ELSEVIER

23 October 1997

Physics Letters B 412 (1997) 1-6

PHYSICS LETTERS B

The collapse of the pn-QRPA as a signal of phase-instabilities

Osvaldo Civitarese [a,1], Peter O. Hess [b,2], Jorge G. Hirsch [c,3]

[a] Departamento de Física, Universidad Nacional de La Plata, c.c. 67 1900, La Plata, Argentina
[b] Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, Apdo. Postal 70-543, México 04510 D.F., Mexico
[c] Departamento de Física, Centro de Investigación y de Estudios Avanzados del IPN. A.P. 14-740, México 07000 D.F., Mexico

Received 23 June 1997; revised 7 August 1997
Editor: W. Haxton

Abstract

Using a schematic model we have investigated the collapse of the proton-neutron quasiparticle random-phase approximation (pn-QRPA) and the occurrence of phase instabilities in the space of solutions of the Hamiltonian. The analysis is based on the use of coherent states within the framework of a boson expansion method. © 1997 Elsevier Science B.V.

PACS: 21.60.Jz; 21.60.Fw; 23.40.Hc

The study of proton-neutron correlations is relevant to the microscopic description of single and double beta decay studies [1,2]. Both the random phase approximation (RPA) and its extension to open shell nuclei, the quasiparticle-random phase approximation (QRPA), [3,4] have been extensively used to calculate double-beta decay observables [2]. One of the best established results in the field, in dealing with results of the QRPA for Gamow-Teller transitions in the case of the two-neutrino double beta decay mode, is the need of renormalized particle-particle (proton-neutron) interactions [5,6] to account for the strong suppression of the nuclear matrix elements, as required by data [7] and theory [1,2]. However, the renormalization of these interactions also induces instabilities in the QRPA spectrum [8] whose physical interpretation has been the source of a lively discussion for nearly a decade. Several attempts have been proposed to avoid the collapse of the QRPA. The use of some of these methods can not be justified from physical arguments [9]. Some others, more elaborately, have tried to prevent the occurence of the QRPA collapse by introducing additional renormalizations of the QRPA matrices to shift the critical values of the coupling constants to unphysical regions [10,11]. In spite of these attempts and regarding the validity of the QRPA results, recent calculations [12] have shown that the suppression of the nuclear matrix elements, for the two-neutrino double beta decay transition, obtained at the QRPA level of approximation is indeed seen in more elaborate shell model basis. As it has been shown extensively, the mechanism responsible for the suppression of the nuclear matrix elements is the renormalization

[1] Fellow of the CONICET, Argentina. E-mail: civitare@venus.fisica.unlp.edu.ar.
[2] E-mail: hess@nuclecu.unam.mx.
[3] E-mail: hirsch@fis.cinvestav.mx.

PII S0370-2693(97)01060-5

of attractive particle-particle (proton-neutron) interactions [5,6]. The inclusion of these renormalized interactions also caused the so-called collapse of the QRPA. In consequence, both effects, i.e: the strong suppression of the nuclear matrix elements and the onset of instabilities at the QRPA level of approximation should not be considered artifacts of the approximation but rather results of the change in the structure of the ground and excited states induced by these interactions. The connection between it and the appearance of a zero-energy state in the spectrum, for certain values of the renormalized coupling constant κ, has been discussed in a series of papers based on group-theoretical methods [13,14].

In the present work we have focussed our attention on the interpretation of the collapse of the proton-neutron QRPA (pn-QRPA) as a signature of a phase transition. The adopted view-point is the one encountered in other areas of physics, e.g. in field theory, which relates phase transitions to symmetry breaking [15].

In order to determine the structure of the phase transition, we have related the order parameter with the number of proton-neutron pairs [16]. We have adopted the schematic, but nonetheless realistic, Hamiltonian of Refs. [17,18] and performed a boson-mapping [19] in a coherent state representation [20] which preserves Pauli's Principle. The coherent states are functionals of a complex order parameter and are used as trial states. The energy surface is given by the expectation value of the Hamiltonian with respect to the coherent states [4]. The dependence of the real and imaginary part of the energy, as a function of the real and imaginary parts of the order parameter, gives information about the position of the phase transition in the parametric (χ-κ) space.

