

# Spontaneous and dynamical breaking of mean field symmetries in the proton-neutron quasiparticle random phase approximation and the description of double $\beta$ decay transitions

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The breakdown of the isospin symmetry, at the level of the quasiparticle mean field approximation, and its partial restoration by effective interactions, at the quasiparticle random phase approximation (QRPA) level of approximation, are studied. The method used to define effective symmetry breaking two-body interactions has been applied previously to particle-number and rotational symmetry violations. The connection between the present approach and the proton-neutron QRPA method with renormalized two-particle interactions is discussed. The formalism is applied to calculate nuclear matrix elements for Fermi double beta decay transitions. [S0556-2813(99)03101-5]

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## I. INTRODUCTION

The formalism of the quasiparticle random phase approximation (QRPA) has been used rather intensively and with great success during four decades since the fundamental paper of Baranger appeared in the literature [1]. The various applications of the formalism, to describe nuclear vibrational modes, are contained in textbooks [2–4]. The use of the QRPA method to describe charge-dependent excitations ( $pn$ -QRPA) was reported by Halbleib and Sorensen [5]. Symmetry properties of the approximation can be found in articles written by Lane and Martorell [6] and by Marshalek and Weneser [7]. In the original version of the QRPA [1] the nuclear many body Hamiltonian consists of short-range pairing interactions and residual two body interactions and it is written in the quasiparticle basis (BCS approach) and diagonalized in the quasiparticle-pair basis. The structure of the ground state correlations generated at the QRPA level of approximation was studied in Ref. [4]. More recently the use of renormalized two-particle channels of the residual interactions at the level of the  $pn$ -QRPA matrix elements was suggested by Vogel and Zirnbauer [8]. The application of these concepts to realistic calculations of double beta decay observables can be found in Ref. [9]. After several years of theoretical efforts centered on the use of the  $pn$ -QRPA method to calculate single- and double- $\beta$ -decay observables [10,11] some questions associated with the consistency of the approach have been revised, partly due to some considerations about the collapse of the QRPA ( $pn$ -QRPA) approximation [12,13]. Studies of this question, performed in the framework of group theoretical models, have been presented in Ref. [14]. The analysis of the  $pn$ -QRPA collapse in terms of a phase transition in a parametric model space was presented in Refs. [15,16]. Recent results based on the separa-

tion of intrinsic and collective variables [17] confirm the notion that the standard formulation of the  $pn$ -QRPA method should be extended. Among the basic theoretical assumptions which should indeed be revisited, in dealing with the explanation of the  $pn$ -QRPA collapse are (a) the separate treatment of proton and neutron isovector pairing correlations, which are usually represented by unrelated BCS mean fields belonging to the initial and final double-even mass nuclei, (b) the onset of isoscalar pairing correlations, affecting both the double-even and double-odd mass nuclei, and (c) the resulting violation of the isospin symmetry once the proton-proton and neutron-neutron BCS procedure is applied to describe, approximately, the pairing correlations. All these effects would certainly become manifest at mean field (quasiparticle) level [18,19]. In addition to these effects, which are generally referred to as spontaneous symmetry violations, one should add the fact that empirical single particle basis are used as input for the  $pn$ -QRPA calculations, thus contributing to undesirable symmetry violations. As it has been pointed out long ago, the QRPA by itself may not be able to cure for the resulting mean field symmetry violations [6]. The relationship between the collapse of the  $pn$ -QRPA and the onset of isoscalar pairing correlations was discussed in Ref. [14], in the framework of the SO(8) global symmetry of the Hamiltonian. The spontaneous breaking of the isospin symmetry, induced by the separate BCS treatment of proton and neutron pairing correlations is rather obvious. In this respect, the inclusion of symmetry violating interactions may be crucial in treating isospin-dependent effects [17]. In the present paper we discuss on symmetry violation effects in the  $pn$ -QRPA by using a method due to Pyatov [20]. The main step of Pyatov's construction is the definition of an effective Hamiltonian which incorporates Dirac's constraints [21] to the original symmetry-breaking Hamiltonian. This method has been used previously in dealing with the violation of particle-number [22], rotational [23] and generalized Galilean invariances [24], and velocity-dependent effects [25]. It is our aim to apply Pyatov's method to isospin-dependent Hamiltonians, written in the quasiparticle basis, in order to explore the link between the collapse of the

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$pn$ -QRPA and the breakdown of the isospin symmetry. Particularly, we would like to determine the dependence of the renormalization applied to two particle interactions, in the  $pn$ -QRPA upon symmetry restoring effects. The paper is organized as follows: a review of the formalism is presented in Sec. II, results for the case of Fermi double- $\beta$ -decay transitions are presented in Sec. III. Conclusions are drawn in Sec. IV.

