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Questioning model-independent estimates of 2 $\nu\beta\beta$ decay rates

O. Civitarese [a], J. Suhonen [b]

[a] Dept. of Physics, Univ. of La Plata, c.c. 67 1900, La Plata, Argentina
[b] Department of Physics, University of Jyväskylä, P.O. Box 35, FIN-40351, Jyväskylä, Finland

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Abstract

In this work we discuss the validity of recently published results, by Rumyantsev and Urin, concerning nuclear matrix elements of the two-neutrino double-beta decay transitions. These authors claim that these matrix elements can be calculated in a model-independent way. We have re-analyzed their results and extended their formalism to account for proton-neutron correlations at the QRPA level of approximation. We have found that the formalism fails in describing the double beta decay observables. © 2000 Elsevier Science B.V. All rights reserved.

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The search of a satisfactory explanation for the observed values of nuclear double-beta-decay matrix elements, particularly for the two-neutrino mode $(2\nu\beta\beta)$ [1,2], has attracted the attention of experimental and theoretical physicists for more than a decade. The reader is kindly referred to [3] for a recent review on the subject. The physics of two-neutrino double-beta-decay transitions is widely viewed as the result of strong cancellations among virtual transitions and/or the dominance of single virtual transitions which exhaust a small fraction of the relevant sum rules [4–6]. In principle, it means that the values of the calculated matrix elements are model dependent, a fact that diminishes the predictive power of the theory. Recently, various attempts were made to eliminate the model-dependence of the $2\nu\beta\beta$ decay amplitudes [7–9]. Unfortunately, as it is shown in [10–12] for the case of the OEM [7,8], the model-independent analysis is strongly limited by the second-order perturbative structure of the matrix elements.

In the work of Rumyantsev and Urin [9] the question about the model independence of the $2\nu\beta\beta$ matrix elements is raised, this time starting from symmetry considerations and from the time dependence of the transition operator. As we shall discuss in detail in the following paragraphs, the method of [9] aims at explaining the smallness of the matrix elements in terms of a soft breaking of the SU(4) symmetry. In their paper the authors have made the analysis in an extreme single-quasiparticle model and they have pointed out the need for a more complete scheme, like the quasiparticle random-phase approximation (QRPA).

We have taken the conclusions of [9] as the motivation for the present study. We have investigated the implications of their approach in terms of the QRPA framework. We have found that the

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method suggested in [9] fails in describing the matrix elements of the $2\nu\beta\beta$ decay mode when the QRPA approximation is adopted to describe nuclear wave functions. This failure is due to some inconsistencies, as we shall show later on, which invalidate the conclusions of [9]. For the sake of completeness we briefly review in the following the formal steps discussed in [9].

The matrix element corresponding to the nuclear $2\nu\beta\beta$ decay for ground-state-to-ground-state transitions is defined by (adopting the notation of [9])

$$M_G = \sum_S \langle f|G^{(-)}|S\rangle \langle S|G^{(-)}|i\rangle \omega_S^{-1}, \tag{1}$$

where

$$\omega_S = E_S - \left(E_i + E_f\right)/2, \tag{2}$$

is the energy denominator of the second-order perturbative matrix element. Here the states $|S\rangle$ form a complete set of virtual 1^+ states in the intermediate double-odd nucleus, and $G^{(\pm)} = \sigma\tau^{\pm}$ is the Gamow–Teller (GT) operator. This equation can be re-written in terms of the shifted Gamow–Teller (or Fermi) operators $V_G^{(-)}$ of [9]

$$V_G^{(-)} = [H, G^{(-)}] - \Delta_G G^{(-)}, \tag{3}$$

where H is the nuclear Hamiltonian and the factor Δ_G is the excitation energy of the giant Gamow–Teller resonance (GTR) relative to the ground state of the initial nucleus. For simplicity we shall discuss hereafter only GT transitions, without losing generality since the inclusion of Fermi transitions can be done in a straightforward way. In Eq. (3) the commutator of the transition operator with the Hamiltonian expresses the time derivative of the GT operator. Next, following the authors of [9], one can use these expressions to cast the matrix element of Eq. (1) in the form

$$M_G = \sum_{S\neq G} \frac{\langle f|V_G^{(-)}|S\rangle \langle S|V_G^{(-)}|i\rangle}{\omega_S\left(\omega_G^2 - \omega_S^2\right)} - \frac{\langle f|V_G^{(-)}|G\rangle \langle G|G^{(-)}|i\rangle}{2\,\omega_G^2}, \tag{4}$$

where following the notation of [9] the ket $|G\rangle$ is the GTR state.

