Theoretical analysis of the second-forbidden electron capture transition $^{123}\mathrm{Te}(1/2^+_{\mathrm{g.s.}}) \rightarrow {}^{123}\mathrm{Sb}(7/2^+_{\mathrm{g.s.}})$

O. Civitarese[1,*] and J. Suhonen[2,†]
[1]*Department of Physics, University of La Plata, c.c. 67 (1900), La Plata, Argentina*
[2]*Department of Physics, University of Jyväskylä, P.O.B 35, FIN-40351 Jyväskylä, Finland*
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The second-forbidden electron-capture (EC) transition $^{123}\mathrm{Te}(1/2^+_{\mathrm{g.s.}}) \rightarrow {}^{123}\mathrm{Sb}(7/2^+_{\mathrm{g.s.}})$ is analyzed in the framework of the proton-neutron quasiparticle random-phase approximation (*pn*-QRPA) and by using the quasiparticle-phonon coupling scheme. The quality of the model used to construct the phonon wave functions is supported by the analysis of the calculated ft values for β-decay transitions feeding states in ^{122}Te. The initial state $J^\pi_i = 1/2^+_{\mathrm{g.s.}}$ is represented as a pure one-quasineutron state in the $2s_{1/2}$ orbital and the final state $J^\pi_f = 7/2^+_{\mathrm{g.s.}}$ is constructed by diagonalizing the quasiparticle-phonon coupling Hamiltonian in a basis consisting of a quasiproton state and quasineutron states coupled to *pn*-QRPA phonons. The influence of high-lying single-particle orbitals, in contributing to the nuclear transition matrix element, is discussed. The obtained theoretical result for the considered second-forbidden EC transition is in good agreement with the data.

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I. INTRODUCTION

The most stringent tests on the predictive power of the currently adopted models of nuclear structure are nowadays provided by the experimental measurements of very rare electroweak decays such as the nuclear double beta decay [1], the nuclear (μ^-, e^-) conversion, and highly forbidden β-decay and electron-capture (EC) processes [2].

Recently, experimental data on second-forbidden EC transitions in ^{123}Te were reported by Alessandrello *et al.* [3]. The measured half-life for the second-forbidden electron-capture transition $^{123}\mathrm{Te}(1/2^+_{\mathrm{g.s.}}) \rightarrow {}^{123}\mathrm{Sb}(7/2^+_{\mathrm{g.s.}})$ is of the order of 10^{19} yr, therefore, comparable to some of the measured half-lives for other very rare electroweak decays such as the two-neutrino nuclear double beta decay [1]. As for the case of nuclear double beta-decay transitions, where the nuclear matrix elements are suppressed, a theoretical explanation of the matrix element of the second-forbidden EC transition in ^{123}Te is a very severe test on the nuclear models used to calculate the observables. We have taken this as a motivation for the present work.

In a manner similar to the one that we have used for the treatment of nuclear double-beta-decay transitions [4], we have considered the various elements entering in the calculations in a systematic approach. We are mainly interested in revealing the basic degrees of freedom participant in the decay process, rather than searching for a casual agreement with data. Like in the case of the nuclear double-beta-decay calculations, the case of second-forbidden EC calculations may require the knowledge of very small components of the participant wave functions, or the final observables may be the result of subtle cancellations among relatively large contributions to individual transition-matrix elements. Considering this point of view, we have adopted as a suitable theoretical framework the combined quasiparticle (BCS method) +proton-neutron quasiparticle random-phase approximation (*pn*-QRPA) approach [4]. Using this scheme we have calculated the spectrum of ^{122}Sb and the first-forbidden β^--decay transitions feeding the ground and excited states of ^{122}Te. In this way we have fixed the parameters of the two-body interaction used to construct the matrix elements of the *pn*-QRPA. Next, without changing these parameters, we have calculated the matrix elements of the quasiparticle-vibration coupling Hamiltonian, for *pn*-QRPA phonons, and diagonalized it to construct the wave function of the final state $J^\pi_f = 7/2^+_{\mathrm{g.s.}}$, which is the ground state of the nucleus ^{123}Sb. With these elements we have calculated the matrix elements of the electroweak operator responsible for the second-forbidden EC transition. Finally, we have studied the dependence of the results on some of the assumptions made in the present calculations and in the calculations of Ref. [5].