## II. FORMALISM

### A. The BCS mean field and the symmetry restoring interactions

The separable monopole pairing interaction can be approximately diagonalized in the BCS quasiparticle representation. The proton and neutron single quasiparticle Hamiltonian can therefore be written as

$$H_{11} = \sum_j (E_{pj}N_{pj} + E_{nj}N_{nj}), \quad (1)$$

where the one-quasiparticle term  $N_{q,j}$  is written in standard notation [3]

$$N_{q,j} = \sum_m \alpha_{q,jm}^\dagger \alpha_{q,jm}. \quad (2)$$

The subindex  $q$  denotes proton ( $p$ ) or neutrons ( $n$ ) states and  $(j,m)$  are single particle angular momentum variables.

The fact that the isospin symmetry is violated by the BCS quasiparticle mean field can be easily demonstrated by expressing the tensorial components (total angular momentum  $\lambda=0$ , isospin  $\tau=1$ ) [26,2] of the isospin one body operator in the quasiparticle basis

$$\begin{aligned} \tau^{(-)} &= \sum_j (t_j A_j^\dagger + \bar{t}_j A_j), \\ \tau^{(+)} &= \sum_j (t_j A_j + \bar{t}_j A_j^\dagger), \end{aligned} \quad (3)$$

and commuting it with the unperturbed BCS Hamiltonian (1). Note that only the two-quasiparticle terms of the isospin operators  $\tau^\pm$

$$\begin{aligned} A_j^\dagger &= \frac{1}{\sqrt{(2j+1)}} \sum_{m>0} \alpha_{p,jm}^\dagger \alpha_{n,j\bar{m}}^\dagger, \\ A_j &= (A_j^\dagger)^\dagger, \end{aligned} \quad (4)$$

will contribute to the expectation value of the commutators.

In the above equations  $E_{pj}$  and  $E_{nj}$  are quasiparticle energies and the reduced matrix elements in Eq. (3) are defined as

$$t_j = \sqrt{(2j+1)} u_{pj} v_{nj}, \quad \bar{t}_j = \sqrt{(2j+1)} u_{nj} v_{pj}. \quad (5)$$

The results corresponding to the commutators

$$[H_{11}, \tau^{(-)}] = \Theta^{(-)}, \quad (6)$$

$$[H_{11}, \tau^{(+)}] = \Theta^{(+)} = -(\Theta^{(-)})^\dagger,$$

lead to the definition of the operator

$$\begin{aligned} \Theta &= \frac{1}{2} (\Theta^{(-)} + \Theta^{(+)}) \\ &= \frac{1}{2} \sum_j E_j (t_j + \bar{t}_j) (A_j^\dagger - A_j), \end{aligned} \quad (7)$$

where  $E_j = E_{pj} + E_{nj}$ . Following Pyatov's method [20], the effective Hamiltonian which exhibits the symmetry can be constructed from the above commutator adding to  $H_{11}$  the induced interaction

$$H_{\text{res}} = -\gamma \Theta^\dagger \Theta. \quad (8)$$

The value of  $\gamma$  is determined by requesting that the zero energy mode [7] is decoupled from the physical spectrum of  $H_{\text{eff}} = H_{11} + H_{\text{res}}$ , as it will be shown below.