This result deserves some comments, namely: i) it contains the same information on virtual intermediate transitions as Eq. (1), and, ii) it separates the contribution due to the GTR from the contributions of the other 1^+ states which belong to the complete set of excitations of the intermediate nucleus. Also, iii) the expression does not depend on any symmetry property of the single-particle field. The authors of [9] have re-written Eq. (4) by assuming that the relations

$$\langle f|\left[V_G^{(-)}, G^{(-)}\right]|i\rangle = 0, \tag{5}$$

and

$$\langle G|V_G^{(-)}|i\rangle = 0, \tag{6}$$

are always valid. As done in [9], the expression for M_G, under the assumptions Eqs. (5) and (6), is the following

$$M_G = \omega_G^{-2} \sum_S \langle f|V_G^{(-)}|S\rangle \langle S|V_G^{(-)}|i\rangle \omega_S^{-1}. \tag{7}$$

Based on Eq. (7) the authors of [9] have performed calculation of matrix elements corresponding to the $2\nu\beta\beta$ mode and arrived at the conclusion that the observables can be calculated in a model-independent way (see also the discussion following Table 1 of [9]). Naturally, it would be desirable to work with such an expression in order to avoid the burden of more detailed calculations involving the structure of the virtual intermediate states. Unfortunately, such a goal has not been reached yet due to the second-order perturbative nature of the relevant matrix elements, among other reasons. This prevents us from removing the sum on intermediate states from Eq. (1) and even from using a sort of cancellation-free summation in evaluating Eq. (1). In fact one can easily understand the results of [9] by looking at the structure of Eq. (7). From the formal steps which we have outlined before we see that the supposedly dominant contribution to the sum, reminiscent of the invoked SU(4) symmetry, must be that given by the GTR as virtual intermediate excitation. The other contributions must be very small, as compared to the GTR one, if the SU(4) symmetry is softly broken as

Table 1
Theoretical and experimental matrix elements corresponding to $2\nu\beta\beta$ decay to the final ground state from the ground state of the nuclei listed in column 1. The theoretical matrix elements M_G of Eq. (16), calculated by using the QRPA method and at the values of g_{pp} listed in column 2, are shown in column 3. The fourth column shows the experimentally extracted values of the matrix elements, $M_G(\exp)$. These values have been obtained from the experimentally determined half-lives, corresponding to the references quoted in brackets, and using the value $g_A = 1$ for the axial-vector coupling constant. The matrix elements are scaled by the electron mass. The final two columns list the quantity S of Eq. (14), for the initial (S_i) and final (S_f) nuclei, in units of MeV.

Nucleus	g_{pp}	M_G	$M_G(\exp)$ [Ref]	S_i	S_f
^{76}Ge	1.04	0.12	0.10 [20]	−20.6	−17.1
^{82}Se	0.70	0.07	0.08 [21]	4.32	1.40
^{96}Zr	1.09	0.06	0.08 [22]	−22.5	−19.2
^{100}Mo	1.0	0.40	0.17 [23]	−20.0	−17.9
^{116}Cd	1.0	0.10	0.10 [24]	−8.93	−0.24