This paper is organized as follows. The basic details of the formalism are presented in Sec. II, the details of the calculations are given in Sec. III and the results are presented and discussed in Sec. IV.

II. THE FORMALISM

The basic elements needed to calculate the matrix elements of a nuclear electroweak transition are introduced in the following sections. They are based on well-known theories and we are referring to them for the sake of completeness. Further details can be found in the literature.

A. The *pn*-QRPA formalism

The standard version of the QRPA [6] was extended several years ago to describe excited states in odd-odd mass nuclei [7] and applied to the study of allowed β-decay transitions. The formalism was later on extended to describe spin-isospin dependent excitations of the first-forbidden type [8] as well as double-beta-decay transitions [9–11]. Concerning double-beta-decay transitions, the formalism was applied

*Email address: civitare@venus.fisica.unlp.edu.ar
†Email address: suhonen@phys.jyu.fi

for different types of interaction [4] and agreement with the weak-coupling shell-model results [12] was obtained after particle-hole and particle-particle channels of the nuclear two-body residual interaction were treated in a consistent manner [10].

In the case of charge-exchanging excitations, the *pn*-QRPA equations [13] are solved for states whose structure is given in terms of the following one-phonon creation operators:

$$\Gamma^{\dagger}(JM,k)=\sum_{pn}\ [X(pn,Jk)A^{\dagger}(pn,JM) + Y(pn,Jk)\overline{A}(pn,JM)], \quad (1)$$

where $X(pn,Jk)$ and $Y(pn,Jk)$ are the forward- and backward-going amplitudes of proton-neutron quasiparticle pairs coupled to angular momentum J and its projection M. These quasiparticle pairs are created (annihilated) by the operators $A^{\dagger}(pn,JM)$ $[A(pn,JM)]$, as defined in Ref. [13]. As usual, the notation $\overline{O}(JM)=(-1)^{J-M}O(J-M)$ is adopted to indicate time reversal [14]. The index k is the eigenvalue index. The one-phonon state with energy ω_k is the result of the action of the one-phonon creation operator $\Gamma^{\dagger}$ on the correlated vacuum

$$|JM,k\rangle=\Gamma^{\dagger}(JM,k)|0\rangle. \quad (2)$$

The *pn*-QRPA method has been reviewed in detail in several previous works (see, e.g., Ref. [4]) and we shall avoid its further discussions here. The relevant feature of the *pn*-QRPA set of eigenvalues, concerning the scope of the present calculations, is that all the parameters entering in the definition of the effective two-body interaction will be fixed. The necessary input consists of the available spectroscopic information about low-lying proton-neutron excitations in ^{122}Sb and the ft values for transitions from this nucleus to states in ^{122}Te. In this manner unphysical effects due to unrealistic renormalizations of strength of the particle-particle and particle-hole channels of the two-body interaction are avoided.

B. The quasiparticle-phonon coupling

The wave function of the final state, in ^{123}Sb, is written as the linear combination

$$|7/2^{+}_{\text{g.s.}}\rangle=a_f(g_{7/2};7/2^{+})|g_{7/2}(p)\rangle + \sum_{nJ}\ a_f(nJ;7/2^{+})|nJ;7/2^{+}\rangle, \quad (3)$$

where the first component corresponds to the contribution of a pure quasiproton state in the $g_{7/2}$ orbit and the terms in the sum represent the contributions of quasineutrons coupled to charge-exchange phonons with $J^{\pi}=1^{+},2^{-},3^{+}$. The phonons included in the basis are the ones with the lowest energy, for each multipolarity, and they represent the lowest states of the spectrum of ^{123}Sb. Their wave functions are obtained by applying the *pn*-QRPA formalism and they are tested by calculating the ft values for β-decay transitions to the ground and excited states of ^{122}Te, as we shall show later.

The coupling of quasiprotons to the lowest quadrupole phonon (charge conserving) will certainly contribute to the higher part of the spectrum of ^{123}Sb. Since only tentative spectroscopic classifications of these levels are available, we cannot test the accuracy of the calculations based on the coupling between the quasiproton in the $g_{7/2}$ and $d_{5/2}$ orbits to the lowest quadrupole phonon in ^{122}Te. However, based on the unperturbed values of the energy for these configurations, we shall neglect the effect of this coupling in dealing with the quasiproton contributions to the ground state of ^{123}Sb. Naturally, this will be a poor approximation if one is dealing with specific properties of the excited $3/2^{+}$ and $5/2^{+}$ states in ^{123}Sb, where the mixing between one and three quasiparticle states may be important [15].

