Spontaneous symmetry breaking and the energy of nuclear isobaric analog states

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The asymmetry between proton and neutron numbers in a nucleus is viewed as a consequence of the spontaneous symmetry breaking of the isospin symmetry. The signatures of this effect, as it was suggested by Danchev, Khanna, and Umezawa and co-workers, may have been seen already in the energetics of the nuclear isobaric analog resonance state and in the systematic of double-odd double-even mass differences. In order to account for finite size effects, not included in Umezawa's approach, we have calculated the mean field term of the Hamiltonian in a realistic nuclear single-particle basis. The nonvanishing value of the I_3 current in the nonperturbative vacuum is explained in terms of ρ meson exchange.

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The realization of low-energy nuclear excitations as the signature of symmetry breaking mechanisms has been pointed out in a series of papers dealing, mostly, with boson excitations [1]. Umezawa and collaborators [2] have suggested the possibility that charge-dependent excitations in nuclei can be described in terms of the spontaneous symmetry breaking of the SU(2) isospin symmetry. The energetic of the nuclear isobaric analog states (IASs) is predicted from the sequence of levels belonging to a certain isospin multiplet by assuming the conservation of the total nuclear isospin. However, as pointed out by Umezawa and co-workers [2], the fact that the ground states of even-even (A,N,Z) and odd-odd $(A,N\mp 1,Z\pm 1)$ nuclei are nearly degenerate, in spite of the fact that the Coulomb interaction should break such a degeneracy, may indicate the presence of another mechanism of dynamical nature.

We have taken the suggestions of Ref. [2] as the motivation for the present work and we have tried to determine the correspondence between the conclusions of Ref. [2] concerning the nuclear matter case and the actual situation in finite nuclei. Our arguments are based on the same arguments advanced by Umezawa and co-workers [2], adjusted to account for nuclear structure effects. As we are going to discuss in the article, the present goal is restricted to a qualitative test of the mechanism of Ref. [2].

The most direct way of relating the degeneracy of the ground state of double-even and double-odd nuclei is based on the assumption that the conservation of the isospin current is broken by a spontaneous symmetry breaking mechanism, i.e: the vacuum expectation value of the isospin current

$$I_3=\left\langle 0\left|\phi^{\dagger}\frac{\tau_3}{2}\phi\right|0\right\rangle \tag{1}$$

is nonvanishing.

Since the nucleon field ϕ is a two component isospinor the vacuum expectation value of the current can be written in terms of the neutron excess $N-Z$. Following Ref. [2] one can introduce an order parameter and relate it to the vacuum expectation value of I_3. In the limit of large neutron excess and in spite of the breaking of the SU(2) isospin symmetry the alignment of the isospin third component is preserved by the change of a neutron (proton) into a proton (neutron), i.e., $I_3\rightarrow I_3\pm 1$. Thus one can think of the nucleus as an axially symmetric rotor in isospin space. In the mean field approximation, the nuclear ground state energy is a quadratic function of I_3 [3] and to this energy one should add the mean field value of the Coulomb interaction [3].

In the presence of the spontaneous symmetry breaking the I_3 current contributes to the mean field value of the energy with an additional linear term of the form

$$\frac{\lambda}{2}I_3:\phi^{\dagger}\tau_3\phi:. \tag{2}$$

In this equation λ is a coupling constant to be determined from the single-particle spectrum of the nuclear Hamiltonian H and I_3 is the vacuum expectation value of the isospin third component (hereafter, for the sake of simplicity, we shall omit the subindex 3).

In the semi-infinite nuclear matter limit considered in Ref. [2] the radial dependences of the nucleon fields ϕ have been replaced by plane waves and surface effects were accounted for by introducing a cutoff at the nuclear surface. After a straightforward calculation the neutron (ϵ_+) and proton (ϵ_-) energies can be written

$$\epsilon_{\pm}=\epsilon+\frac{1}{2}E_c\pm\frac{1}{2}(\lambda I-E_c), \tag{3}$$

thus leading to a proton-neutron energy difference of the order of [2]