The QRPA treatment of  $H_{\text{eff}}$ , is performed by introducing the one-phonon operator  $\Gamma_\nu^\dagger$

$$\Gamma_\nu^\dagger = \sum_k (X_{\nu k} A_k^\dagger - Y_{\nu k} A_k), \quad (9)$$

and in this basis the QRPA equation of motion is written

$$[H_{\text{eff}}, \Gamma_\nu^\dagger] = \omega_\nu \Gamma_\nu^\dagger. \quad (10)$$

The solution of this equation of motion, in the quasiboson approximation  $[A_j, A_k^\dagger] = \delta_{jk}$ , can be cast in the form

$$(1 - s_{11}) = 0. \quad (11)$$

The quantity  $s_{11}$  is defined by

$$s_{11} = \frac{\gamma}{2} \sum_j E_j^2 (t_j + \bar{t}_j)^2 \left( \frac{1}{E_j - \omega_\nu} + \frac{1}{E_j + \omega_\nu} \right). \quad (12)$$

After solving the QRPA system of equations, in order to determine the eigenfrequencies  $\omega_\nu$  and the amplitudes  $X_{\nu k}$  and  $Y_{\nu k}$  of Eq. (9),  $H_{\text{eff}}$  is transformed to the phonon basis

$$H_{\text{eff}} = \text{const} + \sum_\nu \omega_\nu \Gamma_\nu^\dagger \Gamma_\nu. \quad (13)$$

The decoupling, at the QRPA level of approximation, of the zero energy mode can be performed by introducing the transformation due to Marshalek and Weneser [7]

$$\begin{aligned} \hat{P}_\nu &= (\omega_\nu/2)^{1/2} (\Gamma_\nu^\dagger + \Gamma_\nu), \\ \hat{L}_\nu &= -i(2\omega_\nu)^{-1/2} (\Gamma_\nu^\dagger - \Gamma_\nu). \end{aligned} \quad (14)$$

The expression of  $H_{\text{eff}}$  in terms of the operators  $\hat{P}_\nu$  and  $\hat{L}_\nu$  is given by

$$H_{\text{eff}} = \frac{1}{2} \sum_\nu (\hat{P}_\nu^2 + \omega_\nu^2 \hat{L}_\nu^2). \quad (15)$$

This diagonal form of  $H_{\text{eff}}$  is obtained by transforming the pair operators  $A_j^\dagger$  and  $A_j$  to the phonon basis  $(\Gamma_\nu^\dagger, \Gamma_\nu)$  and then to the basis  $(\hat{P}_\nu, \hat{L}_\nu)$ . The explicit expression of  $H_{\text{eff}}$  is

$$H_{\text{eff}} = \sum_{\nu\mu} (d_{\nu\mu} \hat{P}_\nu \hat{P}_\mu + c_{\nu\mu} \omega_\nu \omega_\mu \hat{L}_\nu \hat{L}_\mu), \quad (16)$$

with

$$\begin{aligned} d_{\nu\mu} &= \sum_{kl} [E_k \delta_{kl} + (V_{kl} + W_{kl})] \lambda_{\nu k} \lambda_{\mu l}, \\ c_{\nu\mu} &= \sum_{kl} [E_k \delta_{kl} + (V_{kl} - W_{kl})] \mu_{\nu k} \mu_{\mu l}. \end{aligned} \quad (17)$$

In these equations we have defined

$$\begin{aligned} \lambda_{\nu k} &= \frac{(X_{\nu k} + Y_{\nu k})}{\sqrt{2\omega_\nu}}, \\ \mu_{\nu k} &= \frac{(X_{\nu k} - Y_{\nu k})}{\sqrt{2\omega_\nu}}, \\ V_{kl} &= -W_{kl} = -\frac{\gamma}{4} E_k E_l (t_k + \bar{t}_k)(t_l + \bar{t}_l). \end{aligned} \quad (18)$$

The diagonalization of  $H_{\text{eff}}$  implies  $d_{\mu\nu} = c_{\mu\nu} = \frac{1}{2} \delta_{\mu\nu}$  and the decoupling of the zero energy mode requires that

$$\gamma \equiv \gamma_0 = 1 \left/ \left[ \sum E_k (t_k + \bar{t}_k)^2 \right] \right. \quad (19)$$