The matrix elements of the Hamiltonian that couples quasiparticles and charge-exchanging phonons are constructed in the way described in Ref. [16]. Since we are working with realistic matrix elements of the two body interaction, we must construct the vertex functions accordingly. Thus, we have defined the vertex functions

$$f_s(pn,Jk)=\sum_{p'n'}\ V(p',n',p,n;J)[-u_{p'}u_{n'}X(p'n',Jk) + v_{p'}v_{n'}Y(p'n',Jk)], \quad (4)$$

and

$$g_s(pn,Jk)=\sum_{p'n'}\ V(p',n',n,p;J)[-u_{p'}u_{n'}Y(p'n',Jk) + v_{p'}v_{n'}X(p'n',Jk)], \quad (5)$$

where $V(p',n',n,p;J)$ are the matrix elements of the realistic two-body interaction acting on proton-neutron pair configurations coupled to total angular momentum J [6]. With these vertex functions the off-diagonal matrix elements of the quasiparticle-vibration coupling Hamiltonian acting on the components of the wave functions of the set of $7/2^{+}$ states in ^{123}Sb are written

$$\langle nJ;7/2^{+}|H_{\text{qp-vib}}|p;7/2^{+}\rangle = f_s(pn,Jk)v_nu_p+g_s(pn,Jk)v_pu_n. \quad (6)$$

Similar expressions, for the coupling between quasiparticles and charge-exchanging phonons in the case of separable interactions, can be found in Ref. [17].

It is worth mentioning that the two-body interaction used to construct the matrix elements of the Hamiltonian that couples quasiparticles and phonons is the same that we have used to solve the *pn*-QRPA and QRPA equations. In this manner the present treatment of the vibrational and quasiparticle degrees of freedom is fully consistent.

C. The EC transition operator

The second-forbidden EC transition from the ground state of the nucleus ^{123}Te is governed by the action of the operator

$O_{3M}\tau^+ = i^2 r^2 (Y_2\sigma)_{3M}\tau^+$. After expressing it in the standard second quantization form, it can be transformed to the quasiparticle representation. The terms resulting from this transformation can create or annihilate a quasiproton-quasineutron pair or change a quasineutron into a quasiproton (and vice versa). For the study of the transition we are concerned with, the charge exchange between pure quasiparticle components of the initial $(2s_{1/2})$ quasineutron state in ^{123}Te and the final $(0g_{7/2})$ quasiproton state in ^{123}Te is strictly forbidden by the selection rules of the transition operator. The two-quasiparticle terms of the operator $O_{3M}\tau^+$ are the only ones acting in the transition and they are written as

$$(O_{3M}\tau^+)_{\text{pairs}} = \frac{1}{\sqrt{7}} \sum_{pn} \langle n||O_3||p\rangle [u_n v_p A^\dagger(pn,3M) + v_n u_p \overline{A}(pn,3M)]. \tag{7}$$

These terms can be expressed in terms of one-phonon creation and annihilation operators, for charge-exchanging phonons, as

$$(O_{3M}\tau^+)_{(pn\text{-QRPA})} = \sum_k [\lambda_k \Gamma^\dagger(3M,k) + \mu_k \overline{\Gamma}(3M,k)], \tag{8}$$

where the index k is the eigenvalue index of the *pn*-QRPA phonons and the amplitudes λ_k and μ_k are defined as

$$\lambda_k = \frac{1}{\sqrt{7}} \sum_{pn} \langle n||O_3||p\rangle [u_n v_p X(pn,3k) + v_n u_p Y(pn,3k)], \tag{9}$$

and

$$\mu_k = \frac{1}{\sqrt{7}} \sum_{pn} \langle n||O_3||p\rangle [u_n v_p Y(pn,3k) + v_n u_p X(pn,3k)]. \tag{10}$$

