Temperature dependent BCS-gap equations in the continuum

O. Civitarese,[1,*] R. J. Liotta,[2,†] and T. Vertse[3,‡]
[1]*Department of Physics, University of La Plata, c.c. 67 1900, La Plata, Argentina*
[2]*Royal Institute of Technology, Physics Department Frescati, Frescativagen 24, S-10405 Stockholm, Sweden*
[3]*Institute of Nuclear Research of the Hungarian Academy of Sciences, P. O. Box 51, H-4001 Debrecen, Hungary*
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Pairing correlations for excited nuclei near the proton drip line are described by using finite-temperature BCS equations including the continuum part of the spectrum. The suitability of the proposed method to obtain stable solutions in the presence of single-particle resonances is shown.

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A microscopic analysis of the structure of nuclei close to the neutron or proton drip line constitutes a challenge to existing nuclear models, particularly, when the Fermi level is immersed in the continuum. The main difficulty is that this is a time-dependent problem. Assuming, as in scattering theory, that the processes are stationary one can quantize the continuum by placing the nucleus in an impenetrable box [1–3]. But if active particles move in the continuum there is a radioactive decay and they are permanently emitted. That is, an impenetrable box cannot be a realistic boundary condition for short-living systems. One may thus conclude that a proper way to treat the problem is within the framework of time-dependent quantum mechanics. However, it would be unfeasible to treat the motion of the particles in a nucleus in a time-dependent picture. Besides, it is also very difficult to define the initial conditions in a many-body time-dependent treatment [4]. This feature was already recognized in the beginning of quantum mechanics. Teichman and Wigner tried to reconcile the outgoing character of the decaying process with the conveniences of stationarity by solving the Schrödinger equation with outgoing boundary conditions (for references see [5]). However, in this case one finds that physical quantities, such as energies and probabilities, become complex. One may attempt to give meaning to the imaginary part of these complex quantities. Thus, it is usually assumed that the imaginary part of the energy represents the width of the resonance. One can even find situations where the imaginary part of probabilities (e.g., cross sections) also have a meaning [6]. All these interpretations are valid only if the resonances are isolated and therefore narrow. In these cases one finds that, indeed, the real part of the complex poles of the *S* matrix gives the position of the resonances while the corresponding imaginary parts give the total widths.

The resonances provided by the outgoing solutions of the Schrödinger equation (Gamow resonances) plus bound states and a number of scattering states in the complex energy plane form a representation (Berggren representation) [7]. The inner product (the metric) corresponding to the Berggren representation is giving by the product of the wave function times itself and not its complex conjugate. The theory is, therefore, outside the framework of ordinary quantum mechanics. Nevertheless, one can calculate the resonances and assign to the narrow ones the physical meaning mentioned above. Since resonances with large imaginary parts of the energy are not isolated, only those with small imaginary parts have physical meaning, and the theory itself provides its degree of validity [8].

Once we have defined the way to handle single-particle excitations, we shall focus on the treatment of residual two-body interactions. It is reasonable to expect that pairing properties in drip-line nuclei can be evaluated by using the BCS approximation assuming that the nucleons move either in bound states or in narrow resonances, since wider resonances and scattering states would only contribute to the continuum background [8]. This expectation would be fulfilled for proton excitations or in cases where there is not a $l=0$ bound orbit near threshold [9].

In other words, in order for the particles to feel the interaction among themselves they have to stay in definite states during a time long enough. If this time is too short then the particles do not have time ''to communicate'' with each other (e.g., to exchange pions) and their motion would only contribute to the background energy. Therefore, it is only narrow resonances that have to be included in the representation used to evaluate the nucleus. If such resonances do not exist (as it would be the case in a region of low spin neutron excitations) then the nucleus itself would not exist since the neutrons would escape before there is any time to measure any nuclear property. It is for this reason that in this paper we have chosen to analyze proton drip-line nuclei with well defined narrow resonances.

The treatment of the pairing interaction, in this basis and within the BCS approximation, does not differ much from the standard treatment [10].

