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Collective description of nuclear double beta decay transitions

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Abstract

A consistent treatment of the intrinsic and collective coordinates relevant for the calculation of matrix elements describing nuclear double beta decay transitions is introduced. The method, which was originally developed for the case of nuclear rotations, is adapted to include isospin and number of particles degrees of freedom. To illustrate its main features we apply the formalism to the case of Fermi transitions in a simplified model. From the corresponding results we conclude that the uncertainties found in many existing double beta decay calculations might be largely due to the mixing of physical and spurious effects in the treatment of isospin dependent interactions.

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One can hardly overestimate the importance of the double beta decay as a process explicitly linking the physics of neutrinos with the nuclear structure [1–3]. Nuclear double beta decays are described as second order processes which involve the electroweak decay of two nucleons. These transitions are allowed by the Standard Model if they proceed via the emission of two electron-antineutrino pairs $(2\nu\beta\beta)$ and totally forbidden if they proceed through lepton number violating decays, as the neutrinoless mode $(0\nu\beta\beta)$ which is a unique test of the properties of the neutrino [2,3]. The $(2\nu\beta\beta)$ transitions have been observed [3]. Their correct theoretical description is a necessary step towards the understanding of the neutrinoless mode.

Earlier calculations of $(2\nu\beta\beta)$ were performed within limited shell model spaces [1]. Since the possible double beta decays emitters are heavy nuclei, full scale shell model calculations are unfeasible, and one has to resort to mean field treatments such as the BCS + RPA. Within such approach, it was shown that the inclusion of pairing-type proton-neutron interactions resulted in the suppression of the $(2\nu\beta\beta)$ matrix elements [4,5]. Although this supression was also obtained within several other approaches [6] and it was confirmed by the few

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available shell model calculations [7], the reliability of the theoretical predictions has been hampered by unstabilities in the BCS + RPA treatments. An alternative approach based on group theoretical methods has confirmed the existence of a zero-energy state for certain values of the strength of the proton-neutron, particle-particle, effective interaction [8,9]. The appearance of such a state has been interpreted as a signature of a phase transition [10].

Here we take a point of view based on the fact that the zero-energy state is a consequence of the breakdown of the isospin symmetry implicit in the (separate) neutron (n) and proton (p) BCS solutions [11]. As similar to the case of deformed nuclei, the symmetry may be restored in the laboratory frame through the introduction of collective coordinates. There is, however, a complication due to the fact that in many existing double beta decay calculations the strength of the proton-neutron interactions is taken as an adjustable parameter. As a consequence of this the resulting effective nuclear hamiltonian does not, in general, conserve isospin. In this paper we do not attempt to discuss the derivation of this effective interaction from first principles (Coulomb effects, np mass differences, etc.). However, given such interaction, a central many-body problem that must be solved is to disentangle unphysical isospin violations introduced by the formalism (i.e. BCS approximation for separate protons and neutrons) from those isospin violations produced by isotensor components of the effective nuclear hamiltonian.

The basic aim of this letter is to introduce a formalism which solves this problem through an exact, albeit perturbative, way. Such formalism is based on the treatment of collective coordinates and can be applied to a general realistic nuclear hamiltonian. In order to simplify the presentation, however, we will apply it here to the case of particles moving in a single j-shell and coupled through a charge dependent monopole pairing force. This exactly soluble model has been used [9] for the description of $(2\nu\beta\beta)$ transitions of the Fermi-type and, moreover, it already involves all the complications associated with the collective treatment.

The corresponding hamiltonian is

$$H = \sum_{\upsilon} \left(e_{\upsilon}\tau_{\upsilon} - g_{\upsilon} S^{+}_{\upsilon} S_{\upsilon} \right) - \tfrac{1}{2} g_{\perp} S^{+}_{\perp} S_{\perp} \qquad (1)$$

where $\upsilon = p,n$. Use is made of the operators

$$S^{+}_{\upsilon} = \sum_{m>0} c^{+}_{\upsilon m} c^{+}_{\upsilon \bar{m}}; \quad S^{+}_{\perp} = \sum_{m>0} \left(c^{+}_{pm} c^{+}_{n\bar{m}} + c^{+}_{nm} c^{+}_{p\bar{m}} \right),$$