D. Reduced matrix elements

The collective (one phonon) contribution to the reduced matrix element corresponding to the transition between the ground state (g.s.) and a given one-phonon state is written as

$$\langle 3^+,k||O_3||0\rangle = \sqrt{7}\lambda_k. \tag{11}$$

Following the discussion about the wave functions of the initial and final states, advanced in the previous section, we can write for the reduced matrix element of the transition ^{123}Te$(1/2^+$, g.s.$) \rightarrow {}^{123}$Sb$(7/2^+$, g.s.$)$ the following two contributions:

$$\langle 7/2^+, {}^{123}\text{Sb, g.s.}||O_3||1/2^+, {}^{123}\text{Te, g.s.}\rangle_{\text{collective}} = a_i(s_{1/2}) a_f(s_{1/2} 3_1^+; 7/2^+) \sqrt{\frac{8}{7}} \lambda_1, \tag{12}$$

which corresponds to the process where the initial quasineutron state is a spectator and is not affected by the transition operator which creates the $|3^+ k=1\rangle$ one-phonon state of the final configuration, and

$$\langle 7/2^+, {}^{123}\text{Sb, g.s.}||O_3||1/2^+, {}^{123}\text{Te, g.s.}\rangle_{\text{coupling}} = a_i(s_{1/2}) \sqrt{8} \sum_{nJ} a_f(nJ; 7/2^+) \sum_p X(s_{1/2}p, Jk=1) \times u_n v_p \langle n||O_3||p\rangle \begin{Bmatrix} 1/2 & p & J \\ n & 7/2 & 3 \end{Bmatrix}, \tag{13}$$

which corresponds to the terms where the transition operator is acting upon a quasiproton state of the one-phonon component of the final state.

Both Eqs. (12) and (13) are dependent upon the wave function of the initial state, represented by the amplitude $a_i(s_{1/2})$. The assumption that this is indeed the dominant component of the wave function of the ground state of ^{123}Te is based on the fact that this component exhausts about 80% of the one-neutron transfer data, as stated in Ref. [5]. For the sake of completeness we have considered the unperturbed energies of other potential components of the wave function, such as the $g_{7/2}$ quasiproton state coupled to the 3^+ state of ^{122}Sb, as well as the $d_{5/2}$ and $d_{3/2}$ quasineutron states coupled to the first excited quadrupole state of ^{122}Te. This has been done to estimate the order of magnitude of the coupling that could eventually introduce further admixtures in the wave function of the $1/2^+$ ground state. This mixing can eventually renormalize the amplitude of the pure $s_{1/2}$ quasineutron state. Considering the actual values of their energies, and the fact that they will not contribute directly to the transition due to angular momentum conservation, we can eventually let the coefficient $a_i(1/2)$ vary between 0.9–1.0, if necessary, to account for these extra couplings. As we shall discuss later, this freedom is not affecting the physics of the decay.

III. DETAILS OF THE CALCULATIONS

The single-particle basis used in the calculations consisted of proton and neutron single-particle levels from the shell closure $N=Z=20$ up to the shell closures $N=126$ and $Z=82$. They have been obtained as solutions of the central Woods-Saxon potential, including Coulomb corrections for proton states. The parameters of the potential were taken from Bohr and Mottelson [14]. In this basis we have solved the BCS equations by considering the monopole terms of the two-body interaction constructed from the Bonn one-boson-exchange potential (OBEP) [18], as described in Ref. [10]. Concerning the quality of the results we have compared the obtained values for the BCS gap parameters with the observed (or extrapolated) odd-even mass differences. Few small adjustments in the distribution of single-particle levels around the shell closure $N=Z=50$ were implemented in order to better reproduce the known sequences of one-particle (quasiparticle) states, within the limitations imposed by the near-degeneracy of the lowest portion of the spectrum of these nuclei.

TABLE I. Experimental and calculated log-ft values for transitions from the $J^\pi=2^-$ ground state of ^{122}Sb to low-lying (J^π_k) states in ^{122}Te. Data are taken from Ref. [19].

