Is the single-state dominance realized in double-β-decay transitions?

O. Civitarese
Department of Physics, University of La Plata, C.C. 67, 1900 La Plata, Argentina

J. Suhonen
Department of Physics, University of Jyväskylä, P.O. Box 35, FIN-40351, Jyväskylä, Finland

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In the single-state-dominance hypothesis (SSDH) the decay rate of the two-neutrino double-β decay to the final ground state is solely determined by virtual single-β-decay transitions via the 1^+ ground state of the intermediate nucleus. A very important consequence the SSDH will be that some of nonaccelerator measurements of double-β-decay observables could be circumvented by single-β-decay measurements. To assess the validity of the SSDH, we have carried out a theoretical analysis of all double-β-decay transitions where the spin-parity of the ground-state of the intermediate nucleus is 1^+. The calculations indicate that the double-electron emitters, as well as the less explored double-positron emitters, show evidence of the SSDH. [S0556-2813(98)03009-X]

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Two-neutrino nuclear double-β-decay transitions ($2\nu\beta^{\pm}\beta^{\pm}$) are examined as second-order processes in perturbation theory [1]. The available data on these transitions can be found in Refs. [2,3]. The calculation of decay rates involves a summation over intermediate states, which belong to the spectrum of states of the double-odd nucleus (A, $N\mp1$, $Z\pm1$) in the decay chain connecting the initial (A,N,Z) and final (A, $N\mp2$, $Z\pm2$) nuclei. In this decay Gamow-Teller transitions are the dominant ones allowed by the angular-momentum, parity and isospin selection rules. Some time ago it was suggested by Abad *et al.* [4] that, for those $2\nu\beta^-\beta^-$ transitions where the ground state of the intermediate nucleus is a $J^\pi=1^+$ state, the second-order transition matrix element could be dominated by two virtual transitions, the first going from the initial ground state to this 1^+ state and the second then proceeding from the 1^+ state to the final ground state. From now on we shall call this assumption the single-state-dominance hypothesis (SSDH). Further experimental studies of this effect, restricted to the $2\nu\beta^-\beta^-$ decay of ^{100}Mo and ^{116}Cd, have been reported by García *et al.* [5], Akimune *et al.* [6], and Bhattacharya *et al.* [7]. The role of the Gamow-Teller giant resonance in this context was discussed in [8]. On the theoretical side the importance of lowest-lying 1^+ states was suggested by shell-model calculations [9,10]. However, a systematic theoretical study of these transitions has not been reported yet. The importance of such a study is emphasized by the implications of the SSDH on the extraction of double-β-decay half-lives: the half-lives can be determined from single-β^- and electron-capture measurements.

In this article we show the results of a systematical study, concerning both single- and double-β-decay observables, of all the double-β-decay systems which meet the requirement of the SSDH about the spin of the ground state of the intermediate nucleus. The studies are further extended to double-β-decay transitions to excited final states of spin-parity 0^+ to see if the SSDH can be applied to all 0^+ final states reached in the $2\nu\beta\beta$ decays of the nuclei of interest in the SSDH.

The calculations have been performed by using the proton-neutron quasiparticle random phase approximation (pnQRPA) [11] to describe the states of the double-odd-mass nuclei and the proton-proton and neutron-neutron ($pp+nn$ QRPA) formalism to describe the excited states of the final nuclei participant in a given decay chain. The details of the formalism are reviewed in [12]. The single-particle basis, used to calculate quasiparticle excitations, were constructed with the solutions of the Woods-Saxon central potential for each mass region included in the calculations. The Coulomb correction was added to describe proton states. Matrix elements for the effective two-body interaction were extracted from the G matrix constructed with the Bonn one-boson-exchange potential (OBEP) of [13]. Monopole terms of the interaction between like particles (proton-proton and neutron-neutron channels) were renormalized to account for the observed odd-even mass differences. The empirical values of the pairing gap parameters were reproduced with accuracies of the order of 50–100 keV for all the nuclei included in the systematics. Energies of some single-particle levels around the proton and/or the neutron Fermi surfaces were adjusted to reproduce the observed single-quasiparticle spectrum of even-odd and odd-even mass nuclei in the neighborhood of the initial and final even-even nuclei participant in the double-β decays.