$$\tau_A = \tfrac{1}{2}(\tau_p + \tau_n); \quad \tau_0 = \tfrac{1}{2}(\tau_p - \tau_n); \quad [\tau_{\bar{1}}, \tau_1] = \tau_0,$$

$$T_A = \frac{1}{2}(T_p + T_n); \quad T_0 = \tfrac{1}{2}(T_p - T_n);$$

$$[T_{\bar{1}}, T_1] = -T_0$$

where τ_{υ} are the number operators and $\tau_{\pm 1}$ ($\bar{1} = -1$) the rising and lowering isospin operators in the spherical representation. The $T_{\upsilon}, T_{\pm 1}$ are the corresponding generators of collective rotations in gauge- and isospace. The parameter $g_{\perp}$ plays the role of the renormalization factor g_{pp} introduced in the literature [4,5].

The introduction of collective degrees of freedom is compensated through the appearance of the constraints

$$\tau_z - T_z = 0; \quad (z = n, p, \pm 1), \qquad (2)$$

which express the fact that we can rotate the intrinsic system in one direction or the body in the opposite one without altering the physical situation [12]. Physical states should be annihilated by the four constraints and physical operators should commute with them.

The collective Hilbert space appropriate for an isospin conserving pairing interaction was originally introduced in Refs. [11,13,14]. The states may be labeled by the four quantum numbers $|T_A, T, m, k\rangle$, where T_A is the total number of pairs of particles. The quantum numbers $m \equiv \frac{1}{2}(T + M)$ and $k \equiv \frac{1}{2}(T + T_0)$ determine M and T_0, the isospin projections in the laboratory and intrinsic frame, respectively. We focus on states such that $m \ll T$ and $k = 0$. Hereof we drop the label k from the collective states.

Unphysical violations of the isospin symmetry are allowed in the intrinsic frame. Such frame may be defined, *for instance*, by the condition $\bar{S}_{\perp} = 0$, where the bar denotes the g.s. expectation value [14]. This condition is precisely satisfied by performing the usual separate Bogoliubov transformation for protons and neutrons. The rotations in isospace and gauge space restore the symmetries which are present in the laboratory frame. Thus the *np*-pairing becomes ef-

fectively incorporated, as well as the pairing between identical particles.

However, as different from previous cases where collective coordinates have been used, we are dealing here with an interaction which, in general, does not conserve isospin. Namely, the hamiltonian (1) is not generally an isoscalar. As in most collective treatments, physical isotensor operators must be transformed from the laboratory frame to the intrinsic frame. In the case of the single-particle and pairing hamiltonian this procedure yields

$$
\begin{aligned}
H_{\rm sp}^{\rm (lab)} &= e_A\tau_A + e_0\left(D^1_{00}\tau_0 + D^1_{01}\tau_1 + D^1_{0\bar{1}}\tau_{\bar{1}}\right)\\
H_{\rm pair,0}^{\rm (lab)} &= -g_0\left(S_p^+S_p + S_n^+S_n + \tfrac{1}{2}S_\perp^+S_\perp\right)\\
H_{\rm pair,1}^{\rm (lab)} &= -g_1\Bigg[D^1_{00}\left(S_p^+S_p - S_n^+S_n\right)\\
&\quad - \frac{D^1_{01}}{\sqrt{2}}\left(S_p^+S_\perp + S_\perp^+S_n\right)\\
&\quad + \frac{D^1_{0\bar{1}}}{\sqrt{2}}\left(S_n^+S_\perp + S_\perp^+S_p\right)\Bigg]\\
H_{\rm pair,2}^{\rm (lab)} &= -g_2\Big\{D^2_{00}\left(S_p^+S_p + S_n^+S_n - S_\perp^+S_\perp\right)\\
&\quad - \sqrt{\tfrac{3}{2}}\left[D^2_{01}\left(S_\perp^+S_n - S_p^+S_\perp\right)\right.\\
&\quad \left. + D^2_{0\bar{1}}\left(S_\perp^+S_p - S_n^+S_\perp\right)\right]\\
&\quad + \sqrt{6}\left(D^2_{02}S_p^+S_n + D^2_{0\bar{2}}S_n^+S_p\right)\Big\} \qquad (3)
\end{aligned}
$$

Here the subindices 0,1,2 on the l.h.s. denote isoscalar, isovector and isoquadrupole components, and

$$
e_A = e_p + e_n\,,\quad e_0 = e_p - e_n,
$$

$$
g_0 = \frac{g_p + g_n + g_\perp}{3}\,,\quad g_1 = \frac{g_p - g_n}{2}\,,
$$

$$
g_2 = \frac{g_p + g_n - 2g_\perp}{6}\,.
$$

It is easy to verify that the four components of the hamiltonian (3) commute with the constraints (2) and are therefore physical operators.