J^π_k	log-ft (expt.)	log-ft (th.)
$0^+_{\text{g.s.}}$	9.65	9.65
2^+_1	7.61	7.73
4^+_1	10.43	10.81
2^+_2	7.69	7.66
0^+_1	10.09	9.22

The theoretical spectrum of excited states in the $A=122$ nuclei Sb and Te was constructed by solving the *pn*-QRPA [13] and standard QRPA equations [6] in the above introduced quasiparticle basis. The equations were solved by using two-body matrix elements, for each multipolarity, constructed from the OBEP. Since the parameters of the interaction have been adjusted to reproduce the known energetics of non-charge-exchanging (proton-proton and neutron-neutron) and charge-exchanging (proton-neutron) modes in ^{122}Sn, ^{122}Te, and ^{122}Sb, respectively, no further renormalizations of the interaction were needed. The strength of the particle-hole and particle-particle channels of the interaction were fixed, as described in Ref. [13]. As an indication about the quality of the wave functions, obtained by solving the *pn*-QRPA and QRPA equations, we are showing, in Table I, the comparison between theoretical and experimental ft values for lateral first-forbidden β^- transitions feeding states in ^{122}Te (the data are taken from Ref. [19]). As seen from this table, the agreement between the calculated values and the data is good. We want to point out that these calculations are a necessary test of the wave function of the first $J^\pi=2^-$ state in ^{122}Sb.

IV. RESULTS AND DISCUSSION

In the first part of this section we present and discuss our results and in the second part of it we compare our results with the ones of Ref. [5].

As we have said before, we have done, first, *pn*-QRPA calculations of the set of states with low multipolarities in ^{122}Sb and fixed all parameters of the calculations to setup the spectrum and wave functions of the first $J^\pi=1^+,2^-,3^+$ states. The state with lowest energy obtained in the calculations is a $J^\pi=2^-$ state and the nearest $J^\pi=1^+,3^+$ states were found within 1 MeV respect to the lowest 2^- state, without further changes in the values of the proton-neutron particle-particle coupling strength g_{pp} for any of these channels. Next, we have diagonalized the Hamiltonian (6) with couples a quasiproton state with different configurations of quasineutrons coupled to charge-exchange phonons. These are the dominant terms needed to calculate the wave function of the lowest state state in ^{123}Sb. The dominant contribution of the wave function of the lowest $J^+_f=7/2^+$ state is the quasiproton state in the $0g_{7/2}$ orbital (90%), followed by contributions given by the coupling of the first 2^- state of ^{122}Sb to the $0h_{11/2}$ quasineutron orbital (less than 10%) and by small contributions (of the order of 1%) corresponding to the coupling between the lowest 1^+ state of ^{122}Sb with the quasineutron orbitals $0g_{9/2}$, $0gd_{7/2}$, and the lowest 3^+ state of ^{122}Sb with the quasineutron orbitals $2s_{1/2}$ and $1d_{5/2}$.

TABLE II. Contributions to the matrix element of the transition $^{123}\text{Te}(1/2^+_{\text{g.s.}})\rightarrow{}^{123}\text{Sb}(7/2^+_{\text{g.s.}})$. The results of Eqs. (12) and (13), the total matrix element, and the ratio between the theoretical and experimental half-lives are shown. The matrix elements are given in units of 10^{-2} fm^2. The second and third columns show the results obtained for two different values of $g_{\text{pp}}(J^\pi=3^+)$, as indicated in the table.

Term	$g_{\text{pp}}(J^\pi=3^+)=1.1$	$g_{\text{pp}}(J^\pi=3^+)=1.5$
Collective	2.54	3.11
Coupling	-0.57	-0.62
Total	1.97	2.49
Half-life ratio	2.98	1.76

We have then proceeded with the calculation of the contributions of Eqs. (12) and (13) to the total matrix element. The results are shown in Table II. The involved radial matrix elements have been calculated by using harmonic oscillator radial wave functions. We have performed calculations using different values of $g_{\text{pp}}(J^\pi=3^+)$ and we have studied the effect of the fragmentation induced by high-lying neutron orbitals. The largest contributions to the matrix element are produced by the collective part, Eq. (12), as seen from the results shown in Table II. We have verified that the inclusion of neutrons orbitals above the shell closure at $N=126$ tends to decrease the value of the collective contribution (12) and to increase the value of the contribution (13). The increase of the value of $g_{\text{pp}}(J^\pi=3^+)$ tends to decrease the value of the collective term χ_1 of Eq. (12) but this effect is compensated by the changes of the wave function of the final state, which gets a larger contribution from the $|s_{1/2}3^+_1;7/2^+\rangle$ configuration. This is a trivial effect caused by the decrease in the energy of the first 3^+ state for increasing values of g_{pp}, and it results in an overall increase of the contribution (12), as one can see from the values given in Table II. Since we do not have observables to test the wave function of the first 3^+ state, as we do have to the test the wave function of the 2^- state, we have to rely upon this relative variation to assess the quality of the calculations. The resulting theoretical matrix element, which is the sum of the contributions (12) and (13), varies between 0.020 fm^2 ($g_{\text{pp}}=1.1$) and 0.025 fm^2 ($g_{\text{pp}}=1.5$). The experimentally extracted matrix element is of the order of 0.033 fm^2 [for $(g_A/g_V)=-1.23$]. It is seen that the experimental matrix element can be reproduced within factors of the order of ≈ 1.5. Therefore, one can say that, in principle, all the elements needed to explain the data have been included in the model and that further adjustments of the theory would produce the correct results. Notice that, since the amplitude of the initial quasineutron state $s_{1/2}$ is a common factor of both Eq. (12) and Eq. (13), a renormalization of the amplitude $a_i(1/2)$ by a mere 10% does not change this picture significantly.