Concerning the excitations in the double-odd-mass nuclei, the strength of the residual interaction between protons and neutrons in the particle-hole channel was chosen to reproduce the empirical position of the giant Gamow-Teller resonance within one MeV. The strengths for the monopole and quadrupole interactions between like-particle pairs of quasiparticles in the initial and final double-even nuclei were renormalized in such a way that the observed energies of the first excited one-phonon, states were reproduced by the QRPA calculations. The spurious monopole state at zero energy was removed from the QRPA solutions. With the resulting set of wave functions we have calculated the matrix elements for single-β^--decay and electron-capture transitions feeding low-lying states of the initial and final nuclei. In this

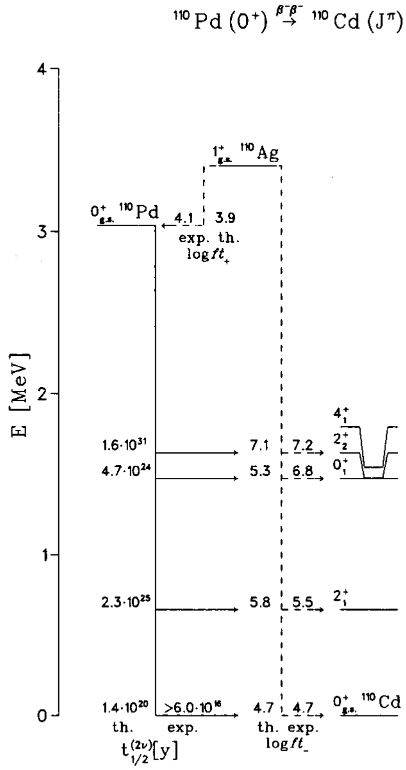

FIG. 1. Decay scheme for $A=110$. Indicated are the electron-capture transition ($\log ft_+$) and the β^- transitions ($\log ft_-$) of ^{110}Ag as well as the half-lives of the double-β^- transitions from ^{110}Pd.

manner the structure of the wave function of the first excited 1^+ state (i.e., the ground state of the intermediate nucleus) was tested independently of the double-β-decay observables. The strength of the particle-particle interaction between protons and neutrons [12] g_{pp} was chosen to reproduce, as well as possible, the experimental $\log ft$ values for transitions feeding the initial ground state and the final ground and excited states.

In the present article we have analyzed single β^--decay and electron-capture data, as well as double-β-decay transitions, in the following double-β^--decay systems: ^{100}Mo $\to^{100}$Ru, ^{110}Pd$\to^{110}$Cd, ^{114}Cd$\to^{114}$Sn, ^{116}Cd$\to^{116}$Sn and ^{128}Te$\to^{128}$Xe. In addition, the double-electron-capture transitions ^{106}Cd$\to^{106}$Pd, ^{136}Ce$\to^{136}$Ba were analyzed together with the single-β-decay observables.

Figure 1 shows the calculated and experimental values of the single- and double-β-decay observables for the double β^- decay of ^{110}Pd. In order to test the validity of the SSDH, we have extracted the matrix elements corresponding to the single-β^- and electron-capture decays from the measured $\log ft$ values. With these values and the corresponding observed transition-energies (Q values) we have calculated the dimensionless double Gamow-Teller (DGT) matrix element connecting the initial and final ground states as

$$M^{(2\nu)}_{\rm DGT}({\rm SSDH})=\frac{6Dm_{\rm e}}{\sqrt{ft_{\rm EC}ft_{\beta^-}}(Q_{\beta^-}+Q_{\rm EC})}, \qquad (1)$$

