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Theoretical results on the double positron decay of ^{106}Cd

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Abstract

Theoretical results for the double positron decay of ^{106}Cd are presented. The calculated values for the lateral single-beta-decay feeding patterns and electromagnetic transitions leading to states in ^{106}Cd and ^{106}Pd are discussed and compared to the data. The results on the double positron decay of ^{106}Cd are compared to new experimental limits. Based on the results of the present calculations, we report on the possibility of detecting the EC/EC or the β^+ /EC decay modes to final $J^\pi = 0^+$ states using the improved sensitivity of the planned measurements. © 2001 Elsevier Science B.V. All rights reserved.

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Neutrino properties and electroweak interactions beyond the standard model can be tested by nuclear double-beta-decay experiments [1]. Direct measurements of about ten of the nuclear double-beta-decay systems have been performed [2–8] or will be performed in the near future. These are the most sensitive experiments ever done, which have lead to measured half-lives of the order of $10^{19} \rightarrow 10^{21}$ years. Direct observations of the decay mode with the emission of two neutrinos have been confirmed and limits on the neutrinoless decay mode have been established [2,3]. Values of the parameters of the models beyond the $\text{SU}(2) \times \text{U}(1)_{\text{elec}}$ model of the electroweak interactions, extracted from these limits, can be found in [3]. Values and limits of the neutrino average mass shift, the right–left and right–right couplings of the weak current, the masses of the left- and right-

handed bosons and the parameters of the supersymmetric Lagrangians, can be extracted from these nuclear measurements. One essential input in the determination of these parameters is the detailed calculation of the structure of the nuclei participating in the decay processes [3].

A fairly good amount of information has been extracted from the analysis of double beta decay observables in the decays of ^{76}Ge [4], ^{82}Se [5], ^{96}Zr [6], ^{100}Mo [7] and ^{116}Cd [8]. Another promising case is the decay of ^{106}Cd [9]. Recently, new results of measurements of the various decay modes of this double positron emitter have been published by Belli et al. [9]. Measurements of the same system are planned to be performed at the Osaka-OTO underground facility [10]. From the nuclear-structure point of view the study of the various decay modes of this particular nucleus, including also single beta decay and electromagnetic multipole transitions, is challenging and necessary because of the obvious implications in particle physics and in astrophysics. In a previous work

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we have focused on the systematics of heavier-mass Sn and Cd isotopes and on their decays [11]. Based on the experience gained from this systematical analysis some preliminary theoretical estimates concerning the ^{106}Cd double beta decay to various final states have been produced [12,13]. Here we extend those studies to the analysis of all the relevant beta-decay side-feeding patterns and the double-beta-decay modes in a realistic single-particle basis, in view of the present improved possibilities of measuring the associated double-beta-decay rates.

In this work we are reporting on the results of microscopic nuclear-structure calculations consisting of the computation of energy levels, electromagnetic transitions, single- and double-beta-decay rates in the mass $A = 106$ Cd–Pd system. These result may be of some use in encouraging further experimental efforts on this system.

The present calculations are based on the quasiparticle random-phase approximation (QRPA) [3,11]. The wave functions of the initial and final states of a given electromagnetic or beta-decay transition are obtained from the diagonalization of a realistic microscopic Hamiltonian consisting of a single-particle mean field, a pairing force and residual two-body interactions. The starting point in the definition of the single-particle mean field is a central potential parametrized as a Woods–Saxon well [14], adjusted to reproduce charge radii and observed single-particle levels near closed shells. This basis we call the Woods–Saxon (WS) basis. The pairing interaction used in the calculations has been constructed from a renormalized, realistic short-range two-nucleon force [15]. The parameters of the force are fixed by the mass dependence of the separation energies in even–odd and odd–even nuclei. Data on the one-quasiparticle spectra of the odd–even and even–odd nuclei are used to fine-tune the mean-field single-particle energies in the vicinity of the proton and neutron Fermi surfaces. This new single-particle basis we call the adjusted basis (adj.), and the resulting new single-particle energies are the ones cited in [13]. It may be noted that the same adjusted basis was used in the analysis of [12] and [13] as can be seen by comparing the theoretical results of the two publications. In the present work we adopt the same changes in the single-particle energies but extend the neutron basis to the full 2p-1f-0h major shell to be sure about the convergence of the re-