With this value of  $\gamma$ , the QRPA secular equation (11) takes the form  $\omega_n^2 F(\omega_\nu) = 0$ , which is obviously satisfied for  $\omega_\nu^2 = 0$  (zero-energy eigenmode) and  $F(\omega_\nu) = 0$  ( $\omega_\nu \neq 0$ ). The explicit form of  $F(\omega_\nu)$  is the following:

$$F(\omega_\nu) = \sum_k \frac{E_k (t_k + \bar{t}_k)^2}{(E_k^2 - \omega_\nu^2)}. \quad (20)$$

### B. Separable particle-hole and particle-particle $pn$ interactions

Results corresponding to realistic proton-neutron interactions in open shell systems have been compared rather successfully with results obtained by using schematic interactions of the form [27,28]

$$H = H_{11} + H_{\text{int}}, \quad (21)$$

with

$$H_{\text{int}} = \chi \tau^{(-)} \tau^{(+)} - \kappa P^{(-)} P^{(+)}. \quad (22)$$

The proton neutron pair operators  $P^{(\pm)}$  are written in the quasiparticle basis as

$$P^{(-)} = \sum_j (p_j A_j^\dagger - \bar{p}_j A_j), \quad (23)$$

$$P^{(+)} = \sum_j (p_j A_j - \bar{p}_j A_j^\dagger),$$

with

$$\begin{aligned} p_j &= \sqrt{(2j+1)} u_{pj} u_{nj}, \\ \bar{p}_j &= \sqrt{(2j+1)} v_{nj} v_{pj}, \end{aligned} \quad (24)$$

and they represent particle-particle (hole-hole) terms of the one particle operator  $\tau$ . The term of the Hamiltonian (22) which contains the pair operators  $P^\pm$  will be referred to as the particle-particle interaction. Solutions corresponding to the Hamiltonian of equation (21) have been obtained both exactly and approximately [15]. Since details of these calculations have been presented previously [15,28] further discussions about this Hamiltonian will be avoided. We shall calculate  $pn$ -QRPA solutions to it in the presence of the symmetry restoring mechanism described in the previous subsection.

Before ending this section we would like to summarize the formalism presented in Secs. II A and II B. In Sec. II A we have shown that the BCS mean field does not preserve the isospin symmetry and we have used Pyatov's Method [20] to partially restore it at the two-quasiparticle level of approximation. In Sec. II B we have introduced an effective Hamiltonian which obviously breaks the isospin symmetry. The  $pn$ -QRPA treatment of this Hamiltonian leads to collapse of the approximation for some values of  $\kappa$ . In this context the question to ask is, of course, to which extent these features survive if the BCS mean field is readjusted in such a way that the spontaneously broken isospin symmetry is partially or totally restored in a dynamical way [i.e., by adding terms such as the ones of  $H_{\text{int}}(\chi, \kappa)$ ]. The first obvious answer to such a question would refer to limitations in the values of the renormalized particle-particle constant  $\kappa$  resulting from the inclusion of terms depending on  $\gamma$  in the Hamiltonian.

### III. RESULTS AND DISCUSSION

In this section we are going to present the results of calculations of proton-neutron  $0^+$  excitations in  $^{76}\text{As}$  and the corresponding matrix elements for the ground state to ground state Fermi-two neutrino-double beta ( $2\nu\beta\beta$ ) decay transition  $^{76}\text{Ge} \rightarrow ^{76}\text{Se}$ . The single particle basis for protons and neutrons has been taken from Ref. [28]. In this basis we have solved BCS equations for protons and neutrons separately. Active particles outside the  $N=Z=20$  closure were considered. The pairing strength parameters, of a separable monopole pairing force interaction, were adjusted in order to reproduce observed odd-even mass differences around  $A=76$ .

*Symmetry restoring effects at the BCS mean field level.* The most general form of the Hamiltonian, at lowest order in the quasiboson expansion (i.e., by keeping terms with  $A^\dagger A$ ,  $A^\dagger A^\dagger$ , and  $AA$ ), contains terms which are proportional to  $\gamma, \chi$ , and  $\kappa$ . Effects associated to the spontaneous symmetry breaking of the isospin symmetry by the BCS quasiparticle mean field are explored by studying the dependence of the QRPA spectrum upon  $\gamma$ . Figure 1 shows the