the authors of [9] claim. On the other hand, the use of the assumptions stated in Eqs. (5) and (6) eliminates the large contribution from the summation in Eq. (7), ending up with what the authors of [9] call a hindrance effect. As it has been shown extensively in the literature this line of argumentation fails because: i) the breaking of the SU(4) symmetry is not a soft one (spin-orbit effects are very large in heavy-mass nuclei) and ii) the experimental values of the matrix element M_G can be explained either by strong cancellations among intermediate contributions [13] or by a single-state dominance from low-lying states [6]. In addition, as we are going to show next, the above presented formalism relies upon the fulfillment of conditions Eqs. (5) and (6) which are not realized in realistic calculations. This statement becomes obvious in the context of the QRPA formalism, as it is shown below. In the following we are going to show that Eq. (7) does not represent the correct value of the matrix element M_G. Particularly, we are going to show that Eq. (5) does not vanish when the operators entering in the commutator are expressed in terms of QRPA phonons and the result of the commutation is given in terms of QRPA transition amplitudes.

In order to write the previous expressions in the QRPA framework, which amounts to introducing proton-neutron two-body correlations which go beyond the single-quasiparticle approximation, we have expressed the GT operator in terms of the QRPA phonons. Details of the formalism can be found in [3]. The starting point is the definition of particle-hole terms of the one-body GT operator, which are converted to the two-quasiparticle basis by means of the BCS transformation. This leads to

$$G_\mu^{(-)} = \sqrt{\tfrac{1}{3}} \sum_{pn} (p \| \sigma\tau^- \| n) \times \left[u_p v_n A^\dagger(pn,1\mu) + v_p u_n \tilde{A}(pn,1\mu) \right] + \text{scattering terms}, \tag{8}$$

where the exact form of the pair-creation and -annihilation operators $A^\dagger(pn,1\mu)$ and $\tilde{A}(pn,1\mu)$ is given in [3]. We can now introduce the phonon operator

$$Q_\mu^\dagger(m) = \sum_{pn} \left[X_{pn}(1^+,m) A^\dagger(pn,1\mu) - Y_{pn}(1^+,m) \tilde{A}(pn,1\mu) \right], \tag{9}$$

which creates a correlated two-quasiparticle state when acting on the QRPA vacuum. The full set of states defined in this manner spans the space of the virtual intermediate 1^+ states which appear in Eq. (1). In terms of these one-phonon creation and annihilation operators the GT operator can be written as

$$\left(G_\mu^{(-)}\right)_{\mathrm{QRPA}} = \sum_m \left[f_m Q_\mu^\dagger(m) + g_m \tilde{Q}_\mu(m) \right], \tag{10}$$

where the QRPA transition amplitudes f_m and g_m are given by

$$f_m = (1^+,m \| G^{(-)} \| \mathrm{QRPA}) \tag{11}$$

and

$$g_m = -(1^+,m \| G^{(+)} \| \mathrm{QRPA}) . \tag{12}$$

The explicit forms of f_m and g_m are given in Eqs. (3.29) and (3.30) of [3]. Within the same approximation the QRPA image of $V_G^{(-)}$ of Eq. (3) reads

$$\left(V_{G\mu}^{(-)}\right)_{\mathrm{QRPA}} = \sum_m \Big[(\Omega_m - \Delta_G) f_m Q_\mu^\dagger(m) - (\Omega_m + \Delta_G) g_m \tilde{Q}_\mu(m) \Big], \qquad (13)$$

where Ω_m is the QRPA eigenvalue of the m-th 1^+ state [3]. Using the above equations one can calculate the QRPA value of the commutator of Eq. (5). The result is

$$S \equiv \tfrac{1}{2} \langle f | \big[(G^{(-)})_{\mathrm{QRPA}}, (V_G^{(-)})_{\mathrm{QRPA}} \big] | i \rangle = \sum_m \Omega_m f_m g_m, \qquad (14)$$

where, for convenience (see the caption to Fig. 1), we have called S the sum in the r.h.s of Eq. (14). From the structure of Eq. (14) it is then obvious that the commutator will not vanish, except if the QRPA energies Ω_m or the transition amplitudes f_m and g_m of Eqs. (11) and (12) vanish. Concerning the other assumption, advanced in Eq. (6), we can now write the corresponding result in the QRPA approach. It reads

$$\langle G | \big(V_G^{(-)}\big)_{\mathrm{RPA}} | i \rangle = (\Omega_G - \Delta_G) f_G. \qquad (15)$$