sults. This means that the present numerical calculations are performed in a single-particle basis consisting of ten single-particle levels for protons (p-f and s-d-g shells) and fifteen single-particle levels for neutrons (p-f, s-d-g and p-f-h shells). The levels can accommodate 26 (28) active protons and 40 (38) active neutrons for Pd (Cd).

The residual two-body interaction is the non-relativistic force resulting from the meson exchange between nucleons. Finite-size effects on this interaction are accounted for by introducing form factors (see [15]). Charge-conserving and charge-changing channels of the interaction were used to construct the correlated nuclear states consisting of like-two-quasiparticle (neutron–neutron (nn) and proton–proton (pp)) and unlike-two-quasiparticle (proton–neutron (pn)) excitations, respectively. The approximate diagonalization of the Hamiltonian was carried out in the general framework of the pp + nn QRPA and pn-QRPA, for double-even and double-odd mass nuclei. Details of the formalism can be found in the review article of [3]. As said before, all free parameters of the model Hamiltonian have been fixed to reproduce observed quantities.

For the sake of completeness we are presenting in the following the relevant expressions which we have used to calculate electromagnetic and beta-decay observables.

The ft values for single-beta-decay transitions, feeding final states in Cd and in Pd, were calculated from the equation

$$ft_{\pm} = \frac{6147 \text{ s}}{g_V^2 |M_F^{\pm}|^2 + g_A^2 |M_{GT}^{\pm}|^2}, \quad (1)$$

where M_F and M_{GT} are the nuclear matrix elements describing Fermi and Gamow–Teller transitions

$$M_F^{\pm} = \frac{\langle J_f || \sum_j \tau^{\pm}(j) || J_i \rangle}{\sqrt{2J_i + 1}}, \quad (2)$$

and

$$M_{GT}^{\pm} = \frac{\langle J_f || \sum_j \sigma(j) \tau^{\pm}(j) || J_i \rangle}{\sqrt{2J_i + 1}}. \quad (3)$$

For the vector (g_V) and axial-vector (g_A) coupling strengths we adopted the values $g_V = g_A = 1.0$. Here we have used the quenched nuclear matter value of g_A . The isospin raising (lowering) operator τ^+ (τ^-)

mediates the β^+ (β^-) decay transitions. Furthermore, $\sigma(j)$ is the Pauli spin operator for the j th nucleon.

The probabilities for electric multipole transitions were calculated from the expression

$$B(E\lambda; J_i \rightarrow J_f) = (e_{\text{eff}}^{(p)} Q_\lambda^{(p)} + e_{\text{eff}}^{(n)} Q_\lambda^{(n)})^2, \quad (4)$$

where $e_{\text{eff}}^{(p)}$ and $e_{\text{eff}}^{(n)}$ are the proton and neutron effective charges [16] and Q_λ are the nuclear matrix elements of the λ -pole operators. They are given by

$$Q_\lambda^{(p)} = \frac{\langle J_f || \sum_j^Z r_j^\lambda Y_\lambda(\hat{r}_j) || J_i \rangle}{\sqrt{2J_i + 1}}, \quad (5)$$

for protons and

$$Q_\lambda^{(n)} = \frac{\langle J_f || \sum_j^N r_j^\lambda Y_\lambda(\hat{r}_j) || J_i \rangle}{\sqrt{2J_i + 1}}, \quad (6)$$

for neutrons, respectively.