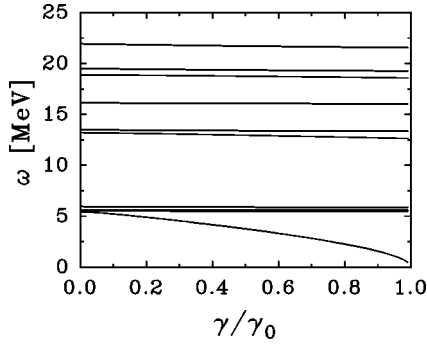


FIG. 1.  $pn$ -QRPA energies  $\omega$  as a function of the ratio  $\gamma/\gamma_0$ , corresponding to the Hamiltonian  $H_{\text{eff}}=H_{11}+H_{\text{res}}$  [see Eqs. (1) and (8)]. The values of  $\omega$  are given in MeV.

$pn$ -QRPA eigenvalues  $\omega$  of  $H_{\text{eff}}=H_{11}+H_{\text{res}}$ , given by Eqs. (1) and (8), for proton-neutron  $J^\pi=0^+$  excitations in  $^{76}\text{As}$ . Note that as  $\gamma \rightarrow \gamma_0=1/[\sum_k E_k(t_k+\bar{t}_k)^2]$  the lowest eigenvalue goes to zero. This result is the direct consequence of the use of Pyatov's prescription [20]. The strength  $\gamma=\gamma_0$  represents the value of the induced interaction (8) which restores the isospin symmetry, broken by the BCS approach, at the quasiboson level. Naturally the breakdown of the symmetry is due to the adoption of separate quasineutron and quasiproton mean fields and it is obviously nonphysical.

Since  $\gamma_0$  represents the value of the induced coupling for which the symmetry is restored, it can be argued that the inclusion of residual interactions ( $\chi, \kappa$ ), for partial restoration ( $\gamma < \gamma_0$ ), would break the symmetry dynamically. An example of this mechanism is given by the well-known fact that renormalized particle-particle ( $\kappa$ ) interactions can produce similar effects (i.e., the vanishing of the lowest  $pn$ -QRPA eigenvalue, as we shall see next). The effects of the partial restoration of the quasiparticle mean field symmetry upon the  $pn$ -QRPA spectrum corresponding to particle-hole ( $\chi$ ) residual interactions is shown in Fig. 2. The Hamiltonian corresponding to this case is  $H=H_{11}+H_{\text{res}}+H_{\text{int}}$ , where  $H_{\text{res}}$  is the above defined [Eq. (8)] symmetry restoring interaction and  $H_{\text{int}}$  is the Fermi separable force, with particle-hole and particle-particle terms included depending on the coupling constants  $\chi$  and  $\kappa$  [Eq. (22)]. Note that the repulsion induced by the particle-hole interaction ( $\chi$ ) is soft-

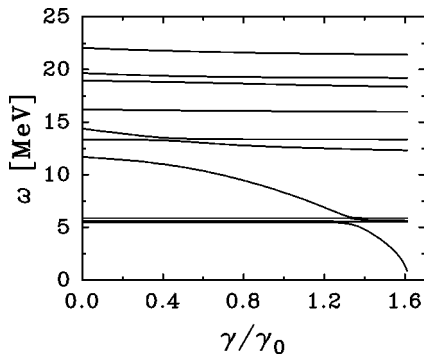


FIG. 2.  $pn$ -QRPA energies  $\omega$  as a function of the ratio  $\gamma/\gamma_0$ , corresponding to the Hamiltonian  $H=H_{11}+H_{\text{res}}+H_{\text{int}}$  [Eqs. (1), (8) and (22)]. The values of the coupling constants of the particle-hole  $\chi$  and particle-particle  $\kappa$  channels of the residual interaction are fixed at 0.6 MeV and 0.0 MeV, respectively.