Up to now the hamiltonian (3) together with the constraints (2) constitute an exact reformulation of the original problem, since the introduction of additional collective coordinates is compensated by the presence of the constraints. Systems of this type can be treated in a perturbative way within an expansion given by the inverse order parameter $1/\bar{S}_\upsilon$, for instance through the BRST procedure [15], as applied to many-body problems in [12], and to the particular case of high angular momentum in [16]. There is, however, a new feature in the present case, namely the presence of the rotational matrices $D^\lambda_{\mu\nu}$ in the hamiltonian. This extra complication can be overcome by means of Marshalek's generalization of the Holstein-Primakoff representation [17], which is amenable to an expansion in powers of T^{-1}. In what follows we will keep only the lowest order terms in such an expansion, assuming $O(\bar{S}_\upsilon) = T$ and $O(g_\nu) = T^{-1}$. Such terms include the two (*pp* and *nn*) pairing hamiltonians in a single j-shell $e_\upsilon\tau_\upsilon - g_\upsilon S_\upsilon^+S_\upsilon$, which are separately treated within the BCS approximation. In doing so, Lagrange multiplier terms $-\lambda_\upsilon(\tau_\upsilon - T_\upsilon)$ have to be added. This treatment yields the independent quasi-particle energies $E_\upsilon = \frac{1}{2}\Omega g_\upsilon$, where Ω is half the value of the shell degeneracy.

The spectrum of the system is ordered into collective bands, each one carrying as quantum numbers the total number of particles and the isospin ($T \le T_A$). The properties of these bands are obtained by adding the remaining leading order terms in (3) to the independent *np* quasi-particle energy terms. To leading order in T^{-1}

$$
H_{\rm (sp+pair)}^{\rm (lab)} = \overline{H} + \omega_d d^+d + H_2\,,
$$

$$
\overline{H} = \sum_\upsilon\left(e_\upsilon\bar{\tau}_\upsilon - g_\upsilon\bar{S}_\upsilon^2\right),
$$

$$
\omega_d = e_0 + \frac{g_1}{T}\left(\bar{S}_p^2 - \bar{S}_n^2\right) + \frac{3g_2}{T}\left(\bar{S}_p^2 + \bar{S}_n^2\right),
$$

$$
\begin{aligned}
&\langle T_A, T-2, m-2|H_2|T_A, T, m\rangle\\
&\quad = -\frac{3g_2}{T}\bar{S}_p\bar{S}_n\sqrt{m(m-1)}\,, \qquad (4)
\end{aligned}
$$

plus null terms proportional to (2). The boson creation operator $d^\dagger$ increases in one unit the value of m [17].

The energy of the band head is given by the BCS expectation value $\overline{H}$. The different members of each band are labeled by the quantum number m and are separated by the distance ω_d, which includes the

difference between the proton and the neutron single-particle energies e_0. Our strategy has been to restore the number of particles T_A and the isospin T as good quantum numbers and, within such a basis, to construct the interband interaction H_2, which allows for the possibility of double-beta decay. In such a way we have been able to disentangle the physical isospin violations from the unphysical ones.

Both within the simple model or in the realistic case, the $\tau_{\pm 1}$ mode disappears [1] from the final physical hamiltonian (4) (to become part of the constraints (2)). This is precisely the (unrenormalizable) phonon that yields a zero frequency root for isoscalar hamiltonians within a naive RPA [9]. From the practical point of view it is as if this (unphysical) RPA boson becomes substituted by the collective boson d^+,d, which is well behaved in the limit of zero frequency. In realistic cases this structure is also maintained, but superimposed to the excitations of the other (physical) RPA modes. This substitution also becomes apparent in the expression for the strong current that appears in the weak hamiltonian, which is proportional to the isospin operator, namely

$$\beta_- = -\sqrt{2}\,\tau_1^{(\mathrm{lab})} = -\sqrt{2}\left(D^1_{11}\tau_1 + D^1_{10}\tau_0 + D^1_{1\bar{1}}\right) \approx \sqrt{2T}\,d^+ + \text{null operator}. \tag{5}$$

From the point of view of the expansion in powers of T^{-1}, the interband interaction H_2 is of the same order (O(1)) as the distance between the states that are mixed by it. Nevertheless, in the following we continue applying perturbation theory by requiring that $|g_2| < g_v$.