In selecting the BCS transformation we have chosen the orientation of the intrinsic frame in such a manner that proton and neutron gaps are different from zero while the proton-neutron gap is zero, as customarily done in the treatment of the pairing interaction in stable nuclei [15]. Also, we have neglected the isoscalar channel of the interaction and we have treated the isovector part of the pairing force. This choice of the interaction is dictated by physical arguments, since convincing physical evidences about isoscalar pairing

*Email address: civitare@venus.fisica.unlp.edu.ar
†Email address: liotta@msi.se
‡Email address: vertse@tigris.klte.hu

are not available, so far. The reader is referred to the work of Ref. [15] for further details about the choice of the interaction.

The only formal difference between the formalism presented here and the standard BCS treatment is that now the scalar product between two quasiparticle wave functions is the integral of their product (for details see, e.g., Ref. [11]). Therefore, all physical quantities, when expressed in terms of BCS parameters, have the same form as the one that one would obtain by using real functions [10]. The BCS transformations can be performed without any reference to the complex character of the involved single-particle configurations. The matrix elements of the monopole separable force are the same, for all pairs, as usual. Notice that the pairing interaction acting on states belonging to the continuum are included through the narrow resonances. Since there is a limited number of these resonances, the convergence problem found in other treatments [12] does not appear here. The gap equation reads

$$\Delta = G \sum_i \Omega_i u_i v_i , \tag{1}$$

where G is the average matrix element of the separable monopole pairing interaction and $u_i(v_i)$ are the BCS amplitudes of the quasiparticle transformations. The quasiparticle energies are defined as the solutions of the standard dispersion relation

$$1 = G \sum_i \Omega_i / E_i . \tag{2}$$

In both equations the index i represents the complete set of quantum numbers needed to define a single-particle state belonging to the adopted (Berggren) basis.

Since we are interested in the description of drip-line nuclei and in potential applications of the BCS formalism under extreme conditions (finite temperatures) we shall also introduce the thermal averaging procedure on the correlated quasiparticle vacuum [13]

$$\langle \alpha_i^\dagger \alpha_i \rangle = f_i(T), \tag{3}$$

where T is the temperature and $f_i(T)$ is the quasiparticle thermal occupation factor defined in [13]. The thermal averaging procedure, for bound single-particle (quasiparticle) states, can be extended to include resonant single-particle

TABLE I. Proton single-particle states corresponding to $N=50$. The complex energies are in units of MeV.

State	Energy
$d_{5/2}$	(2.744,-0.000)
$s_{1/2}$	(4.447,-0.015)
$g_{7/2}$	(4.584,-0.000)
$d_{3/2}$	(5.544,-0.018)
$h_{11/2}$	(5.975,-0.000)
$f_{7/2}$	(9.147,-0.385)
$i_{13/2}$	(13.658,-0.090)

TABLE II. Proton single-particle states corresponding to $N=114$. The complex energies are in units of MeV.

State	Energy
$h_{9/2}$	(1.691,-0.000)
$f_{7/2}$	(1.880,-0.000)
$i_{13/2}$	(3.162,-0.000)
$p_{3/2}$	(4.520,-0.000)
$f_{5/2}$	(5.015,-0.000)
$p_{1/2}$	(5.769,-0.000)
$g_{9/2}$	(8.548,-0.004)
$j_{15/2}$	(10.161,-0.000)
$i_{11/2}$	(10.307,-0.000)