The Hamiltonian adopted for the present calculations includes a single particle term, both for protons and neutrons, a pairing interaction between like-nucleons and a proton-neutron two body interaction which is furthermore parametrized in terms of particle-hole and particle-particle channels. This form of H has been used previously both in realistic and in schematic calculations [18]. The schematic hamitonian reads

$$H = H_p + H_n + H_{\text{res}}, \tag{1}$$

where

$$H_p = \sum_p e_p a_p^\dagger a_p - G_p S_p^\dagger S_p, \quad H_n = \sum_n e_n a_n^\dagger a_n - G_n S_n^\dagger S_n, \quad H_{\text{res}} = 2\chi \beta_J^- \cdot \beta_J^+ - 2\kappa P_J^- \cdot P_J^+ . \tag{2}$$

For the definitions of the operators and more details, please see Ref. [13]. The operators $S_{p(n)}$ are monopole pair operators, $G_{p(n)}$ are the pairing coupling constants, $\beta_J^\pm$ and $P_J^\pm$ are particle-hole and particle-particle proton-neutron operators, χ and κ are the coupling constants of the separable proton-neutron two body interaction. We shall consider the one-shell limit of this Hamiltonian. Pairing effects will be accounted for by quasiparticle mean fields, for protons and neutrons, which represent the dominant contributions to the pairing correlation energies, as given by the separable monopole pairing interactions.

This Hamiltonian has been used both for the description of Fermi ($J=0$) and Gamow-Teller ($J=1$) excitations and the corresponding transitions. The solutions for $J=0$ have been studied in [13] while the case of Gamow-Teller excitations have been presented in [14]. For the sake of simplicity and without loss of generality, we shall proceed with the case of $J=0$. In the BCS representation the quasiparticle-pair operators have the form

$$A^\dagger = \left[\alpha_p^\dagger \otimes \alpha_n^\dagger\right]_{M=0}^{J=0}. \tag{3}$$

Thus, at the QRPA order of approximation, e.g. by keeping bilinear products of $A^\dagger$ and A, we arrive at the expression

$$H = \epsilon\, C + \lambda_1 A^\dagger A + \lambda_2 (A^\dagger A^\dagger + AA), \tag{4}$$

where the proton and neutron quasiparticle energies have been replaced by a common value ϵ. The operator C and the coupling constants λ_1 and λ_2 of Eq. (4) are given by

$$C = \sum_{m_p} \alpha_{pm_p}^\dagger \alpha_{pm_p} + \sum_{m_n} \alpha_{nm_n}^\dagger \alpha_{nm_n}, \quad \lambda_1 = 4\Omega\left[\chi\left(u_p^2 v_n^2 + v_p^2 u_n^2\right) - \kappa\left(u_p^2 u_n^2 + v_p^2 v_n^2\right)\right],$$

$$\lambda_2 = 4\Omega(\chi + \kappa) u_p v_p u_n v_n, \tag{5}$$

where 2Ω is the degeneracy of the shell. The adopted QRPA treatment of this Hamiltonian [13] leads to the collapse of the QRPA collective excitation for $2\lambda_2 = 2\epsilon + \lambda_1$. This result, which is also found in the exact solution of the model, does not show-up in the renormalized QRPA treatment of Refs. [10,11]. The collapse of the QRPA excitation energy has also been found in the extension of the present model to a larger group representation [14]. Since, as we have said before, it remains valid in the more realistic shell model treatment of [12] we shall attempt to understand the underlying physical mechanism from a more direct and simple picture where it can be featured as the signature of a phase transition. Consequently, we shall introduce a boson representation which transforms the combination of fermionic degrees of freedom into bosonic ones and which preserves Pauli's Principle. The link with the phase-transition mechanism is established by introducing, in this boson basis, coherent states and an order parameter.

