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Nuclear Structure and Double Beta Decay

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Nuclear structure aspects of nuclear double beta decay are reviewed. The validity of some of the approximations adopted in the description of nuclear double beta decay observables is discussed. Results for nuclear matrix elements, weak interaction parameters and neutrino properties are presented.

1 Introduction

The determination of neutrino properties from nuclear double beta decay measurements, which started long ago [1], shows a revival since direct measurements of nuclear-double-beta-decay half-lives are available [2] [3] [4] [5].

From the side of theory, the link of nuclear and particle physics in the physics of double beta decay transitions was emphasized years ago by Vergados [6]. Specific aspects of the problem have been reviewed in [7, 8, 9, 10]. For a more recent compilation of theoretical and experimental results see please [11] and [12].

Since the sensitivity of the theoretical predictions, for relevant nuclear matrix elements is commonly viewed as a major source of uncertainties in the determination of nuclear double beta decay observables, I would like to start with this talk by given a brief summary on the status of some of the approximations. I will shortly discuss, afterwards, the possibilities offered by the measurements of double-beta-decay transitions to excited states. The material is taken from recent works done in collaboration with Jouni Suhonen, of the University of Jyvaskyla, Finland. The last part of the talk will be devoted to the discussion of the role played by collective and intrinsic degrees of freedom in the description of nuclear double beta decay matrix elements. These results are taken from work done in collaboration with Peter Hess and Jorge Hirsch, from the UNAM and CINVESTAV (Mexico).

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2 Brief review on the status of the calculations

The decay rates for the two-neutrino nuclear double beta decay are deduced by assuming that the leptons are in an s-wave state and that lepton energies and nuclear mass differences are replaced by the electron rest-mass energy and the double-beta Q-value [8]. After integrating over lepton coordinates and summing over nuclear states one obtains for the double-beta-decay half-life

$$[t_{1/2}^{(2\nu)}]^{-1} = \mathrm{G}^{(2\nu)} \left|M_{\mathrm{DGT}}^{(2\nu)}\right|^2 , \tag{1}$$

where the double Gamow–Teller matrix element (DGT) between the initial ($0_{\mathrm{g.s.}}^{(i)}$) and final ($0_{\mathrm{g.s.}}^{(f)}$) ground states is given by

$$M_{\mathrm{DGT}}^{(2\nu)} = \sum_m \frac{(0_{\mathrm{g.s.}}^{(f)} \parallel \sum_i \sigma(i)\tau^{\pm}(i) \parallel 1_m^+)(1_m^+ \parallel \sum_i \sigma(i)\tau^{\pm}(i) \parallel 0_{\mathrm{g.s.}}^{(i)})}{\left[\frac{1}{2}Q_{\beta\beta}(0_{\mathrm{g.s.}}^{(f)}) + E(1_m^+) - M_i\right]/m_e + 1} , \tag{2}$$

$E(1_m^+) - M_i$ is the energy difference between the mth intermediate 1^+ state and the initial ground state and τ^- (τ^+) corresponds to the β^- (β^+/EC) decay. In the above equation all the masses are expressed in energy units and the Q value has been specified for the final ground state. The factors $G^{(2\nu)}$ are the phase-space integrals, associated with $\beta^-\beta^-$, $\beta^+\beta^+$, β^+EC and ECEC modes. A compilation of experimental results has been published recently [12]. The overall trend exhibited by data shows that the half-lives for the positive results of the $2\nu\beta\beta$ processes vary between 10^{19} to 10^{21} years. That this variation cannot be understood only in terms of phase-space factors has been known already from the earlier studies. The authors of [7] have used leptonic phase-space factors and simplified shell-model matrix elements (the weak-coupling limit) to evaluate (1) and found far too short half-lives. By taking calculated leptonic phase-space factors the extracted matrix elements are in many cases (^{128}Te, ^{130}Te, ^{136}Xe, ^{150}Nd) of the order of one tenth of the single-particle estimate. This fact immediately raises the question about the mechanism responsible for this suppression [13]. Possible explanations are configuration mixing, collective effects and Q-value effects. In the following we shall summarize some of the features found in these transitions, both from data and theory, obtained from the existing literature. For details see [11]. The example of the ground state to ground state double beta decay of ^{76}Ge suggests that the decay amplitude is to certain extent model independent and most likely depends on small components of the involved wave functions. Thus, the question about ways of testing the theoretical predictions rises naturally and has lead to suggestions that the decay to excited states might be less sensitive to small components of the wave functions, an observation first made by us [14] and later confirmed by others. Relatively large values of the extracted matrix elements are found in the analysis of the decay to the first excited 0^+ state, a result which is particularly significant for the analysis of the decay of ^{100}Mo, ^{82}Se and ^{116}Cd [15]. The possibility of identifying relevant correlations, as emerging from the features of the different decays, namely to the final ground state and to final excited states, together with the simultaneous description of electromagnetic transitions from these exited states of the final nucleus, has raised considerable attention, both experimentally and theoretically. We shall talk more on this in the next subsection. The features which can not be accounted for by using the spherical QRPA method are certainly related to deformations in the single-particle potential. This may be the case for the transitions in some Mo and Ru isotopes, where, as a function of increasing neutron number and approaching the double beta decay system at $A = 100$, deformation effects have been identified.