The inverse half-life for two-neutrino double-beta-decay transitions to a final state J_f reads

$$[t_{1/2}^{(2\nu)}(0_i^+ \rightarrow J_f^+)]^{-1} = G_{\text{DGT}}^{(2\nu)}(J_f^+) |M_{\text{DGT}}^{(2\nu)}(J_f^+)|^2, \quad (7)$$

where $G_{\text{DGT}}^{(2\nu)}(J_f^+)$ is the integral over the phase space of the leptonic variables [3]. The nuclear matrix elements $M_{\text{DGT}}^{(2\nu)}(J_f^+)$ for the double positron emission can be written as

$$\begin{aligned} M_{\text{DGT}}^{(2\nu)}(J_f^+) &= \sum_{m,n} \left(\langle J_f^+ || \sum_j \sigma(j) \tau_j^+ || 1_m^+ \rangle \right) \\ &\quad \times \langle 1_m^+ | 1_n^+ \rangle \left(\langle 1_n^+ || \sum_j \sigma(j) \tau_j^+ || 0_i^+ \rangle \right) \\ &\quad \times \left[\left(\frac{1}{2} Q_{\beta\beta} + E_m - M_i \right) / m_e + 1 \right]^{-s}. \end{aligned} \quad (8)$$

The power s assumes the values $s = 1$ for $J_f = 0$ and $s = 3$ for $J_f = 2$. The overlap $\langle 1_m^+ | 1_n^+ \rangle$ between the two sets of 1^+ states, which are pn-QRPA solutions based on the initial and final ground states, helps in matching the two branches of virtual excitations.

By using the above described formalism we have produced a detailed description of the two-neutrino (2ν) decay channels of the $A = 106$ system. The results are shown and discussed next.

Spectroscopy of ^{106}Pd

With the above theoretical input we have calculated the single- β^- decay of ^{106}Rh and the single- β^+ decay of ^{106}Ag feeding states in ^{106}Pd . The experimental values of the measured transitions are shown in Table 1 while the corresponding theoretical results are listed in Table 2 for the two basis sets used. The overall agreement for the $\log ft$ values of the single- β^-

Table 1

Experimental level scheme of ^{106}Pd . Observed levels (J^π) and the corresponding energies are listed together with the $\log ft$ values for transitions feeding these states from the 1^+ ground state of ^{106}Rh (third column) and from the 1^+ ground state of ^{106}Ag (fourth column). The data are taken from Ref. [17]

J^π	E (MeV)	$\log ft_-$	$\log ft_+$
0^+	0.0	5.2	4.9
2^+	0.512	5.9	5.2
2^+	1.128	6.6	–
0^+	1.134	5.4	6.5
2^+	1.562	5.8	6.5
0^+	1.706	7.0	7.0
2^+	1.909	8.2	7.9
0^+	2.001	5.9	6.1
2^+	2.242	6.7	6.9
0^+	2.278	6.6	7.6
2^+	2.308	6.6	6.8
2^+	2.439	6.7	7.8
0^+	2.624	5.8	–
2^+	2.784	7.3	–
2^+	2.821	6.4	–
0^+	2.828	6.4	7.2
0^+	2.878	5.8	6.6
2^+	2.902	6.2	–
2^+	2.918	5.8	–
2^+	3.161	6.6	–
0^+	3.221	5.5	–
2^+	3.250	–	–
2^+	3.252	6.6	–
0^+	3.321	5.6	–

Table 2

Calculated level scheme of ^{106}Pd . The states (J^π) and their energies (in MeV), E (2nd and 3rd columns), are shown together with the calculated $\log ft$ values for transitions feeding them from the ground state of ^{106}Rh (4th and 5th columns) and from the ground state of ^{106}Ag (6th and 7th columns), as depicted in the previous table. The symbols (adj.) and (WS) refer to the two different basis sets used in the calculations. For more information see the text

