The two resultant values of g_2 are 538 keV/A and 236 keV/A (c.f. Eq. (118)), which are much too small to yield significant Fermi contributions to the double beta-decay process.

Table 7
Empirical values of the contribution from pairing effects to the coefficient c in the Eq. (116). All values are given in units of keV

Nucleus	$c_{\text{empirical}}$	E_{Coulomb}	c_{pairing}	$c_{\text{excited states}}$	c_{pairing}
He^6	335			266	69
Be^{10}	362	228	134	299	63
C^{14}	337	193	144	258	79
Ne^{18}	347	170	177	223	124
Ne^{22}	314	154	160	246	68
Al^{26}	302	141	160	228	74
Se^{30}	275	131	144	210	65
S^{34}	286	123	163	233	53
Ar^{38}	284	117	167	199	85
Ca^{42}	287	111	176	211	76
Ti^{46}	276	196	170		
Cr^{50}	259	102	157		
Fe^{54}	276	98	178		
Ni^{58}	260	94	166		
			163 ± 14		71 ± 19

References

[1] V.I. Tretryak, Yu.G. Zdesenko, At. Data. Nucl. Tables 61 (1995) 43.
[2] M.K. Moe, P. Vogel, Ann. Rev. Nucl. Part. Sci. 44 (1994) 247.
[3] S. Stoica, H.V. Klapdor-Kleingrothaus, Eur. Phys. J. A 9 (2000) 345.
[4] K. Zuber, Phys. Rep. 305 (1998) 295.
[5] M. Doi, T. Kotani, E. Takasugi, Prog. Theor. Phys. Suppl. 83 (1985) 1.
[6] H. Ejiri, Phys. Rep. 338 (2000) 265.
[7] J. Suhonen, O. Civitarese, Phys. Rep. 300 (1998) 123.
[8] J.D. Vergados, Phys. Rep. 111 (1986) 1.
[9] E. Caurier, F. Nowacki, A. Poves, J. Retamosa, Phys. Rev. Lett. 77 (1996) 1954.
[10] A. Faessler, F. Simkovic, Phys. Atom. Nucl. 63 (2000) 1165;
A. Faessler, F. Simkovic, J. Phys. G 24 (1998) 2139.
[11] O. Civitarese, A. Faessler, T. Tomoda, Phys. Lett. B 194 (1987) 11.
[12] O. Civitarese, P. Hess, J. Hirsch, Phys. Lett. B 412 (1997) 1.
[13] D.R. Bes, O. Civitarese, N.N. Scoccola, Phys. Lett. B 446 (1999) 93.
[14] C. Becchi et al., Phys. Lett. B 52 (1974) 344.
[15] M. Henneaux, C. Teitelboim, Quantization of Gauge Systems, Princeton Univ. Press, Princeton, NJ, 1992.
[16] D.R. Bes, J. Kurchan, The Treatment of Collective Coordinates in Many-Body Systems, World Scientific, Singapore, 1990.
[17] D.R. Bes, O. Civitarese, Am. J. Phys., 2002, in press.
[18] D.R. Bes, O. Civitarese, Phys. Rev. C 63 (2001) 044323.
[19] Aa. Bohr, B.R. Mottelson, Nuclear Structure, Vol. II, Benjamin, Massachusetts, 1975.
[20] D.R. Bes, R.A. Broglia, Nucl. Phys. A 80 (1966) 289.
[21] J. Ginocchio, J. Weneser, Phys. Rev. 170 (1968) 859.
[22] G.G. Dussel et al., Nucl. Phys. A 175 (1971) 513;
G.G. Dussel et al., Nucl. Phys. A 183 (1972) 298;
D.R. Bes et al., Nucl. Phys. A 217 (1973) 93.
[23] N. Auerbach, J. Hufner, A.K. Kerman, C.M. Shakin, Rev. Mod. Phys. 44 (1972) 481;
A.M. Lane, A.Z. Mekjian, Adv. Nucl. Phys. 7 (1973) 97.
[24] N. Auerbach, V. Bernard, N. van Giai, Nucl. Phys. A 337 (1980) 143.

[25] E.C.G. Sudarshan, N. Mukunda, Classical Dynamics. A Modern Perspective, Wiley, New York, 1974.
[26] P. Vogel, nucl-th/9805015;
O.A. Macchiavelli et al., nucl-th/9907121.
[27] D.R. Bes et al., Phys. Rev. C 61 (2000) 24315.
[28] E.R. Marshalek, Phys. Rev. C 11 (1975) 1426.
[29] E.R. Marshalek, J. Weneser, Phys. Rev. C 2 (1970) 1682.
[30] D.R. Bes et al., Phys. Rep. 34 (1977) 1.
[31] E.R. Marshalek, Nucl. Phys. A 275 (1977) 416;
E.R. Marshalek, Nucl. Phys. A 381 (1982) 240.
[32] J. Kurchan, D.R. Bes, S. Cruz Barrios, Nucl. Phys. A 509 (1990) 306;
J.P. Garrahan, D.R. Bes, Nucl. Phys. A 473 (1994) 448.
[33] J.P. Garrahan, private communication.
[34] D.R. Bes et al., Phys. Lett. B 52 (1974) 253;
D.R. Bes, Prog. Theor. Phys. Suppl. 74 (1983) 1.
[35] S.T. Belyaev, V.G. Zelevinski, Nucl. Phys. 39 (1962) 583;
E.R. Marshalek, Nucl. Phys. A 224 (1974) 221, 245.
[36] I. Britz, A. Pape, M.S. Antony, Nucl. Data Nucl. Data Tables 68 (1) (1998) 126.