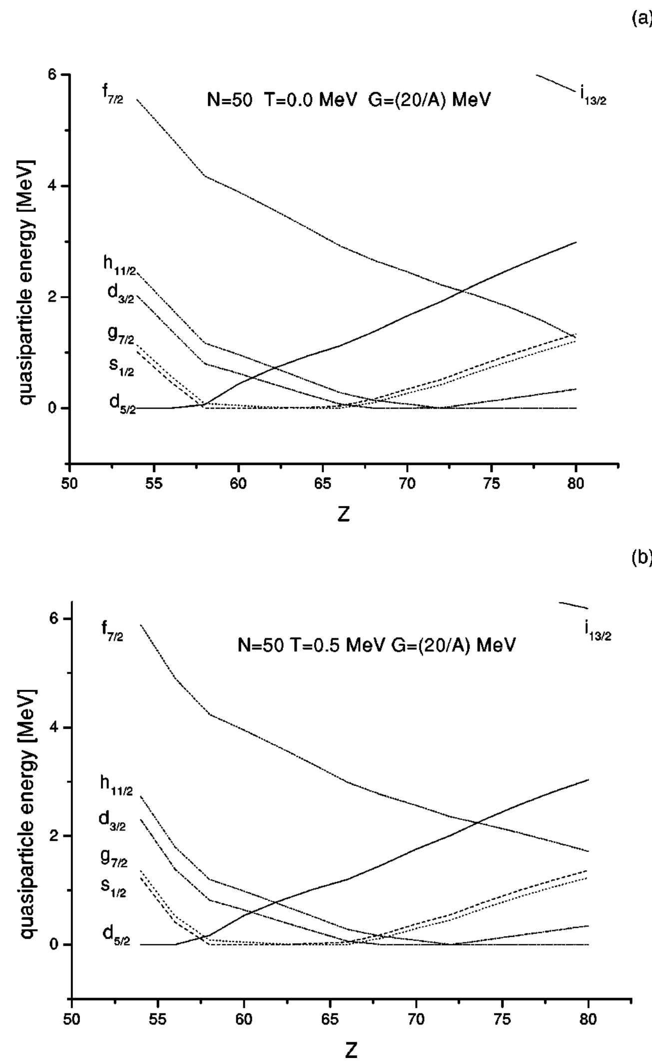

FIG. 1. Proton-quasiparticle spectrum, as a function of the proton number Z, for nuclei with $N=50$ neutrons. Case (a) corresponds to the zero temperature results $(T=0)$ and case (b) corresponds to $T=0.5$ MeV. The quasiparticle energies are given in units of MeV and the quantum numbers of each single-particle orbital are shown in the curves.

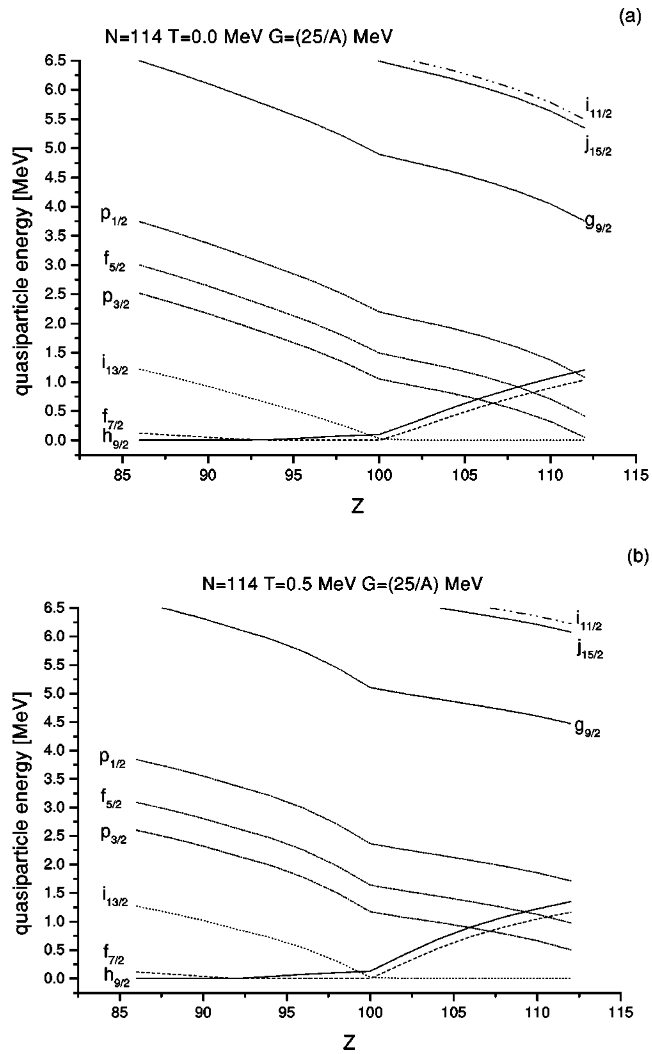

FIG. 2. Proton-quasiparticle spectrum for nuclei with $N=124$. Cases (a) and (b) show the results corresponding to proton-quasiparticle energies at temperatures $T=0$ and $T=0.5$ MeV, respectively.