J^π	E (adj.)	E (WS)	$\log ft_-$ (adj.)	$\log ft_-$ (WS)	$\log ft_+$ (adj.)	$\log ft_+$ (WS)
0^+	0.0	0.0	5.2	5.1	3.7	3.8
2^+	0.515	0.507	5.7	6.4	5.1	5.1
2^+	1.030	1.014	7.1	6.0	6.6	5.6
0^+	1.030	1.014	5.5	4.7	5.7	4.8
0^+	1.997	2.048	5.4	4.8	3.9	4.0
2^+	2.276	2.493	7.6	7.1	4.5	5.1
0^+	2.535	2.636	4.4	4.0	6.6	4.3
0^+	2.669	2.781	7.2	6.7	6.1	7.7
2^+	2.684	2.751	6.3	6.2	6.1	5.2
2^+	2.760	2.784	5.3	4.3	5.0	4.4
0^+	2.978	–	5.1	–	–	–
2^+	3.084	2.863	5.1	5.1	–	5.3
2^+	3.089	–	4.4	–	–	–
2^+	3.177	–	6.9	–	–	–
0^+	3.288	–	4.0	–	–	–
2^+	3.467	–	6.9	–	–	–

decay of ^{106}Rh to the ground- and low-lying excited quadrupole states is rather good, especially for the adjusted basis, while faster transitions are obtained for the single- β^+ decay of ^{106}Ag for both basis sets (the transition rates for the adjusted, again, being closer to the experiment). For the β^+ decay the theoretical matrix elements for the feeding of monopole states are larger than the experimentally extracted values by factors of the order of 3. For this decay branch the available decay energy is lower than for the β^- branch and as a result, there is no feeding of states beyond 2.9 MeV. At this point it is worth of mentioning that in the adjusted basis of [13], in Fig. 9 of [13] (the same basis was also applied in [12]) almost the same results as quoted in Table 2 for the adjusted basis are obtained (in [13], Fig. 9, the $\log ft_+$ values for the 2_2^+ and 0_1^+ states have been erroneously interchanged). The similarity comes from the previously mentioned fact that our adjusted energies coincide with the ones of Ref. [13], and the small difference from the fact

that we have an expanded neutron valence space with respect to the one of [13].

Concerning the quality of the calculated energy levels for the low-lying states of ^{106}Pd , the excitation energies shown in Table 2 indicate that there cannot be found a one-to-one correspondence between the experimental energies and the energy levels predicted by the calculation. This is due to the fact that some more complex (shell model) configurations enter the energy spectrum below 3 MeV, part of which could be assigned to collective four-quasiparticle states (orthogonal to the two-phonon excitations accounted for by our model). There is also a possibility for a soft deformation of the Pd nuclei leading to co-existing deformed “intruder” states in the low-energy spectrum of these nuclei. This deformation could be partly associated to the existence of collective four-quasiparticle states. In fact, our calculations suggest, both for the adjusted and the WS basis, that the two-quasiparticle states start at 2 MeV (excluding the first 2^+ state, which is a very

low-lying collective two-quasiparticle state) implying a more complex structure, e.g., four-quasiparticle contributions, for the levels between 1.128 MeV and 1.909 MeV. It has to be noted that the calculated level density below 3.5 MeV cannot be increased by expanding the single-particle valence space; the effect of this expansion is only seen at high excitation energies. However, the single-particle level density at the Fermi surface seems to affect the level density at around 3–3.5 MeV as seen from Table 2 when comparing the results obtained in the adjusted and WS basis sets.

As a further test of the quality of the corresponding wave functions we have calculated E2 and E3 transitions to the ground state from the first excited quadrupole and octupole states in ^{106}Pd . The obtained values for the reduced transition probabilities are 46.2 W.u. (adj.), 42.7 W.u. (WS) for the E2, and 11.3 (adj.), 34.8 W.u. (WS) for the E3, which should be compared with the experimental ones of 44.7 W.u. and 27 W.u., respectively [18]. The effective charges used to calculate these values are $e_{\text{eff}}^{(p)} = 1.2e$ for protons and $e_{\text{eff}}^{(n)} = 0.2e$ for neutrons. From the above one sees that if one wants to reproduce the experimental energy (2.08 MeV) of the first 3^- state in ^{106}Pd using the adjusted basis one obtains a QRPA state of too low collectivity. The calculated energy has to be pushed a few hundreds of keV downward in order to reach the experimental $B(\text{E}3)$ value.