To make the comparison of Eqs. (1) and (7) as explicit as possible, we write them at the QRPA level. The result corresponding to Eq. (1) reads

$$M_G = \sum_m \frac{f_m g_m}{\Omega_m - E_0}, \qquad (16)$$

and the result corresponding to Eq. (7) is given by

$$M_G = \frac{1}{(\Omega_G - E_0)^2} \sum_m \frac{(\Omega_G - E_i)^2 - \Omega_m^2}{\Omega_m - E_0} f_m g_m, \qquad (17)$$

where $E_0 = \frac{1}{2}(E_i + E_f)$ is the average between the initial and final ground state energies.

As it becomes obvious from Eqs. (14) and (15), which have been obtained by using the QRPA approach, they do not support the main conclusions upon which the calculations presented in Ref. [9] are based, namely: that Eqs. (5) and (6) are always valid. Their QRPA values, Eqs. (14) and (15), do not vanish. Notice that the validity of both Eqs. (5) and (6) is the crucial step needed to rewrite Eq. (4) in the form of Eq. (7). In the following we shall show and discuss the results of realistic calculations which we have performed as a quantitative test of the above expressions and assumptions. The $2\nu\beta\beta$ decay transitions which we have analyzed are the ground-state-to-ground-state decays in ^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo and ^{116}Cd. Among the set of $2\nu\beta\beta$ decay candidates we have selected these 5 examples since there exist accurate experimental data on half-lives of their transitions to the final ground state. The single-particle levels correspond to a Coulomb-corrected Woods–Saxon central potential, including the spin-orbit interaction, with parameters taken from [14]. The matrix elements of the two-body interaction were constructed from the OBEP-G-matrix [15], adjusted to account for nuclear finite-size effects. Pairing matrix elements were adjusted to reproduce observed odd-even mass differences, as described in [16].

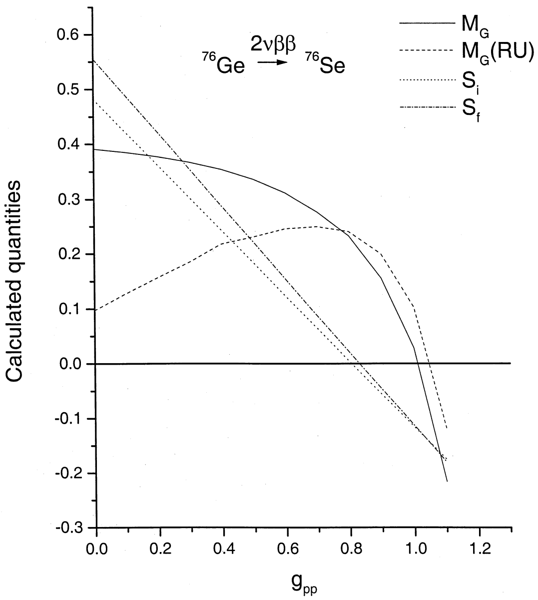

Fig. 1. Theoretical matrix elements corresponding to the $2\nu\beta\beta$ decay of ^{76}Ge, given by Eq. (16) [M_G] and Eq. (17) [M_G(RU)], are displayed as functions of g_{pp}. Both matrix elements are scaled by the electron mass. The quantity S of Eq. (14), calculated for the initial [S_i] and final [S_f] nuclei, is also shown. The values of S_i and S_f are given in units of MeV and multiplied by the factor 10^{-2}.

The residual two-body proton-neutron interaction was parametrized in terms of particle-hole and particle-particle channels, as described in [17]. The strength of the 1^+ component of the particle-hole proton-neutron interaction was adjusted to reproduce the energy of the GTR, for each of the considered cases. The strength of the proton-neutron particle-particle 1^+ channel (g_{pp}) of the two-body interaction [18], which is usually determined by reproducing known single-beta decay observables [19,6], was taken as a free parameter. This is done to show the sensitivity of the results upon variations of this parameter. The eigenvalues and eigenvectors of the QRPA equations, which describe the set of virtual 1^+ states of the intermediate double-odd nucleus belonging to the double β^- decay chain $(A,N,Z)-(A,N-1,Z+1)-(A,N-2,Z+2)$, were solved in single-particle basis consisting of two oscillator major shells around the proton and neutron Fermi surfaces, including intruder orbits and their spin-orbit partners. The above described procedure is by now a rather well-known one and the reader is kindly referred to [3] for further details of the calculations.