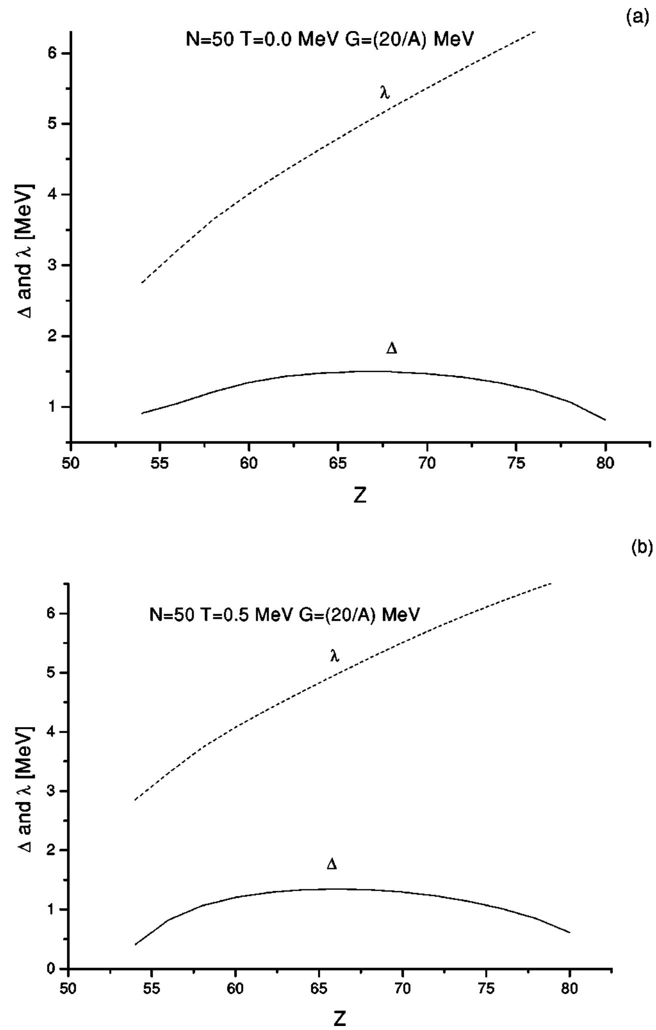

FIG. 3. Temperature dependence of the gap, Δ, corresponding to $N=50$ and $Z=56$, case (a), and $N=114$, $Z=90$, case (b). The chemical potential λ is shown in the figure.

states. This is done by considering that one is dealing with complex quantities. The resulting picture is rather appealing because the quasiparticle motion describing an effective mean field built by pairing correlations can "hit" resonant single-particle states by two different mechanisms, namely, (a) by the smearing out of the Fermi surface, due to pairing correlations and, (b) by thermal occupation of high lying single-particle states.

In the present context the inclusion of resonant states in the single-particle basis is meant to account for effects due to the continuum on the quasiparticle mean field. The inclusion of temperature-dependent effects aims at considering both ground state and excited states distributions on the construction of the quasiparticle mean field. We shall show that the BCS method can be applied to describe pairing correlations in drip-line nuclei both at zero ($T=0$) and finite ($T\neq 0$) temperatures. In the following we shall present and discuss the results of our calculations. We have performed BCS calculations for open shell proton-drip-line nuclei around two neutron shell closure, namely, $N=50$ and $N=114$. The proton-single-particle basis used in the calculations, for each of these neutron closed shell configurations, are given in Tables I and II, respectively. The proton-single-particle states are the solutions of a Woods-Saxon nuclear central potential that includes the Coulomb, the spin-orbit and the centrifugal terms. Wave functions of resonant states are constructed as described in Ref. [14]. The separable monopole pairing interaction of Eq. (1) is parametrized by the strength G. The BCS equations were solved in the basis that includes both bound states and resonant states, in the way that has been described previously. The corresponding quasiparticle energies, for the active single-particle states, were obtained for different proton numbers. The results are shown in Figs. 1 and 2. Cases (a) and (b) of these figures show the results corresponding to different temperatures. One sees that the inclusion of thermal excitations does not affect the results considerably. The quasiparticle spectrum shows a limited spreading even for very large values of Z. The temperature dependence of the gap, Δ, for the case with $N=50$ and $Z=56$, case (a), and $N=114$ and $Z=90$, case (b), is shown in