where m_e is the electron rest mass, and we adopt the values $D=6147$ and $g_A^2=1.0$ for the β-decay constant and the axial-vector coupling strength, respectively. The general theoretical expression for the matrix element governing the two-neutrino mode of the nuclear double-β decay, from an initial ground state, $0^+_{\rm I}$, in (A, N, Z) to a final $0^+_{\rm F}$ state in (A, $N\mp2$, $Z\pm2$) is given by [12]

$$M^{(2\nu)}_{\rm DGT}(0^+_F)=\sum_m \frac{M^{(\rm I)}_m M^{(\rm F)}_m}{[(1/2)\,Q_{\beta\beta}+E_m-M_{\rm I}]/m_{\rm e}+1}, \qquad (2)$$

where the sum extends over all 1^+ states of the intermediate nucleus. The denominator of this equation consists of the energy E_m of the mth intermediate 1^+ state and the mass energy $M_{\rm I}$ of the parent nucleus, as well as of the double-β-decay Q value $Q_{\beta\beta}$. The virtual single-$\beta^\pm$ decay matrix elements for the initial-branch ($M^{(\rm I)}_m$) and final-branch ($M^{(\rm F)}_m$) are given by

$$M^{(\rm I)}_m=(1^+_m\|\hat{\beta}^\pm\|0^+_{\rm I}), \qquad (3)$$

$$M^{(\rm F)}_m=\sum_{m'}(0^+_{\rm F}\|\hat{\beta}^\pm\|1^+_{m'})\langle 1^+_{m'}|1^+_m\rangle, \qquad (4)$$

where the overlaps $\langle 1^+_{m'}|1^+_m\rangle$ between the two sets of *pn*-QRPA solutions are added to account for the matching of the intermediate 1^+ states [12]. The structure of the matrix elements, for virtual single-β-decay transitions from a *pn*-phonon state to a $pp+nn$-phonon state, is given in [12].

The double-β-decay half-life $t^{(2\nu)}_{1/2}$ can be obtained from the expression

$$[t^{(2\nu)}_{1/2}(0^+_{\rm g.s.}\to0^+_F)]^{-1}=|M^{(2\nu)}_{\rm DGT}(0^+_F)|^2G^{(2\nu)}_{\rm DGT}(0^+_F), \qquad (5)$$

where 0^+_F can either be the ground state or an excited 0^+ state of the double-β-decay daughter, and $G^{(2\nu)}_{\rm DGT}(0^+_F)$ is the corresponding integrated kinematical factor [14,15]. In correspondence with the matrix element (1), the SSDH is realized in the numerical calculations by restricting the summation in Eq. (2) to the first *pn*-QRPA eigenstate.

The results for the transitions to the final ground state are shown in Table I both for the double-β-decay electron and positron emitters. In this table the extracted matrix elements are compared with the calculated ones. Both the contributions of the first excited 1^+ state and the total contribution [sum over all m in Eq. (2)] are shown. One can see from these results that the theory predicts the correct order of magnitude for the $M^{(2\nu)}_{\rm DGT}$ in spite of differences in the single-electron-capture matrix elements. It is also noted that the SSDH matrix element (1) agrees quite well with the calculated total matrix element (2).

From the table one notices that one can divide the discussed $2\nu\beta\beta$ decays into two categories, namely, (1) decays with clear theoretical single-state dominance (decays of ^{110}Pd, ^{114}Cd, ^{116}Cd, ^{128}Te, and ^{136}Ce) and (2) decays with no theoretical single-state dominance but where the calculated total DGT matrix element evolves towards the extracted DGT matrix element of the SSDH due to interfering contributions from the higher-lying 1^+ states in the intermediate nucleus (^{106}Cd and possibly ^{100}Mo). Concerning the scarce data on half-lives, both the theoretical and the extracted SSDH values are consistent with the experimental

TABLE I. The double Gamow-Teller matrix element $M^{(2\nu)}_{\mathrm{DGT}}$ and the corresponding double-β-decay half-life $t^{(2\nu)}_{1/2}$ in different approximation schemes for the nuclei with potential SSDH characteristics. The experimental double-β-decay half-life $t^{(2\nu)}_{1/2}$ (exp.) is given for reference whenever known. The matrix elements $M^{(\mathrm{I})}_1$ and $M^{(\mathrm{F})}_1$ correspond to the first intermediate 1^+ state [$m=1$ in Eq. (4)]. Case A: Extracted values for $M^{(\mathrm{I})}_1$ and $M^{(\mathrm{F})}_1$ from observed ft values. Case B: Calculated values by including only the first 1^+ state into the sum of Eq. (2). Case C: Calculated total matrix element of Eq. (2).