Another interesting possibility, from the experimental side, is the determination of the single-state dominance [16] by combining measurements of single-beta-decay and EC transitions with double-beta-decay measurements. Examples of this can be found in refs. [16] [17]. In some selected cases, like in the decays of ^{100}Mo and ^{116}Cd, it has been confirmed, by using effective matrix elements (for single beta decay and electron capture) extracted from the lateral (one-step) transitions and effective energy denominators, that the extracted matrix elements for the two-neutrino double-beta-decay mode can be approximated by a single virtual two-step process involving only the ground state of the participant doubly-odd nucleus. However, for some other cases where 1^+ is not the multipolarity of the ground-state, like the decay of Ge, the "single-state" dominance does not manifest itself in the final value of the two-neutrino double-beta-decay matrix elements. Rather, the participation of few low-lying 1^+ states of the intermediate nucleus is needed to produce the observed suppression of the final matrix element. Theoretical results show that the fragmentation of the β^+ strength is perhaps the most sensitive quantity in a double-beta-decay calculation and also the one with the largest theoretical uncertainties. Concerning the quenching of the beta-decay intensities on the proton-rich side of the nuclear chart, this mechanism does not influence significantly the final values of the involved nuclear matrix elements the magnitude of which, as said before, seems to be the result of a cancellation between attractive and repulsive proton-neutron interactions [18]. The predicting power of the QRPA can certainly be improved and a good amount of work has been done already leading to this direction. However, most of the attempted fine tunings of the model fail in one respect or the other. The need to bring the theory to a "cancellation-free" status seems to be out of context because of the features exhibited by the shell-model results. Apparently the most difficult task is to correct for the prediction of the β^+ strengths, which is the place where the theoretical description has failed thus far.

How do all these reflect upon neutrinoless transitions? Results of the calculations show a less drastic dependence on nuclear-model assumptions than in the case of the two-neutrino decay but still they are sensitive to the models used to describe the neutrino-physics sector of the calculations. The overall trend extracted from data (which are only upper or lower limits for the weak-interaction parameters and the half-lives, respectively) shows a sort of universality in the extracted (and predicted) mass term of the weak lagrangian, establishing an upper limit for the neutrino-mass shift of the order of 1 eV. This limit is compatible with the relative strength of the parity-violating effective left-right coupling, which for most models has a sort of universal upper limit of the order of 10^{-7}. The same can be said about the singlet-majoron-nucleon coupling constant, which for most cases has an upper limit of the order of 10^{-4}, implying a scale of about 100 keV for the spontaneous symmetry breaking of the U(1) B-L symmetry. The corresponding matrix element seems to be of the order of 2.8. From these considerations and from the fact that the mixing with a right-handed second generation of gauge bosons is predicted in a similar fashion by nuclear-structure studies and studies of muon decay and supernova neutrinos, one can conclude by saying that nuclear structure approximations entering neutrinoless nuclear double beta decay studies are not too bad.

3 Double beta decay to excited states

QRPA calculations of double-β decays have not been able to reproduce data in the A=100 system. The A=116 system, because of its smaller deformation, is a good candidate to test QRPA calculations. In the following we shall present the preliminarly analysis of two experiments that determine the electron capture decay branch of ^{116}In [17]. The measured 2ν-decay half-life of ^{116}Cd, $t_{1/2} = (2.2^{+0.7}_{-0.4}) \times 10^{19}$ y. [19], can be compared to an estimation of the contribution of the virtual transition via only the ground state of ^{116}In. We estimate $M^{2\nu}_{GT}$ as

$$M^{2\nu}_{GT} = \frac{<^{116}\,\mathrm{Sn(g.s.)}|\sigma\tau^+|^{116}\mathrm{In(g.s.)}><^{116}\,\mathrm{In(g.s.)}|\sigma\tau^+|^{116}\mathrm{Cd}>}{(Q_{EC}+Q_{\beta^-})/2} \tag{3}$$