To achieve this goal, in a first step, we have performed the Dyson's mapping of the Hamiltonian (4) by replacing the quasiparticle-pair operators by [19]

$$A^\dagger \to b^\dagger\left(1 - \frac{b^\dagger b}{2\Omega}\right), \quad A \to b, \quad C \to 2b^\dagger b. \tag{6}$$

The operators $b^\dagger$ and b are boson creation and annihilation operators, which obey exact boson-commutation relations. The number of bosons n_b is restricted by the condition $n_b \leq 2\Omega$. This restriction guarantees that spuriousities due to non-physical states with a larger number of bosons will not be present in the basis.

The transformed Hamiltonian corresponding to Eq. (4) is written

$$H = (2\epsilon + \lambda_1) b^\dagger b - \frac{\lambda_1}{2\Omega} b^{\dagger 2} b^2 + \lambda_2\left(1 - \frac{1}{2\Omega}\right) b^{\dagger 2} - \frac{\lambda_2}{\Omega}\left(1 - \frac{1}{2\Omega}\right) b^{\dagger 3} b + \frac{\lambda_2}{4\Omega^2} b^{\dagger 4} b^2 + \lambda_2 b^2. \tag{7}$$

The second step consists of the introduction of coherent states [20]. We chose the ansatz

$$|\alpha\rangle = N_0 \sum_{l=0}^{2\Omega} \frac{\alpha^l}{l!} b^{\dagger l} |0\rangle,$$

where α is a complex parameter

$$\alpha = \alpha_0 e^{i\phi} \tag{8}$$

and N_0 is a normalization factor. In the following we shall distinguished between a first ansatz (all powers are retained in the above sum) which mixes even and odd number of proton-neutron pairs and a second ansatz (only even powers are retained in the sum appearing in the definition of $|\alpha\rangle$) which takes into account the fact that the Hamiltonian (Eq. (7)) only mixes even (odd) number of pairs within themselves. The expectation value of the transformed Hamiltonian, Eq. (7), gives the potential energy surface, $E(\alpha) = E_r + iE_i$, which depends both on the real and imaginary parts of the order parameter as well as on the actual values of the coupling constants of the model. The minima can be identified by performing a search in the parametric (χ, κ)-space subject to the variation of the order parameter. Different regimes of the solution will therefore be determined by non-trivial values of the order parameter.

In the following we will present numerical results which correspond to the model parameters

$$\Omega = 5, \quad N_p = 4, \quad N_n = 6, \quad \epsilon = 1.0\,\mathrm{MeV}. \tag{9}$$

The particle-hole strength χ is absorbed in the value of ϵ and the particle-particle strength κ is varied in the range $0 \leq \kappa \leq 0.2$ MeV.

The potential-energy surface $E(\alpha)$ was minimized as a function of the order parameter α. The dependence of the expectation value of the number of proton-neutron pairs (upper box) and of the order parameter α (lower box) at the minimum and as a function of κ in the captions to Fig. 1. This behaviour demostrates that for the expectation value of the proton-neutron pairs a sudden change of correlations occurs around some critical value

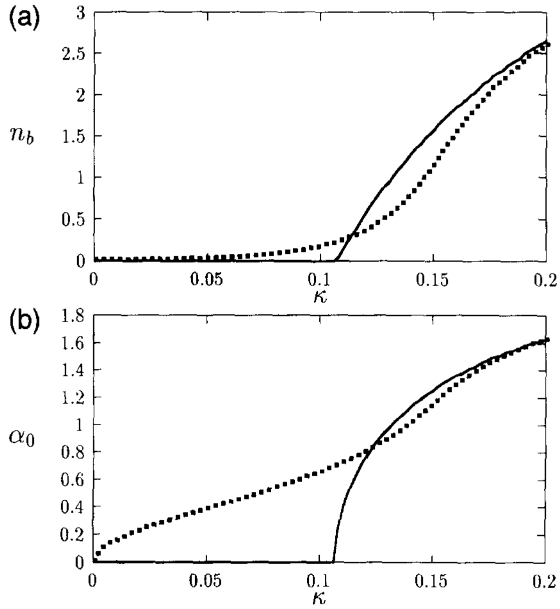

Fig. 1. Upper box: The expectation value of $b^\dagger b$ as a function of the coupling constant κ at the minimum of the energy surface. The values of κ are given in units of MeV. The solid line corresponds to the first and the dotted line to the second ansatz described in the text. Lower box: The same for the real part of the order parameter α.