Spectroscopy of ^{106}Cd

Similar analysis as the one discussed above was performed for the case of the structure of low-lying states of ^{106}Cd . These states are fed by the single- β^+ decay of ^{106}In . For the single- β^- decay from ^{106}Ag not definite ft -values are available. The experimental values of the measured transitions are shown in Table 3. The theoretical results are given in Table 4. In this case the results for the adjusted basis are much better than for the WS basis producing a nice quantitative agreement with the experimental $\log ft_+$ values. Also the level density below 3 MeV seems to be correct, the energies of the 4^+ states being shifted some 0.3 MeV upward relative to the experimental energies. As seen from the energies, the states which we are describing as vibrational two-phonon states of ^{106}Cd show

Table 3

Experimental level scheme of ^{106}Cd . Observed levels (J^π) and energies, E , are listed together with the $\log ft$ values for transitions leading to these states from the $J^\pi = 3^+$ ground state of ^{106}In (third column). The data are taken from Ref. [17]

J^π	E (MeV)	$\log ft_+$
2^+	0.633	6.3
4^+	1.494	6.6
2^+	1.717	6.6
4^+	2.104	6.6
(4^+)	2.252	6.3
4^+	2.305	7.0
(2^+)	2.347	5.8
2^+	2.371	–
4^+	2.468	–
$2^+, 3^+, 4^+$	2.491	–
2^+	2.566	5.9
2^+	2.630	6.5
$1, 2^+, 3$	2.721	6.4
$2, 3^+$	2.890	6.0

Table 4

Calculated level scheme of ^{106}Cd . The theoretical spectrum (J^π, E) is shown with the calculated $\log ft$ values for transitions from the ground state of ^{106}In . The notation is the same as in Table 2

J^π	E (adj.)	E (WS)	$\log ft_+$ (adj.)	$\log ft_+$ (WS)
2^+	0.633	0.648	6.1	5.9
4^+	1.266	1.296	6.9	5.4
2^+	1.266	1.296	6.4	5.9
4^+	2.087	2.008	7.0	5.0
2^+	2.406	2.366	5.9	5.4
2^+	2.570	2.586	5.4	4.2
4^+	2.663	2.530	7.0	5.9
2^+	2.767	2.631	5.7	6.3
4^+	2.785	2.560	6.1	5.5
2^+	2.810	2.906	6.0	5.9
4^+	2.833	2.744	7.0	4.0
4^+	2.897	3.066	6.8	6.3
2^+	2.967	3.746	5.9	7.1

a larger anharmonicity than in the case of ^{106}Pd . We have calculated the E2 transition to the ground state from the first excited quadrupole state of ^{106}Cd . The obtained value, for the reduced transition probability, is 25.9 W.u. (adj.), 26.4 W.u. (WS) and the experimental one is 25.8 W.u. [18]. The effective charges used to calculate these values are, as before, $e_p^{\text{eff}} = 1.2e$ for protons and $e_n^{\text{eff}} = 0.2e$ for neutrons.