Let us proceed now with the discussion of some calculations. We assume $g_p = g_n = g$. The excitation energy ω_d is displayed in Fig. 1 for the cases $j = \frac{9}{2}$, $T_A = 5$, $T = 3$, $e_0 = 0.8$ MeV, $g = 0.4$ MeV and $j = \frac{19}{2}$, $T_A = 10$, $T = 4$, $e_0 = 0.63$ MeV, $g = 0.2$ MeV, as function of the ratio g_2/g (upper boxes). We predict the exact results for $g_2 = 0$ and very satisfactory ones for the other values, in spite of the fact that for these results we have neglected the interband interaction. The matrix element of double beta decay

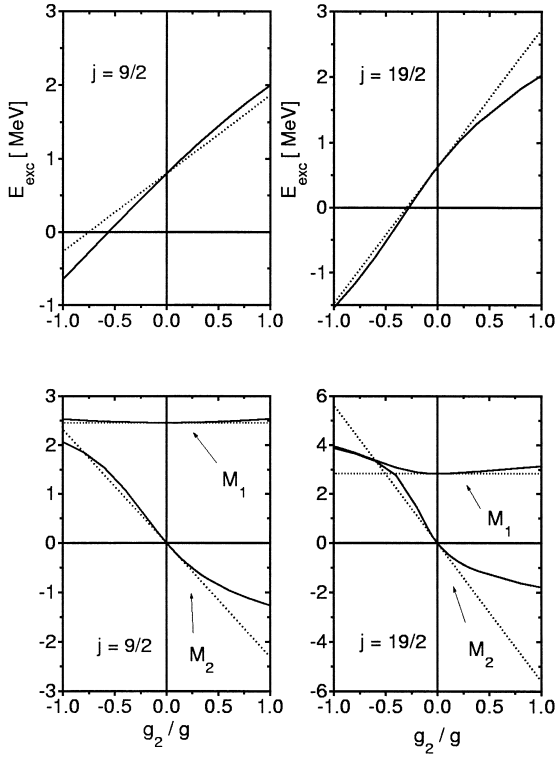

Fig. 1. Excitation energy and transition matrix elements. Exact (solid lines) and collective (dotted lines) results for the excitation energy (upper boxes) and transition matrix elements M_1 and M_2 (lower boxes) corresponding to the two different sets of parameters ($j = 9/2$ and $j = 19/2$) discussed in the text.

transitions, which for the present case correspond to pure Fermi transitions (cf. [9]), is proportional to the product of the two matrix elements

$$M_1 = \langle T_A,T,1|\,\beta_-|T_A,T,0\rangle \approx \sqrt{2T}$$

$$M_2 = \langle T_A,T-2,0|\,\beta_-|T_A,T,1\rangle \approx -\frac{2\sqrt{T}\,\langle T_A,T-2,0|H_2|T_A,T,2\rangle}{\bar{H}(T_A,T,2) - \bar{H}(T_A,T-2,0)}. \tag{6}$$

These matrix elements are displayed in the lower boxes of Fig. 1 for the same parameters as in the upper boxes. The expression for the interband matrix element in (4) does not distinguish whether the r.h.s. should be calculated for the initial or the final value of T, since it is valid for $T \gg 1$. Therefore, the effective interband matrix element has been chosen as the geometric average of the values obtained for each of the two connected bands.

[1] To leading order, the isospin operators have a boson structure since $[\tau_{\bar{1}},\tau_1] \approx T$ and $\tau_{\bar{1}}$ annihilates the state with $\bar{\tau}_0 = -T$. In higher orders of the expansion in powers of T^{-1} this mode appears explicitly [12].