The β^- ft value of ^{116}In to the $g.s.$ of ^{116}Sn is known from the half life and decay branch to the $g.s.$ but the electron capture (EC) decay branch is not known. If we assume it to be similar to the corresponding transition in the neighbouring nucleus ^{114}Cd, i.e. $\log ft = 4.2$, we obtain $t^{2\nu}_{1/2} = 1.17 \times 10^{19}$ y. This shows that the contribution from the ground state of ^{116}In alone could account for the total decay rate. The situation could be common to all $0^+ \to 0^+$ double β- decaying nuclei in which the ground state of the intermediate nucleus has $J^\pi = 1^+$. The fact that the transition through the ground state of the intermediate nucleus dominates the double-β- decay rate shows the importance for the calculations to reproduce the single-β-decay matrix elements that connect this state to the double-β-decay initial and final states. Experiments were performed using natural In (95.7%^{115}In + 4.3%^{113}In) targets and ^{115}In targets of 99.99% isotopic purity. The results are given in [17].

The calculations have been perfomed in a basis consisting of two complete oscillator shells around the double-shell-closure $N = Z = 50$, assuming ^{40}Ca as the inert core. The set of virtual intermediate states, needed to calculate the double-beta-decay matrix elements, are described as the superposition of quasiproton and quasineutron pairs coupled to angular momentum $J^{\pi} = 1^{+}$ and with eigenvalues and amplitudes given by the pn-QRPA model. The strength of the proton-neutron particle-hole channels is adjusted to reproduce the energy of the giant Gamow-Teller resonance. The present theoretical value, for the energy of the GT resonance in ^{116}In is about E_{GT}=15 MeV, measured from the ground state of In. The strength of proton-neutron particle-particle channels of the interaction, g_{pp}, is determined via the known-single-beta decay transitions in ^{116}In. As usual in this sort of calculations two sets of pn-QRPA states have to be built, one describing the excitations starting from the initial nucleus and the other corresponding to excitations starting from the final nucleus, both interpreted as states of the intermediate nucleus. Wave functions and overlaps between both sets of states are treated as in [18]. The excited states of ^{116}Sn are described as superposition of two-quasiprotons and two-quasineutrons. The QRPA matrix equations are diagonalized to determine amplitudes and eigenvalues for monopole and quadrupole excitations. The energy of the first excited quadrupole state and the value of the measured B(E2) transition from this state to the ground state are reproduced in the calculations by adjusting the coupling constant of the quadrupole channels of the two-body interaction and by introducing effective charges. The results of the present calculation are $B(\mathrm{E2}, 2_1^+ \to 0_{\mathrm{g.s}}^+)$= 10.6 W.u, for $e_{\mathrm{eff}}^{(\mathrm{p})} = 1.39e$ and $e_{\mathrm{eff}}^{(\mathrm{n})} = 0.39e$. For the monopole excitations it has been verified that the first QRPA eigenvalue, at zero energy, is just the solution of the pairing-gap equation corresponding to the same interaction. The pn-QRPA calculations, for g_{pp}=1.0, yield a final matrix element $M_{\mathrm{GT}}^{(2\nu)}$= 0.120 (in units of inverse electron mass) for the transition to the ground state of ^{116}Sn. This result is practically given by a the contribution of a single virtual excitation. Slightly weaker dominance is found in the results corresponding to g_{pp}=0.75. In the framework of the pn-QRPA it corresponds to the contribution of the first excited 1^+ state of ^{116}In, relative to the ground state of both the initial and final nuclei. Since the ground state of ^{116}In is a 1^+ state this result supports the above mentioned single-state dominance. In practice two effects are contributing to this dominance, namely: a) that the contribution of the virtual 1^+ excitation has a small energy denominator when this state is also the ground state, and b) that the product of single-beta-decay matrix elements entering in the definition of $M_{\mathrm{GT}}^{(2\nu)}$ is mostly governed by the virtual β^+ transition. When the matrix element $M_{\mathrm{GT}}^{(2\nu)}$ is approximated by the product of the matrix elements extracted from the measured β^- and EC transitions one gets the value $(M_{\mathrm{GT}}^{(2\nu)})_{\mathrm{approx}} = 0.11 \pm 0.03$, which is quite similar to the theoretical value. To conclude with the analysis of the QRPA results it can be said that the overall agreement between data and the calculations, both for the single- and double-beta-decay transitions, supports the notion that theoretical approximations are working in the A=116 system better than in the A=100 system where the QRPA was seen to fail in predicting data. This may be due to the fact that both ^{116}Cd and ^{116}Sn are spherical nuclei, a condition which might not be found in the A=100 system.