Next, we can turn to the discussion of some features of the present work in relation with the results reported in the

work of Ref. [5]. The authors of Ref. [5] have restricted their discussion to the collective part [see Eq. (12)], which in their work was calculated making use of a Skyrme-type of effective interaction. They have performed a *pn*-QRPA analysis of the wave function of the first 3^+ state in ^{123}Sb, by varying the strength of the particle-particle proton-neutron interaction ($g_{\rm pp}$). They found a strong dependence of the results upon this degree of freedom. As they mention in their conclusions, by slightly renormalizing the particle-particle interaction, they were able to reproduce the experimental half-life. However, this good agreement was based on the crucial assumption that in the final wave function the amplitude $a_f(s_{1/2}3_1^+;7/2^+)$, of Eq. (12), would be of the order of 0.4. Actually, this assumption is not supported by the results obtained in the present calculation, which yields a smaller value for this amplitude (less than 0.01). Also, as we have discussed before, both the contributions (12) and (13) to the total nuclear matrix element are relevant. In addition, they both are dependent on the used parametrization of the residual interaction. Furthermore, the ratio between these two contributions [Eqs. (12) and (13)] is dependent on the chosen value of $g_{\rm pp}$. Following the philosophy of the authors of Ref. [5], we have also investigated the sensitivity of the results by changing the value of $g_{\rm pp}$ in the $J^\pi=3^+$ channel of the proton-neutron interaction. In our experience, this variation alone is not sufficient to reproduce the experimental value of the decay half-life. While it is indeed true, as the authors of Ref. [5] have pointed out, that one can eventually produce a strong cancellation of the term (12), by varying $g_{\rm pp}$, it is difficult to justify the physical meaning of the procedure. As it is described in Ref. [4], the meaning of a renormalization of the value $g_{\rm pp}$ for proton-neutron interactions in the $J^\pi=1^+$ channel can be understood in terms of symmetry effects. In this context, the renormalization of $g_{\rm pp}$ for $J^\pi=3^+$ interactions lacks a physical interpretation. We felt that it should not be taken as a free parameter adjusted to explain the EC data, since there are other effects to be accounted for. While the contribution (12) varies within a factor of the order of 30% by changing $g_{\rm pp}$ from $g_{\rm pp}=1.1$ to $g_{\rm pp}=1.5$, the contribution (13) remains roughly the same and it amounts to approximately 20% the magnitude of Eq. (12) and with opposite sign.

Considering all these, we think that a consistent theoretical analysis of the transition must include the following elements: (i) the test of the quality of the *pn*-QRPA wave function of the $J^\pi=2^-_{\rm g.s.}$ on ^{122}Sb, (ii) the consistent calculation of the wave function of the low lying states in ^{123}Sb, by using the same interaction and model parameters used to describe the states in ^{122}Sb, and (iii) the inclusion of high-lying neutron orbits, in addition to the shell closure $h_{11/2}$ at $N=126$, which are needed to collect intensity.

V. CONCLUSIONS

We may conclude this analysis by saying that the agreement between data and theoretical results supports the notion that the physics governing highly forbidden EC and β-decay transitions very much resembles the classical example of $E1$ transitions [20]. As in that case, also here the total nuclear matrix elements are controlled by very small components of the participant wave functions that can contribute coherently. Therefore, the theoretical analysis of highly forbidden EC transitions, like the one considered in this work, may be strongly dependent also upon the number of single-particle states included in the calculations.

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