System	Case	$M^{(\mathrm{I})}_1$ (SSDH)	$M^{(\mathrm{F})}_1$ (SSDH)	$M^{(2\nu)}_{\mathrm{DGT}}$	$t^{(2\nu)}_{1/2}$	$t^{(2\nu)}_{1/2}$(exp.)
	A	0.86	0.68	0.18	8.6×10^{18}	
^{100}Mo	B	2.05	0.68	0.25	4.4×10^{18}	
	C			0.12	1.9×10^{19}	9.5×10^{18}
	A	1.21	0.61	0.19	1.8×10^{20}	
^{110}Pd	B	1.55	0.58	0.24	1.2×10^{20}	
	C			0.21	1.4×10^{20}	
	A	0.54	0.76	0.12	1.2×10^{25}	
^{114}Cd	B	1.36	0.61	0.25	2.8×10^{24}	
	C			0.25	2.8×10^{24}	
	A	0.93	0.63	0.16	1.3×10^{19}	
^{116}Cd	B	1.36	0.32	0.12	2.3×10^{19}	
	C			0.12	2.3×10^{19}	3.75×10^{19}
	A	0.38	0.12	0.013	1.7×10^{25}	
^{128}Te	B	1.21	0.027	0.010	2.9×10^{25}	
	C			0.012	2.0×10^{25}	$>2\times10^{24}$
	A	$\leqslant$0.48	1.08	$\leqslant$0.17	$\geqslant5.3\times10^{21}$	
^{106}Cd	B	0.29	2.98	0.28	2.0×10^{21}	
	C			0.17	5.3×10^{21}	
	A	$\sim$0.68[a]	0.68	$\sim$0.14	$\sim3.2\times10^{21}$	
^{136}Ce	B	0.19	1.81	0.11	5.1×10^{21}	
	C			0.14	3.2×10^{21}	

[a]Not measured; $M^{(\mathrm{I})}_1$ assumed to be of the order of $M^{(\mathrm{F})}_1$.

half-lives. In the case of the ^{100}Mo and ^{106}Cd decay, the theoretical total matrix element is about half the contribution due to the first 1^+ state but those contributions do not differ much from the extracted values. As a matter of fact, a good description of the ^{100}Mo double β decay has proved to be unfeasible by the present type of theoretical framework [16,17], most likely due to deformation effects.

In light of the previous discussion it seems that there is both experimental and theoretical evidence on the realization of the SSDH for the ground-state transitions. One has to stress that the theoretical values displayed in Table I were obtained with a value of g_{pp} which reproduces as well as possible the known EC and β^- decay rates. As a measure of the accuracy of the used method we refer to the results of Fig. 1 where the theoretical log ft values are compared with the available data. For the present calculations the adopted values of g_{pp} are of the order of unity for the double electron emitters and some 30% less for the double positron emitters.

In spite of the absence of data on double-β-decay transitions to excited states of the final nucleus, the presence of the same sort of dominance upon these transitions can be determined by performing a similar analysis as was done for the ground-state-to-ground-state transitions. According to Barabash [18] the matrix elements governing double-β-decay transitions to the first excited 0^+ state should compare favorably with the matrix elements for the ground-state-to-ground-state transitions. However, the comparison of theoretical results with the few available pieces of experimental information (decay of ^{100}Mo) does not support the statements of Ref. [18], in particular when the excited 0^+ states are described as members of the two-quadrupole-phonon triplet [12]. It thus becomes natural to investigate what is the structure of these states and whether the SSDH applies also to the double-β-decay transitions to them.