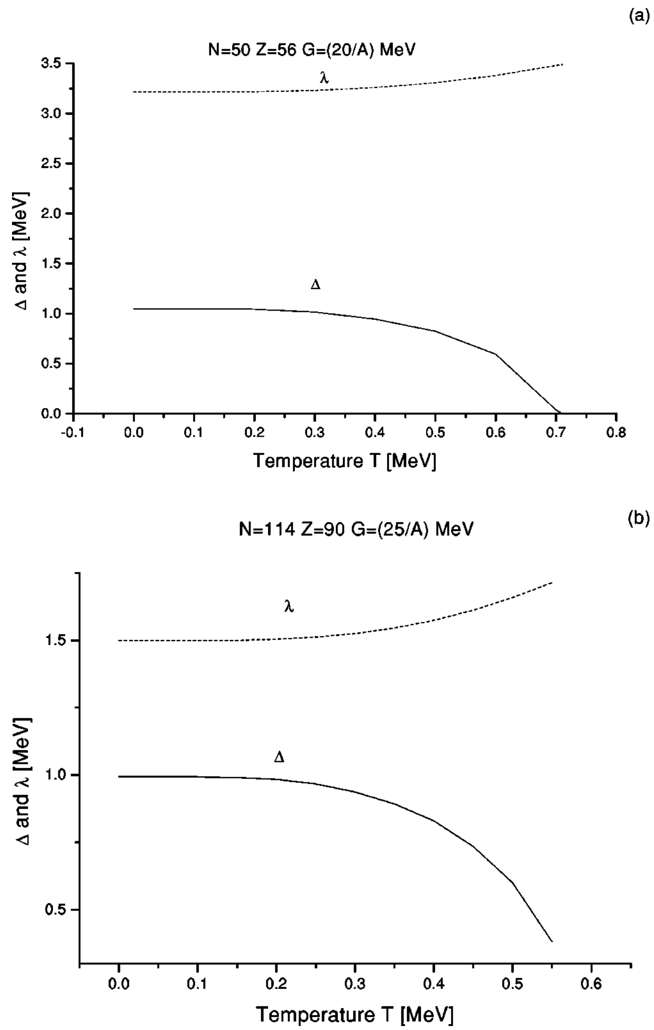

FIG. 4. Mass dependence of the gap, Δ, and of the chemical potential, λ, for nuclei with $N=50$, as a function of the proton number Z, for temperatures $T=0.0$ MeV, case (a), and $T=0.5$ MeV, case (b), respectively.

Fig. 3. The behavior of Δ versus T shows the well-known suppression of pairing correlations by thermal blocking effects [13]. One also sees in these cases a smooth temperature dependence of the chemical potential λ. Figure 4 shows the mass dependence of the gap Δ and of the chemical potential λ, as functions of the proton number, for $N=50$ and temperatures $T=0$ and $T=0.5$ MeV, respectively. The relatively constant value of the gap indicates that the quasiparticle mean field is well defined, even for the very extreme situations considered in the present calculations. In all cases shown here the imaginary part of the physical quantities are small and therefore they do not affect the interpretation of the corresponding real parts.

Although some of the examples that we have considered correspond to nuclei with very short half-lives, they illustrate rather nicely the main scope of the calculations, that is to treat bound and resonant single-particle states as components of the configurations activated by the pairing interaction.

The picture that emerges from these results supports the notion that the BCS method can be used to describe pairing correlations in proton-drip-line nuclei. The description of the associated single-particle basis, as composed by bound states and resonant single-particle states with complex energies, does not invalidate the use of the BCS formalism. The results that we have obtained, both for the zero and finite-temperature cases, show that the features of the solutions very much resemble the solutions corresponding to nuclei along the stability line. We think that these results are useful in dealing with the calculation of the structure of drip-line nuclei since the feasibility of BCS-type of calculations largely simplify the task of including residual interactions and collective effects.

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