The double positron decay of ^{106}Cd

The matrix elements of Eq. (8), for the double positron decay of ^{106}Cd to the ground and excited states of ^{106}Pd , were used to calculate the half-lives listed in Table 5. The results calculated in the two different basis sets are basically the same for the 0_1^+ transition but differ considerably for the ground-state transition. In the adjusted-basis calculation of [12,13] (using a smaller neutron valence space than in this work) slightly bigger matrix elements for both the ground-state and 0_1^+ transitions were obtained leading to some three times smaller half-lives for these processes. This means that the effect of the larger neutron valence space is more visible in the double-beta-decay rates than in the single-beta-decay rates discussed earlier in this article. The comparison with the experimental limits of [9] shows that, in general, the calculated half-lives are much larger than the experimentally reached limits. Even in the best case, which is the β^+/EC ground-state-to-ground-state transition, the theoretical result is one (adj.) or two (WS) orders of magnitude larger than the experimental limit.

In the fourth column of Table 5 we give the results of M. Hirsch et al. [19] for comparison. In [19] only the ground-state transitions are calculated by using the pn-QRPA with the G matrix based on the Paris potential. They use two oscillator major shells for the proton and neutron valence space in their calculation adopting Coulomb-corrected Woods–Saxon single-particle energies. In the case of the ^{106}Cd decay the authors of [19] have to use an arbitrary shift in the spin-orbit part of the WS mean-field potential to avoid a curious behaviour of the double-beta-decay matrix element for values of g_{pp} close to the collapse of the pn-QRPA. No justification of this procedure starting from the quasiparticle spectra is given and thus the predictive power of the calculation is reduced. Nevertheless, as can be seen from Table 5, the numerical results of [19] are quite close to our adjusted-basis results.

As discussed in the context of Tables 1–4, the spectroscopic analysis very much favours the use of the adjusted basis in the double-beta-decay calculation. In this basis the present theoretical value for the double electron capture (EC/EC) to the ground state is $t_{1/2}^{(2\nu)} = 3.0 \times 10^{20}$ yr, and the value for the β^+/EC transition to the ground state reads $t_{1/2}^{(2\nu)} = 2.4 \times 10^{21}$ yr (see Table 5). These theoretical predictions are expected to be reached by new experiments, as the one proposed by the authors of [9] or the planned Osaka-OTO experiment [10]. We think that this is the most relevant result of the present calculations since it can be confirmed experimentally.

To conclude, in this Letter we have discussed the results of calculations for the double positron decay

Table 5

Calculated half-lives (in years) for the double positron decay of ^{106}Cd . The first column indicates the decay modes. The present calculated values are listed in the second and third columns. On the fourth column we quote the results of M. Hirsch et al. [19] for comparison. The experimental limits, from Ref. [9], are given in the last column of the table. For more information on the notation see the caption of Table 2

Decay mode	$t_{1/2}^{(2\nu)}$ (adj.)	$t_{1/2}^{(2\nu)}$ (WS)	Hirsch et al. [19]	Exp. limit
$\beta^+/\text{EC}(2\nu)$ g.s \rightarrow g.s	2.4×10^{21}	4.4×10^{22}	4.1×10^{21}	4.1×10^{20}
$\beta^+/\text{EC}(2\nu)$ g.s \rightarrow 0_1^+	5.1×10^{26}	5.8×10^{26}		1.1×10^{20}
$2\beta^+(2\nu)$ g.s \rightarrow g.s	9.5×10^{25}	1.8×10^{27}	4.2×10^{26}	2.4×10^{20}
EC/EC(2ν) g.s \rightarrow g.s	3.0×10^{20}	5.5×10^{21}	8.7×10^{20}	
EC/EC(2ν) g.s \rightarrow 0_1^+	3.0×10^{23}	3.4×10^{23}		7.3×10^{19}

channels of ^{106}Cd , leading to the ground and excited 0^+ states of ^{106}Pd . To test the quality of the proposed nuclear-structure model we have performed a detailed comparison between theoretical and experimental results for the available observables, both for beta decay and electromagnetic transitions. The presently reported theoretical values of the half-lives for the double positron decay channels are larger than the measured limits given in [9]. However, the results for the EC/EC and β^+ /EC transitions to the ground state show that some of the predicted double-beta-decay rates may be reached in the near future by more sensitive measurements.

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