Our results are summarized in Table 1 and in Fig. 1. The dependence of the matrix elements M_G, of Eqs. (16) and (17), for the case of ^{76}Ge, upon the variation of the strength parameter g_{pp} is shown in Fig. 1. In the same figure we show the results corresponding to the quantity S of Eq. (14), for this transition. The sum of Eq. (14) was calculated for the initial (S_i) and final (S_f) nuclei, separately. The qualitative behaviour of the quantities of Fig. 1 is the same for the other cases of Table 1. As it is seen from this figure, the sum of Eq. (14) vanishes at only one point in the parametric space. For the case of the decay of ^{76}Ge it corresponds to the value $g_{pp}=0.8$, a value which yields a matrix element M_G twice as large as the experimentally determined one [2]. Considering that the magnitude of the commutator of Eq. (5) is 2×10^2 the magnitude of the quantities S_i and S_f depicted in the figure, the claim of [9] that it vanishes in a model-independent way appears to be unrealistic.

In Table 1 we show the theoretical matrix elements, calculated in the QRPA formalism, for the considered $2\nu\beta\beta$ ground-state-to-ground-state transitions. The theoretical values of M_G have been obtained by fixing the parameter g_{pp} at the values indicated in the table. For comparison, the experimental values of the matrix elements are also listed in the table. Both experimental and theoretical values of M_G listed in Table 1 are scaled by the electron mass. It may be noted that in all cases, except for the decay of ^{100}Mo, the experimental values of M_G can be reproduced by the QRPA calculations. For ^{100}Mo the QRPA breaks down slightly beyond $g_{pp}=1.0$ and the experimental value of M_G can not be reached with the same accuracy as for the other cases.

The results of Eq. (14) are shown in the last two columns of Table 1, both for the initial and final nuclei participant in the transitions. With the exception of the final branch S_f of the decay scheme of ^{116}Cd, the absolute values of S_i and S_f are large and none of them vanish in the vicinity of the values of g_{pp} listed in the table. Here it is worth pointing out that the magnitude of the sum S of Eq. (14) is just half the magnitude of the quantity $\langle f\|[V_G^{(-)},G^{(-)}]\|i\rangle$ of Eq. (5) showing that for the case of a realistic interaction and within the QRPA formalism Eqs. (5) and (6) are not realized. Therefore, the transformation of Eq. (4), which is generally valid, into Eq. (7), which depends on the validity of Eqs. (5) and (6), is not justified. Finally, and concerning the transition matrix element of Eq. (15), it is seen that it does not vanish unless $N=Z$ and the energies of the proton and neutron levels are the same. These conditions are never satisfied by double-beta-decay systems.

To end up with the discussion of the results we would like to comment briefly on the values given by Eq. (17). We have compared the results given by Eq. (16), which is the correct QRPA expression for the matrix element M_G, and the ones given by the QRPA version of Eq. (7) (e.g. Eq. (17)), which is the approximation advocated by Urin and Rumyantsev. The results are shown in Fig. 1 and they correspond to the transition in ^{76}Ge. It is seen from this Figure that the results of Eq. (17) differ from the results of Eq. (16) and that they are also model dependent. At the point where Eq. (14) vanishes, i.e, the point where the approximation of [9] must work, Eqs. (16) and (17) give similar results, as expected.

To conclude, we have discussed the consequences of the results of Ref. [9] in the context of the QRPA formalism. We have found that the conditions under which the matrix elements of the $2\nu\beta\beta$ decay mode can be treated as model-independent ones, according to the claim of Rumyantsev and Urin [9], are not realized in realistic calculations.

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