In the present study we have extended the hypothesis of [18] by assuming that some of the lowest-lying excited 0^+ states can be of a single-phonon character, and we have calculated single- and double-β-decay rates to these states accordingly. At the same time we have performed detailed calculations of the decay of the 1^+_1 intermediate ground state to excited 2^+ and 0^+ states both for the electron as well as the positron emitters. A detailed discussion of the results will be presented elsewhere [19]. As examples we shall discuss the cases of ^{106}Cd, ^{110}Pd, and ^{136}Ce. The corresponding results for transitions to excited 0^+ states are shown in Table II. We have computed those final states as monopole vibrations and as two-quadrupole-phonon states. As seen from these results the matrix elements for transitions leading to one-phonon states have larger matrix elements than the transitions to two-quadrupole phonon states and in all cases the SSDH is clearly present.

TABLE II. The double Gamow-Teller matrix element $M_{\mathrm{DGT}}^{(2\nu)}$ for transitions to excited 0^+ final states. The excitation energies of the states and their log ft-values are shown together with the extracted (SSDH) and theoretical values of the double Gamow-Teller matrix elements. The quantities listed in the last two columns are the theoretical matrix elements corresponding to the first intermediate 1^+ state and to the complete set of 1^+ states, respectively. The assumed theoretical structure of each state is given in parenthesis and it indicates a single one-phonon state $(1\text{-}ph)$ or a two-quadrupole phonon state $(2\text{-}ph)$.

System	$E(O_k^+)$[Mev]	log $ft_-(O_k^+)$	$M_{\mathrm{DGT}}^{(2\nu)}$(SSDH)	$M_{\mathrm{DGT}}^{(2\nu)}(1_1^+)$	$M_{\mathrm{DGT}}^{(2\nu)}$(all)
	1.134	6.5	$\leqslant$0.041	0.075(2-ph)	0.086(2-ph)
^{106}Cd	2.002	6.1	0.113	0.616(1-ph)	0.570(1-ph)
^{110}Pd	1.473	6.8	0.029	0.187(2-ph)	0.209(2-ph)
	1.579	6.1	0.048	0.015(2-ph)	0.014(2-ph)
^{136}Ce	2.141	5.7	0.112	0.128(1-ph)	0.132(1-ph)

In conclusion, it is found that the single-state-dominance hypothesis introduced by Abad *et al.* [4] and explored in the works of Akimune *et al.* [6] and García *et al.* [5], is strongly supported by the present theoretical results since in the majority of the studied cases either a clear theoretical single-state dominance or an evolution of the calculated total DGT matrix element towards the extracted SSDH value of the matrix element (for double positron emitters) was observed. An even more striking theoretical single-state dominance was observed for the DGT decays to excited 0^+ states. In this study both one-phonon and two-phonon monopole states of spherical nature were examined. The nuclei considered in the present paper are located in a specific region of the nuclear chart, namely, the proton and neutron Fermi surfaces are found near $g_{9/2}$ and $g_{7/2}$ orbitals, respectively. In this region a 1^+ state appears as the ground state or a very low-lying state in odd-odd nuclei, and much of the Gamow-Teller transition strength, except that to the giant resonance, is concentrated on these states. It might be that the single-state dominance does not apply to other double-β-decay systems where the locations of the Fermi surfaces do not favor low-lying 1^+ excitations in the intermediate nucleus.

It is evident that more experimental information about double-β-decay transitions to excited states is needed. Particularly it would be important to gain information on transitions to excited 0^+ states because the calculated matrix elements are of the same order of magnitude or even larger than the matrix elements corresponding to transitions to the final ground state. Furthermore, the single-state dominance, if realized, would be useful for experiments on nuclei whose half-lives of two-neutrino double-β decay are not known. Even if not being exact, but only very approximate, the SSDH would still help in giving order-of-magnitude estimates for the double-β-decay half-lives.

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