$$\delta E=\epsilon_- -\epsilon_+=E_c-\lambda I. \tag{4}$$

This result means that, in principle, the proton-neutron energy difference is not solely determined by the Coulomb energy. In Eq. (3) the quantity ϵ is the energy associated to the nucleonic motion in a charge-independent density-dependent central potential. To compute the energy shift δE in a consistent manner the coupling constant λ should be determined as a function of the order parameter associated to the breaking of the isospin symmetry, for each value of I and for each value of the density. Assuming that protons and neutrons are confined inside the same volume and that they have uniform densities (nuclear matter limit) one can calculate λ easily. The values obtained for λ using this approximation are given by the expression [2]

$$\lambda = \frac{1}{I}(E_c - \delta E_{\bar{n}}), \tag{5}$$

where $\bar{n}$ is the density. The information on the average Fermi energy, for protons and neutrons, is given by the average density $\bar{n}$ corresponding to the motion in an average, charge independent, mean field [3]. In the following we shall test the validity of these approximations, as they have been presented in Ref. [2], by extracting λ from the known energetics of the IAS [4]. The data are fitted by the expression [4]

$$E_{\mathrm{GT}} - E_{\mathrm{IAS}} = \left(-30.0\frac{(N-Z)}{A} + 6.7 \right) \ \mathrm{MeV}, \tag{6}$$

where E_{GT} and E_{IAS} are the energies of the Gamow-Teller ($J^{\pi}=1^{+}$) and isobaric analog state ($J^{\pi}=0^{+}$) resonances, respectively. From these values and from the known energetic of the Gamow-Teller giant resonances [5–7] one can extract the energetic of the IAS. The obtained values are shown in Fig. 1 (solid line) and compared with the results of direct measurements. The experimental points of Fig. 1 (denoted by dots) are taken from Ref. [5] (for $A=90$), Ref. [6] (for $A=100$ and 116), and Ref. [7] (for $A=208$). Experimental error bars are also shown in Fig. 1. The solid line of Fig. 1 corresponds to energy of the isobaric analog state (E_{IAS}) extracted from Eq. (6). From Fig. 1, it is seen that, within error bars, the extracted values of IAS agree rather well with data. To determine the value of λ we can now calculate the energy E_{IAS} as a function of the isospin variable I. The results are shown in Fig. 2. The experimental values can be reproduced, within error bars, for values of λ in the interval $10^{-2}<\lambda<10^{-1}$, in units of MeV. Because of the relatively large error bars (which are of the order of 0.5 MeV) a more precise determination of the value of λ seems to be a rather difficult task. The best fit to data corresponds to $\lambda(\chi^2)=3.5\times10^{-2}$ MeV. The value indicated in Fig. 2 as λ_{ρ} is the value which we have obtained from the ρ-meson exchange mechanism, as we shall discuss below. In spite of the abovementioned deviation of the experimental results, one can still set up limits on the value of λ by performing both the analysis of data and some microscopic calculations, as we shall discuss next.

We have performed some calculations on two different levels: (a) by performing nuclear structure calculations to determine the energetic of the IAS and GT modes, in order to extract the value of λ as we have done by using the data, and (b) by constructing the proton-neutron interaction at the level of a Lagrangian, thus fixing the value of λ from an effective field theory.

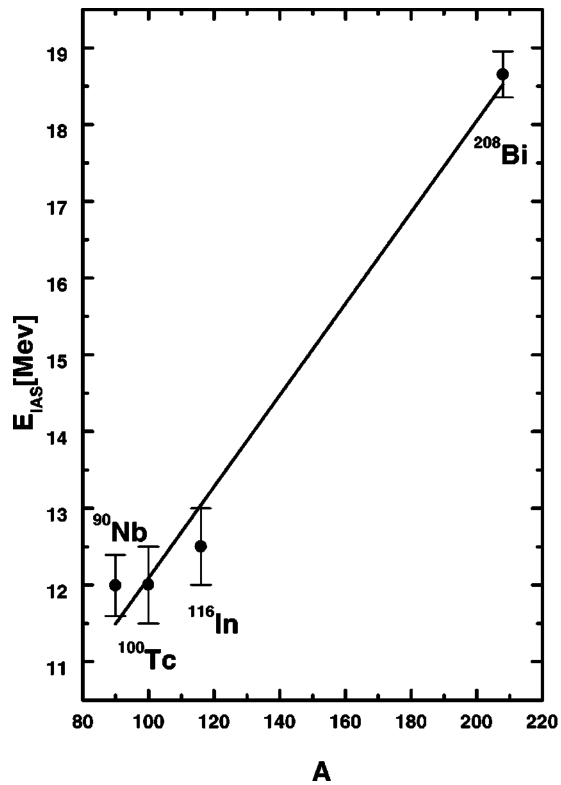

FIG. 1. Comparison between extracted (solid line) and measured (dots and error bars) energy of IAS states, for some of the nuclei considered in the text. The data are taken from Refs. [5–7].