of the coupling constant κ (κ_c). The onset of the phase transition is observed at values of κ just before the point where $2\lambda_2 = \lambda_1 + 2\epsilon$. As the number of particles approaches the limit $N_{p(n)} \to 2\Omega$ the phase transition vanishes.

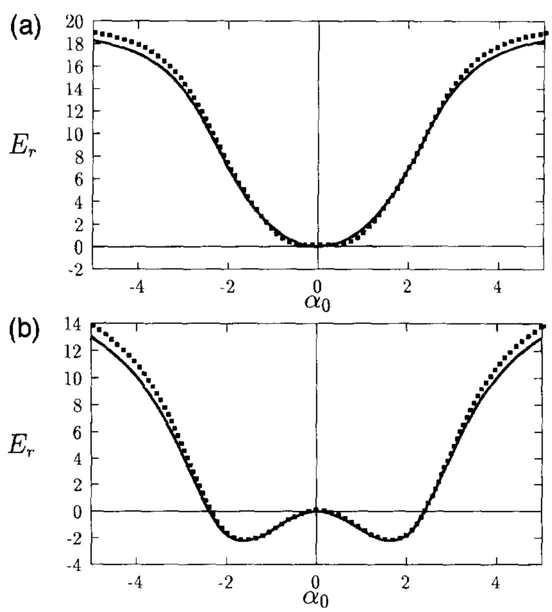

Fig. 2. Real, E_r, part of the energy, as a function of the order parameter α, for $\kappa < \kappa_c$ (upper box) and for $\kappa > \kappa_c$ (lower box). The angle ϕ of Eq. (8) is equal to $\pi/2$ and κ_c is the critical value ($\kappa_c \approx 0.1$ MeV) shown in Fig. 1. The values of E_r are given in units of MeV. The solid line corresponds to the first and the dotted line to the second ansatz.

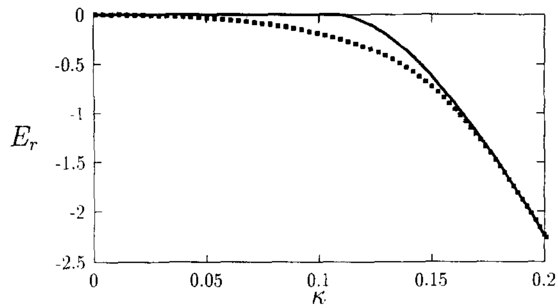

Fig. 3. Evolution of the energy corresponding to the minimum, as a function of κ. Both the E_r and κ are given in units of MeV. The solid line corresponds to the first and the dotted line to the second ansatz.

of the potential-energy surfaces is well demostrated by the results shown in Fig. 2. The upper box shows the harmonic dependence of the energy, as a function of α_0, reflecting the dominance of the QRPA harmonic expansion around the minimum corresponding to $\alpha_0 = 0$. This regime is, of course, to be related with values of κ smaller than the critical value shown in Fig. 1, which is $\kappa_c \approx 0.10$ MeV. For the example shown in the upper box of Fig. 2 we have used the value $\kappa = 0.02$ MeV. The lower box of Fig. 2, where the value $\kappa = 0.20$ MeV is used, shows the characteristic shape of a double-well potential with symmetric minima located at non-zero values of α_0. This is a clear evidence of the symmetry breaking induced by the renormalized particle-particle interactions.

ndence of the real part of the potential-energy (E_r) versus κ at the minimum. Clearly, for the first ansatz (solid line) and for values of κ lower than the critical one, the energy remains closer to the spherical minimum, e.g. the harmonic one with a single minimum at zero which is the parametric region where the standard QRPA is valid. For values of κ larger than the critical value the shape of the real part of the energy at the minimum shows a change in the slope, around κ_c, followed by a steady decrease.