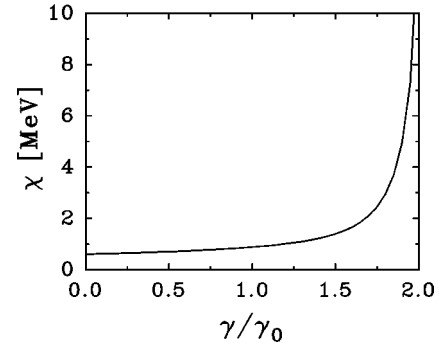


FIG. 3. Self-consistent value of  $\chi$ , in units of MeV, as a function of the ratio  $\gamma/\gamma_0$  and for  $\kappa=0.0$  MeV. All values of  $\chi$  on the curve give  $E_{\text{IAS}} \approx 11$  MeV.

ened by the attractions induced by the symmetry restoring interaction ( $\gamma$ ). The effect is particularly important for the eigenvalue of lowest energy and for the collective mode, which corresponds to the isobaric analog state (IAS). In the standard application of the  $pn$ -QRPA method the coupling constant  $\chi$  is fixed in such a way that the position of the IAS is reproduced. If  $\chi$  is varied, to reproduce a constant value of the energy of the IAS ( $E_{\text{IAS}}$ ) while changing  $\gamma$ , then the results shown in Figs. 3 and 4 are obtained. In Fig. 3 the value of  $\chi$  which reproduces the energy  $E_{\text{IAS}}$  is shown as a function of the ratio  $\gamma/\gamma_0$ . These values are practically constant up to  $\gamma=\gamma_0$  and show a fast increase for larger values of  $\gamma$ . The  $pn$ -QRPA spectrum obtained by fixing the energy of the IAS, by changing  $\chi$  along the curve of Fig. 3, is shown in Fig. 4. Next, in Fig. 5 we show results corresponding to the  $pn$ -QRPA spectrum, as a function of  $\kappa$ , obtained for the coupling constants  $\chi=0.6$  MeV and for different values of the ratio  $\gamma/\gamma_0$ . From these results it is seen that the behavior of the lowest eigenvalue is quite similar to the one corresponding to Fig. 1, and that the tendency to shift the other eigenvalues to lower energies is more evident for this case. By increasing the ratio  $\gamma/\gamma_0$  the value of  $\kappa$  which produces the collapse of the  $pn$ -QRPA spectrum decreases. It means that the renormalization of the  $P^-P^+$  term of the Hamiltonian is limited by the break down of the isospin symmetry at the level of the quasiparticle mean field. Critical values of  $\kappa$ , for  $\chi=0.6$  MeV and as a function of the ratio  $\gamma/\gamma_0$ , are shown in Fig. 6. From these results it is therefore concluded that the induced symmetry restoring interaction produces a

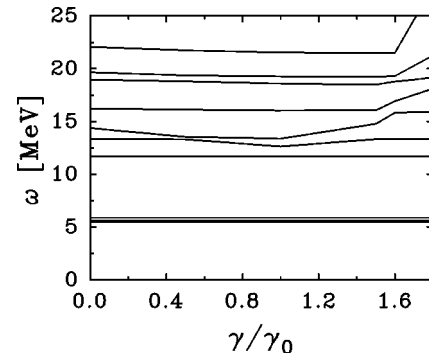


FIG. 4.  $pn$ -QRPA spectrum, corresponding to the self-consistent value of particle-hole constant  $\chi$  (see Fig. 3), as a function of the ratio  $\gamma/\gamma_0$ .