Concerning point (a), conventional nuclear structure calculations of the IAS and GT modes need the definition of proton and neutron single particle basis, the treatment of the pairing components of the two body interaction, and the diagonalization of residual two body proton-neutron forces. This procedure is a matter for textbooks [8] and we shall omit a more detailed discussion. Instead, we shall indicate the main steps of the calculations. We have introduced residual proton-neutron interactions to describe the position of the IAS. The calculations were performed by using separable proton-neutron interactions treated in the framework of the random phase approximation (RPA) for close shell nuclei, and the quasiparticle random phase approximation (QRPA) for the case of open shell nuclei [8]. In performing the calculations we have used spherical harmonic oscillator single particle basis and separable monopole pairing interactions [9,10]. An example of the calculations is given in Ref. [11], where the formalism is applied to describe proton-neutron excitations of a even-even open shell nucleus. Hereafter, as an example, we shall discuss the case of proton-neutron excitations of the ground state of ^{116}Cd leading to the IAS resonance in ^{116}In. The results shown in Fig. 3 have been obtained by performing proton-neutron QRPA calculations [10] with a separable two body interaction of the type de-

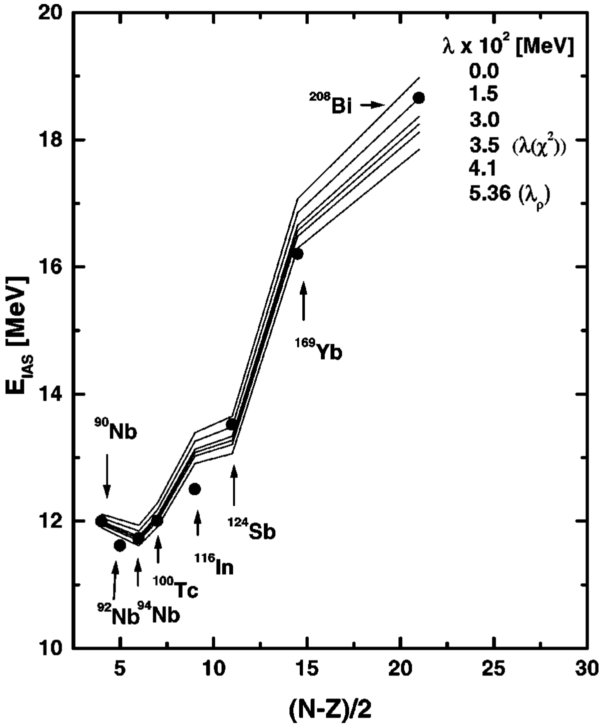

FIG. 2. Calculated values $E_{\mathrm{IAS}}=E_{\mathrm{Coulomb}}$-$\lambda I$, with $I=(N-Z)/2$. The solid lines correspond to different values of λ. Experimental values are shown by dots and they represent the energy centroids. The line denoted by $\lambda(\chi^2)$, corresponds to the value of λ which has been determined by a χ^2 fit to the data. The line denoted by λ_ρ corresponds to the value of λ determined from the ρ-meson exchange mechanism.