From the above discussed results it is concluded that the renormalization of attractive particle-particle interactions, at the QRPA level of approximation, counter-balance the effects due to repulsive particle-hole interactions by inducing permanent deformations in gauge space, e.g. the space where the number of proton-neutron pairs in vacuum can be defined in terms of an order parameter. This has been shown by introducing the concept of coherent states. In general the order parameter is complex, but the values of α at the minima of the potential-energy surface are located on the real axis. The spontaneous breaking of the proton-neutron-pair symmetry can only be produced in the presence of the particle-particle interactions and it manifests itself in the appearance of a zero-energy state. It means that the orientation of the QRPA wave-function in gauge space can not be eliminated by perturbatively expanding around the spherical minima. At this point, the analogy with the situation found in systems with permanent intrinsic deformations can be drawn by noticing that transitions from the spherical to the deformed ground states are non-perturbative [21].

Acknowledgements

This work has been partially supported by the CONACYT (Mexico), and the FONCYT (Argentina) project PMT-PICT0079. Discussions with K. Kaneko are grateful acknowledged.

References

[1] F. Boehm, P. Vogel, Physics of Massive Neutrinos (Cambridge Univ. Press, Cambridge. 1987).
[2] M.K. Moe, P. Vogel, Ann. Rev. Nucl. Part. Sci. 44 (1994) 247.

[3] D.J. Rowe, Nuclear Collective Motion, Methuen, Co. Ltd., London (1970).
[4] P. Ring, P. Schuck, The Nuclear Many-Body Problem (Springer-Verlag, New York, 1980).
[5] P. Vogel, M.R. Zirnbauer, Phys. Rev. Lett. 57 (1986) 3148.
[6] O. Civitarese, A. Faessler, T. Tomoda, Phys. Lett. B 194 (1987) 11.
[7] V.I. Tretyak, Yu.G. Zdesenko, Atomic and Nuclear Data Tables 61 (1995) 43.
[8] O. Civitarese, A. Faessler, J. Suhonen, X.R. Wu, J. Phys. G: Nucl. Part. Phys. 17 (1991) 943.
[9] X.R. Wu et al., Phys. Lett. B 272 (1991) 169; B 276 (1992) 274.
[10] F. Catara, N. Dinh Dang, M. Sambataro, Nucl. Phys. A 579 (1994) 1.
[11] J. Toivanen, J. Suhonen, Phys. Rev. Lett. 75 (1995) 410.
[12] E. Caurier, A. Poves, A.P. Zuker; Phys. Lett. B 252 (1990) 13;
E. Caurier, F. Nowacki, A. Poves, J. Retamosa, Phys. Rev. Lett. 77 (1996) 1954.
[13] J.G. Hirsch, P.O. Hess, O. Civitarese, Phys. Rev. C 54 (1996) 1976; Phys. Lett. B 390 (1997) 36; Phys. Rev. C (in press).
[14] J. Engel et al., Phys. Rev. C 55 (1997) 1781.
[15] E. Leader, E. Predazzi, Gauge Theories and New Physics (Cambridge Univ. Press, London, 1983).
[16] A. Bohr, B.R. Mottelson, Nuclear Structure, Vol. II (Benjamin, Reading, Mass. 1975).
[17] V.A. Kuz'min, V.G. Soloviev, Nucl. Phys. A 486 (1988) 118.
[18] O. Civitarese, J. Suhonen, Nucl. Phys. A 578 (1994) 62.
[19] A. Klein, E.R. Marshalek, Rev. Mod. Phys. 63 (1991) 375.
[20] K.T. Hecht, Vector Coherent State Method and its Application to Problems of Higher Symmetries, Lecture Notes in Physics (Springer-Verlag, Heidelberg, 1987).
[21] D.R. Bes, J. Kurchan, The treatment of collective coordinates in mechanical systems; an application to the BRST invariance. Lecture Notes in Physics, Vol. 34 (World Scientific, Singapore, 1990).