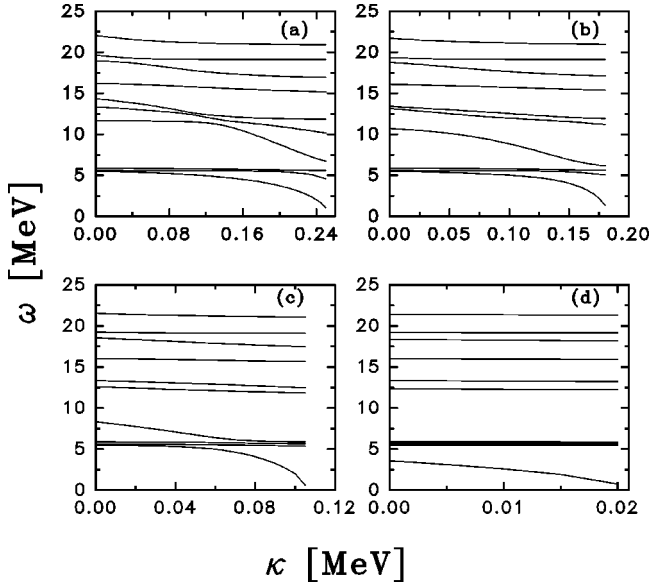


FIG. 5.  $pn$ -QRPA spectrum of  $H=H_{11}+H_{\text{res}}+H_{\text{int}}$  as a function of the coupling constant  $\kappa$  [see Eq. (22)]. The energies  $\omega$  are given in units of MeV, the coupling constant  $\chi$  is fixed at the value  $\chi=0.6$  MeV, and the ratio  $\gamma/\gamma_0$  has the value 0 (a), 0.5 (b), 1.0 (c), and 1.5 (d), respectively.

strong renormalization of the  $\kappa$ -dependent interaction. This strong renormalization is by far more important than the one needed to produce the collapse of the  $pn$ -QRPA for  $\gamma=0$ .

In the previously shown figures the induced symmetry restoring interaction has been parametrized in terms of the ratio  $\gamma/\gamma_0$ . Actual values of  $\gamma_0$ , as a function of the neutron excess, are shown in Fig. 7. It is observed that the values of  $\gamma_0$  are distributed around  $0.8 \times 10^{-2} \rightarrow 1.1 \times 10^{-2}$  MeV $^{-1}$ .

To conclude with the analysis of the results presented until now we can say that the collapse of the  $pn$ -QRPA produced by particle-particle interactions is strongly dependent upon the spontaneous breaking of the isospin symmetry, which is forced by the BCS approximation. It was shown that the partial restoration of the symmetry can strongly reduce the value of  $\kappa$  for which the collapse is produced. It also means that the crossing of eigenvalues induced by  $\kappa$  [15] and the appearance of a zero eigenvalue associated to the symmetry ( $\gamma=\gamma_0$ ) are different phenomena [6].

In the following we shall present the results corresponding to the matrix element

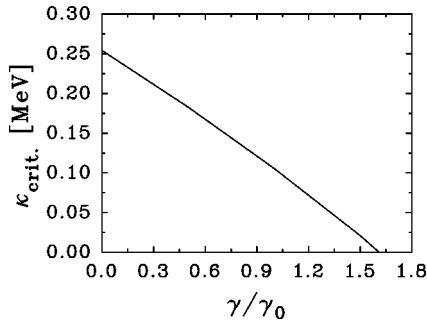


FIG. 6. Critical values of the particle-particle coupling constant  $\kappa$ , in units of MeV, as a function of  $\gamma/\gamma_0$  and for  $\chi=0.6$  MeV.

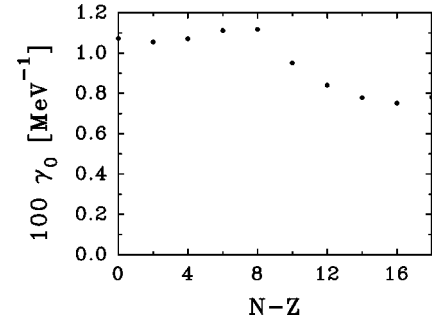


FIG. 7. Strength of the symmetry restoring interaction  $\gamma_0$ , in units of MeV $^{-1}$ , as function of the neutron excess  $N-Z$ , for isotopes of Ge.

$$M_{2\nu} = \sum_n \frac{\langle f || \tau^- || n \rangle \langle n || \tau^- || i \rangle}{E_n + E_0}, \quad (25)$$