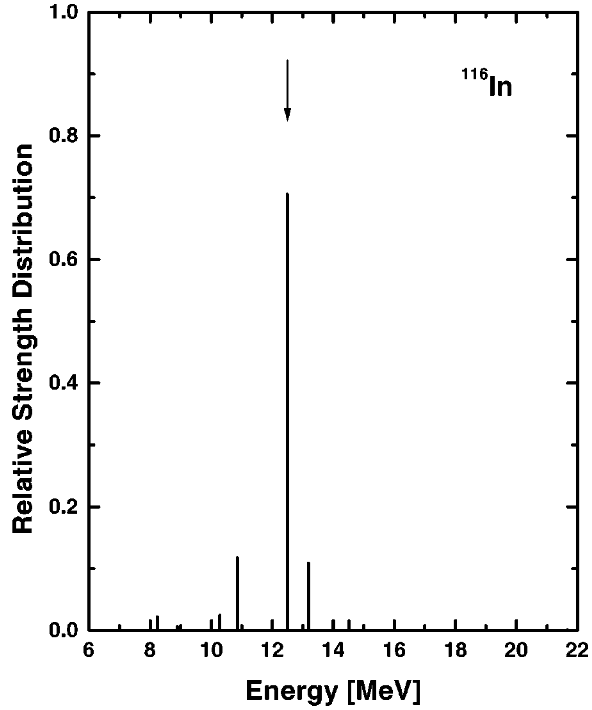

FIG. 3. Calculated strength distribution for the case of isospin ($J^\pi=0^+$) excitations in ^{116}In. The arrow indicates the position of the experimentally determined energy of the IAS resonance.

scribed in Ref. [11]. The results displayed in Fig. 3 show a narrow distribution of strength on three states around the position of the IAS resonance. The energy differences between these states are of the order of 1 MeV. This result is consistent with the assumption advanced by Umezawa *et al.* [2], about the above-mentioned symmetry related effects upon the energetic of the IAS. Similar results are obtained for the case of the IAS resonance in ^{100}Tc. If the calculated energy weighted centroid of the IAS are used to extract the value of λ one gets a value similar to the empirical value, namely: $\lambda \cong 3\times 10^{-2}$ MeV. However, when the isospin symmetry is approximately restored by performing a projection to fix the intrinsic isospin frame [11] the strength goes to a low-lying state which is the lowest proton-neutron excitation [11]. The addition of a residual (repulsive) interaction moves up this state to higher energies but not to the observed position of the IAS resonance. Again this result is fully consistent with the conjecture of Umezawa *et al.* about the nature of the IAS based on the dynamical symmetry rearrangement [2]. From the results of our calculations we have seen that this is indeed the case of the other nuclei considered in this work.

An alternative way of viewing the symmetry breaking mechanism advocated in Ref. [2] is the ρ-meson exchange between nucleus. By using the Lagrangian $L_{\rho NN}$ introduced in Ref. [12], with the coupling constant given in Table IV of Ref. [12], one gets an in-medium value of λ of the order of $\lambda_\rho = 5.4\times 10^{-2}$ MeV. This value should be compared with the value used in relativistic mean field calculations [15], which is of the order of $0.5-1.0\times 10^{-2}$ MeV. In dealing with the estimate of λ we have assumed that the ρ-meson exchange takes place between $A(A-1)/2$ pairs of nucleons, where A is the nuclear mass number. The structure of the Lagrangian used to extract the just quoted value of λ is given in Eqs. 2-29 and 2-30 of Ref. [12]. The values of the energy of the IAS resonances, corresponding to this value of λ are shown in Fig. 2.

From the above reported results we can conclude that the realization of the neutron excess in terms of the spontaneous breaking of the isospin symmetry [2], in analogy with the symmetry breaking mechanism of field theory [13], can bring in some new theoretical elements in dealing with the description of properties of the collective modes associated to charge exchange nuclear excitations [9]. In the present work we have adapted the formalism developed by Umezawa and co-workers to the case of single particle and collective excitations in finite nuclei. We have found that the breaking of the isospin symmetry and the dynamical rearrangement of it can in fact produce measurable effects upon the energy differences of double-even- and double-odd-mass nuclei as well as on the energy of the IAS. The analogy with the case of a field theory with ρ meson exchange (Refs. [12,15]) between nucleons was used and we found that the same concepts can be used to introduce both the breaking of the isospin symmetry and the associated Goldstone boson. Although none of the previously discussed concepts is by itself new in the description of nuclear single particle and collective motions [14] their use can facilitate the connection between the standard nuclear models and the emergent effective nuclear field theory. Due to the deviation of the data and other theoretical uncertainties, such as the form of the proposed ρ-NN Lagrangian and the momentum dependence of the interaction vertex at tree level, we may say that the present analysis supports the concepts advanced in Ref. [2] in a qualitative way.

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