Fig. 2 displays Fermi double beta decay matrix elements, corresponding to transitions from the initial to the final ground states. It has been calculated using the expression

$$M_{2\nu} = \frac{M_1 M_2}{\omega_d + \Delta} \tag{7}$$

where the energy released Δ is taken to be 0.5 MeV, as in [9]. In addition to the exact and collective values of these matrix elements, we have included in this figure the results obtained by using some other approximations. As expected from the fact that Fermi transitions only connect states with the same value of isospin, the exact result shows the suppression of the matrix element around the point where the strength of the *np* symmetry breaking interaction approaches the value of the fully symmetric interaction. This result is reproduced both in the naive QRPA and in the collective approach. The other approximation badly misses this cancellation. A detailed comparison between the results of exact, naive QRPA and renormalized QRPA (RQRPA) calculations can be found in [9]. It is worth to note that in the collective approach the corresponding sum rule (Ikeda's sum rule) is exactly observed. This is not the case of other approaches, as the RQRPA. The collective approach, as seen in Figs. 1 and 2, not only reproduces exact results very satisfactorily but it also gives some insight about the mechanism responsible for the suppression of the matrix elements. As found in the calculations, the value of the matrix element M_2 depends critically on the strength of the physical symmetry breaking term H_2. On the other hand, the values of M_1 are not very much dependent on this interaction. Finally, it should be observed that the point where the excitation energy vanishes and the point where the symmetry is completely restored are different (cf. Fig. 1). This result, also obtained in the exact diagonalization of the full hamiltonian, cannot be reproduced by other means as shown in [9].

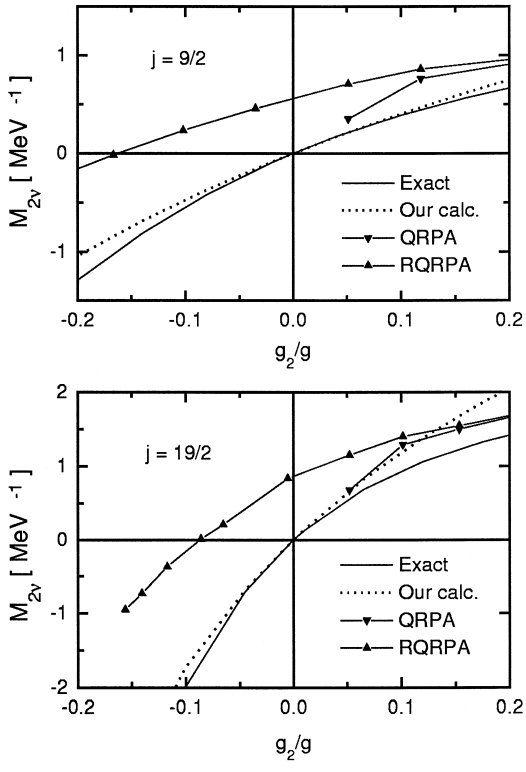

Fig. 2. Matrix elements for Fermi double beta decay transitions calculated in several different approximations. The meaning of the QRPA and RQRPA approximations is explained in the text.

In conclusion, it is found that a correct treatment of collective effects induced by isospin dependent residual interactions in a superfluid system is feasible: physical effects due to the isospin symmetry-breaking terms in the hamiltonian are obtained even in the presence of the BCS mean field built upon separate proton and neutron pairing interactions. The interplay of intrinsic and collective coordinates guarantees that the isospin symmetry is restored and that spurious contributions to the wave functions are decoupled from physical ones. Particularly, the problem of the unstabilities found in the standard *np* QRPA are avoided by the explicit elimination of the zero frequency mode from the physical spectrum (but keeping it in the perturbative expansion). The appearance of this mode cannot be avoided by the inclusion of higher order terms in the QRPA expansion or by any other ad-hoc renormalization procedure, like the RQRPA, once the BCS procedure is adopted for the separate treatment of *pp*- and *nn*-pairing correlations [9].

From the point of view of the expansion in powers of T^{-1}, the results shown in this letter are encouraging, in spite of the fact that we have not used very large values of T. Further details will be presented in a longer publication, in which the case $T \ll O(\bar{S}_\nu)$ will also be treated. We will also report

there on the extension of the formalism to include any number of non-degenerate j-shells as well as the effects of the $S = 1, T = 0$ pairing interaction on the Gamow-Teller transitions. In spite of these complications, the main features of the formalism remain essentially the same, albeit the expressions become more cumbersome to handle.

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