4 Symmetry breaking effects

Since the introduction of renormalized particle-particle interactions in the treatment of proton-neutron correlations in open shell systems [13, 18] the physical interpretation of double-beta decay transitions becomes heavily dependent upon the adopted values of the model parameters. The sensitivity of the two-neutrino double-beta decay mode upon g_{pp} was found to be present both in schematic and in realistic models. Several methods have been proposed to cure for this severe dependence since it become obvious that the renormalization was needed and that physical values of the coupling constants were hard to obtain from first principles. However, these approaches suffer from a common disease, e.g. the violation of the Ikeda Sum Rule [20, 21]. Both the collapse of the QRPA and the violation of the Ikeda Sum Rule can be interpreted in terms of a "phase-transition" rather similar to the pairing one, with the number of pn-pairs playing the role of an order parameter. We shall extent the similarities between the "critical" behaviour of the QRPA against renormalized particle-particle interactions and the more familiar concept of symmetry breaking. Details are given in [22].

The starting Hamiltonian is written

$$H = H_p + H_n + H_{res} \tag{4}$$

where

$$\begin{gathered} H_p = \sum_p e_p a_p^\dagger a_p - G_p S_p^\dagger S_p \qquad H_n = \sum_n e_n a_n^\dagger a_n - G_n S_n^\dagger S_n \\ H_{res} = 2\chi \beta_J^- \cdot \beta_J^+ - 2\kappa P_J^- \cdot P_J^+ \end{gathered} \tag{5}$$

where the operators $S_{p(n)}$ are monopole pair operators, $G_{p(n)}$ are the pairing coupling constants, $\beta_J^\pm$ and $P_J^\pm$ are particle-hole and particle-particle proton-neutron operators, χ and κ are the coupling constants of the separable proton-neutron two body interaction. We shall consider the one-shell limit of this Hamiltonian. After performing BCS transformations, for neutrons and protons separately, at the QRPA order of approximation, e.g. by keeping bilinear products of two quasiparticle creation and annihilation operators, $A^\dagger$ and A, we arrive at the expression

$$H = \epsilon\, C + \lambda_1 A^\dagger A + \lambda_2 (A^\dagger A^\dagger + AA), \tag{6}$$

The currently adopted QRPA treatment of this Hamiltonian [20] has shown that the collapse of the QRPA is controlled by the ratio between κ and the natural scale of the model (e.g. G or the quasiparticle energies). The value of χ is fixed by the position of the resonance associated to the decay mode while κ is mostly responsible for the fragmentation of the low-lying intensities. We shall introduce a boson representation which transforms the combination of fermionic degrees of freedom into bosonic ones and which preserves Pauli's Principle. The link with the phase-transition mechanism is established by introducing, in this boson basis, coherent states and an order parameter. The Dyson's mapping of the Hamiltonian is performed by replacing the quasiparticle-pair operators by

$$A^\dagger \to b^\dagger \left(1 - \frac{b^\dagger b}{2\Omega}\right) , \quad A \to b , \quad C \to 2b^\dagger b . \tag{7}$$

The operators $b^\dagger$ and b are boson creation and annihilation operators, which obey exact boson-commutation relations. The number of bosons n_b is restricted by the condition $n_b \leq 2\Omega$. This restriction guarantees that spuriousities due to non-physical states with a larger number of bosons will not be present in the basis.

The transformed Hamiltonian is written

$$\begin{aligned} H = {} & (2\epsilon + \lambda_1)b^\dagger b - \tfrac{\lambda_1}{2\Omega}b^{\dagger 2}b^2 + \lambda_2(1 - \tfrac{1}{2\Omega})b^{\dagger 2} \\ & - \tfrac{\lambda_2}{\Omega}(1 - \tfrac{1}{2\Omega})b^{\dagger 3}b + \tfrac{\lambda_2}{4\Omega^2}b^{\dagger 4}b^2 + \lambda_2 b^2. \end{aligned} \tag{8}$$