which represents the nuclear matrix element associated to  $2\nu\beta\beta$ -Fermi transitions between the initial ( $i$ ) and final ( $f$ ) ground states [15]. We shall show results for  $M_{2\nu}$  obtained with the  $pn$ -QRPA treatment of three different Hamiltonians, namely, (i) BCS+induced symmetry restoring  $\gamma$  interactions, (ii) BCS+particle-hole ( $\chi$ ) and particle-particle ( $\kappa$ ) interactions, and (iii) BCS + particle-hole ( $\chi$ ) and particle-particle ( $\kappa$ ) interactions in the presence of a partial restoration of the symmetry. The results for case (i) are shown in Fig. 8. The dependence of the matrix element  $M_{2\nu}$  upon  $\gamma$  shown in this figure is particularly relevant because it demonstrates that the matrix element vanishes for a value of  $\gamma$  which is not  $\gamma_0$  ( $\gamma/\gamma_0 \approx 0.92$ ), i.e., the value of  $\gamma$  for which the symmetry is restored. This is indeed true at the QRPA level of approximation, which has been observed both in the definition of  $\Theta$  and in the solutions of the commutator of Eq. (10). This feature remains also when the  $ph$  channel of the interaction is activated (see Fig. 8). Again for this case the matrix element vanishes at  $\gamma \neq \gamma_0$  and the overall reduction of its value for  $\gamma=0$  is due to the quenching induced by the repulsive particle-hole interaction. The suppression of the matrix element  $M_{2\nu}$  induced by renormalized particle-particle interactions ( $\kappa$ ) between quasiprotons and quasineutrons is shown in Fig. 9 for different values of the

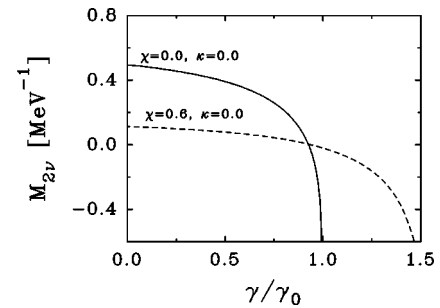


FIG. 8. Matrix elements  $M_{2\nu}$ , for Fermi  $2\nu\beta\beta$  ground state to ground state transition in  $^{76}\text{Ge}$ , as a function of  $\gamma/\gamma_0$ . The values of the coupling constants  $\chi$  and  $\kappa$  used in the  $pn$ -QRPA calculations are  $\chi=0.0$  MeV and  $\kappa=0.0$  MeV (solid-line), and  $\chi=0.6$  MeV and  $\kappa=0.0$  MeV (dashed-line), respectively.  $M_{2\nu}$  is given in units of MeV $^{-1}$  and the constant  $E_0$  of Eq. (25) was fixed at  $E_0=0.5$  MeV.

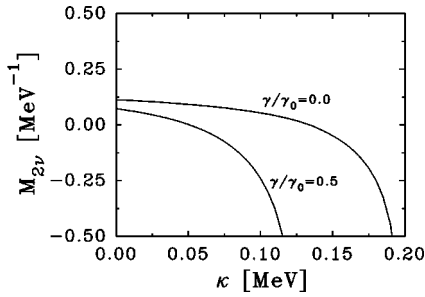


FIG. 9. The matrix element  $M_{2\nu}$ , of Eq. (25), as a function of the particle-particle constant  $\kappa$ , for  $\chi=0.6$  MeV and  $\gamma/\gamma_0=0.0$  and 0.5, as indicated in the figure.

ratio  $\gamma/\gamma_0$ . Note that the value of  $\kappa$  which produces the total suppression of the matrix elements, for  $\gamma/\gamma_0=0.5$ , is three times smaller than the value corresponding to  $\gamma/\gamma_0=0.0$ . It is therefore concluded that the suppression of  $M_{2\nu}$  induced by renormalized particle-particle interactions ( $\kappa$ ) depends upon symmetry breaking mean field effects.

#### IV. CONCLUSIONS

In this work we have investigated some consequences of the spontaneous breaking of the isospin symmetry produced

by the adoption of the BCS quasiparticle mean field, constructed from proton-proton and neutron-neutron pairing correlations. We have applied the method due to Pyatov [20] to define induced effective interactions which can dynamically restore the symmetry. We have compared the results of this approach with the results of the conventional QRPA method and found that the collapse of the  $pn$ -QRPA lowest eigenvalue produced by renormalized particle-particle interactions depend strongly on the induced symmetry restoring interactions. It is also found that the collapse of the  $pn$ -QRPA solutions and the cancellation of the matrix elements  $M_{2\nu}$  occur at different values of the induced couplings  $\gamma/\gamma_0$ . This feature remains valid when renormalized particle-particle interactions are turned on. It is seen, from the above discussed results, that the  $pn$ -QRPA collapse induced by  $\kappa$  is restricted to values of  $\kappa$  which are strongly limited by symmetry considerations.

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