The meaning of the QRPA collapse as a phase transition is better illustrated with the help of coherent states

$$\mid \alpha >= N_0 \sum_{l=0}^{2\Omega} \frac{\alpha^l}{l!} b^{\dagger l} \mid 0 >, \tag{9}$$

where α is a complex parameter and N_0 is a normalization factor. Different regimes of the solution will therefore be determined by non-trivial values of the order parameter. The potential-energy surface $E(\alpha)$, i.e. the expectation value of H on the coherent state, was minimized as a function of the order parameter α. Then the dependence of α with the coupling constant κ, at the minimum, was determined. The results are shown in Figure 1. This behaviour demostrates that a sudden change of correlations occurs around some critical value of the coupling constant κ (κ_c). The onset of the phase transition is observed at values of κ just before the point where $2\epsilon + \lambda_1 - 2\lambda_2$ vanishes. The critical behaviour of the potential-energy surfaces is well demostrated by the results shown in Figure 2, where the harmonic dependence of the energy, as a function of α_0, and thus the validity of the QRPA harmonic expansion fails for non-vanishing values of the order parameter $\alpha_0 = 0$. In this example the interesting analogy existing between symmetry breaking mechanisms, either spontaneous like the breaking of the number of particle symmetry by the BCS vacuum or dynamical like the breaking of the isospin by the residual particle-particle interactions, can be established. Own to this analogy, the non-perturbative nature of the expansions around the critical point in the parametric space where the QRPA collapses, indicates that efforts to correct it based on perturbative methods can yield to non-physical solutions. By the other side, the double well shape of the potential energy surfaces for coupling constants passing by the critical point indicates that non-perturbative methods should be applied to calculate matrix elements involved in nuclear double beta decay transitions.

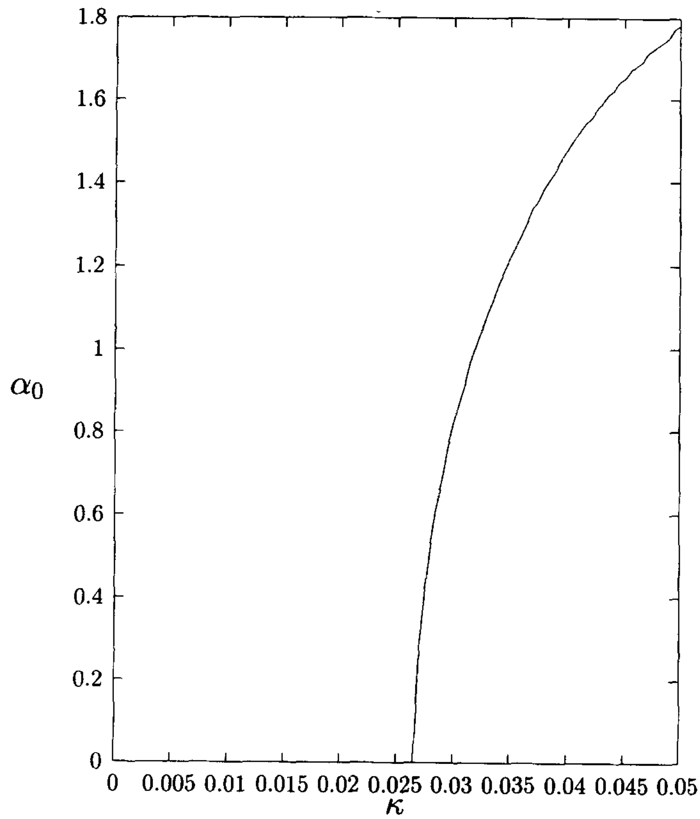

Figure 1. Real part of the order parameter α, as a function of the coupling constant κ, for a fixed value of χ.

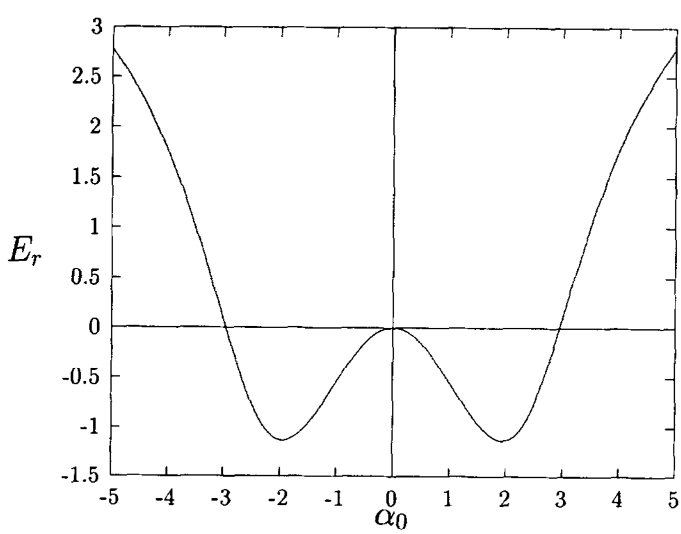

Figure 2. Real part (E_r) of the energy, as a function of the order parameter α, for $\kappa > \kappa_c$

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