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Temperature Dependence of the Nuclear Binding Energy: effects on the EOS for Hot Nuclear Matter Using Different Models

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Abstract

Effects due to the temperature dependence of the nuclear binding energy upon the equation of state (EOS) for hot nuclear matter are studied. Nuclear contributions to the free energy are represented by temperature dependent liquid drop model terms.

Phase coexistence is assumed for temperatures of the order of $1\,\mathrm{MeV} \leqslant T \leqslant 6\,\mathrm{MeV}$, baryon number densities ρ of the order of $10^{-4}\,\mathrm{fm}^{-3} \leqslant \rho \leqslant 10^{-1}\,\mathrm{fm}^{-3}$ and lepton fractions of the order of $0.2 \leqslant y_l \leqslant 0.4$. It is found that the total pressure of the system is not affected by the temperature dependence of the nuclear free energy, in spite of changes observed in the nuclear pressure due to the different parametrizations used to represent the nuclear binding energy.

1. Introduction

The study of nuclear properties at finite temperatures is relevant both for applications in astrophysics [1–11] and for models of finite nuclei and nuclear matter at high excitation energy.

In this work we have investigated the influence of the temperature dependence of the nuclear free energy [12] on the statistical equilibrium between nuclei and nucleons in a nuclear matter phase. Different methods has been proposed during the last decades to compute the equation of state (EOS) of system with free nucleons, nuclei and leptons [1–9], mostly to describe conditions found in astrophysics and in highly energetic nuclear collisions . Since the currently adopted methods are well known we shall use them as the framework to test less known aspects of nuclear structure at finite temperature [12–18]. From this point of view we shall use the EOS as a tool to determine the validity of different approximations which have been proposed to describe nuclear matter properties at finite temperature, rather than exploring already solved questions related with the way in which the EOS is calculated [1–8,19–21]. Similar approaches can be found in [1–8], where the temperature dependence of the nuclear free energy has been parametrized using modified versions of the Fermi Gas Model. In the present treatment we have used a realistic nuclear free energy [13], based on temperature dependent mean field calculations, and another one [22], constructed as an extrapolation from the data on nuclear ground state and excited state energies, to determine physical consequences of both approximations upon the EOS.

In order to introduce the present formalism let us briefly review some of the approximations which are used in dealing with the calculation of the EOS, namely:

(*i*) Bulk nuclear matter properties at finite temperature have been described by using density dependent Hamiltonians, of the Skyrme's type [14, 23, 24], treated in the plane wave limit [2] and in the context of the Thomas–Fermi approximation [15, 25, 26];

(*ii*) The EOS, under constraints which are suitable for the description of astrophysical processes, has been obtained for electrically neutral systems with electrons, neutrinos, protons and neutrons in a drip phase and nuclei [1, 3, 5, 6, 19];

(*iii*) The drip phase, at equilibrium with the nuclear phase, has been treated as an interacting Fermi Gas [2] and the nuclear phase has been described in terms of a distribution over a mass range with constant, temperature independent, free energies [5].

In this picture one can think of a phase transition between nucleons in nuclei (nuclear phase) and in nuclear matter (drip phase). Another phase transition (the liquid to gas phase transition) can occur as a consequence of the thermal collapse of the interaction between nucleons in the drip phase [4]. The temperature scale of this phase transition is fixed by the energy of ordinary nuclear matter at saturation [14, 23] and it is determined by the residual two body interaction used to describe the drip phase [1–6, 9].

The temperature domain for these transitions varies with the force used to describe nuclear interactions. Nuclear partition functions yield a critical temperature of the order of $T = 8$–$10\,\mathrm{MeV}$ [21] for the melting of nuclei while a standard partition function for nuclear matter yields a critical temperature of the order of $T = 18$–$20\,\mathrm{MeV}$ [1, 5] for the liquid to gas phase-transition, respectively. Recently, limiting temperatures of the order of $T = 4$–$8\,\mathrm{MeV}$, for the nuclear phase, have been reported by Baldo *et al.* [12].

The liquid to gas phase-transition at $T = 18$–$20\,\mathrm{MeV}$ has not been observed. From crude arguments, based on nuclear structure concepts, a critical temperature of the order of $T = 8$–$10\,\mathrm{MeV}$ can be associated with a transition from bound to unbound nucleons, as mentioned above. However, this value is too high to be consistent with data extracted from the analysis of compound nucleus reactions [27–30]. Experimental results suggest that $T = 4$–$6\,\mathrm{MeV}$ is a more reliable value [27]. In this respect the theoretical values reported by Baldo *et al.* [12] are more realistic than the values extracted from mean field calculations [6]. It implies that the temperature which a compound nucleus can reach can be lower than the theoretical value obtained in a mean field approach [6].

The work of Ref. [13], about the temperature dependence of the nuclear free energy, has shown that effects associated with it are important in dealing with the calculation of nuclear partition functions. The parametrization of [13] is consistent with the experimental information [27–30] and with theoretical evidences [12] about limiting temperatures. Another parametrization of the nuclear binding energy, at finite temperatures, has been proposed more recently in [22]. As we shall discuss in detail, the parametrization of Davidson *et al.* [22] shows a more pronounced temperature dependence of the associated Liquid Drop Model coefficients. It has also induced some speculations about the observation of signals on "phase transitions" [22]. Following the conclusions of [22] these effects should strongly influence the EOS. As it is said in [22], the low temperature behavior of the volume coefficient differs from the one used to compute the equation of state (EOS) for symmetric nuclear matter. In view of this we would like to explore the consequences upon the EOS once a given temperature dependence of the nuclear binding energy is assumed. In the present work the results corresponding to the EOS calculated by using the parametrization proposed in [22] and the one of Guet et al. [13] will be compared. In order to test the accuracy of the method adopted to compute the EOS the present results will be compared, also, with the results of Lattimer *et al.* [5].

The equations which are discussed in Section 2 represent a system where:

(*i*) a fraction of nucleons is bound in a nuclear phase. The nuclear free energy is assumed to be temperature dependent and it has been parametrized as in Ref. [13]. In this parametrization the usual Liquid Drop Model (LDM) coefficients [31], which are temperature independent, are replaced by temperature dependent ones, instead;

(*ii*) a fraction of nucleons are in a drip phase [1]. The microscopic description of this drip phase is obtained from the finite temperature treatment of Skyrme's Sk^* Hamiltonian in the nuclear matter limit [14, 23, 24];

(*iii*) nuclei and drip nucleons are at thermal equilibrium; and

(*iv*) Coulomb lattice effects are included, to account for the interaction between nuclei and the background of electrons, following the method of Ref. [10].

The equilibrium conditions are fulfilled by imposing constraints on chemical potentials and baryon number densities at fixed volume [8]. Changes in nuclear properties due to a neutron vapor phase have not been considered; intrinsic nuclear excitations are also neglected in constructing the nuclear partition function except for a renormalization of the nuclear level density parameter [31]. Results for the EOS of the system and related parameters are shown and discussed in section 3. Conclusions are drawn in Section 4.

2. Formalism

The system is described by the following parameters: the neutron density ρ_n, the proton density ρ_p and the electron density ρ_e. These quantities can also be written in terms of a total baryon number density ρ and a charge fraction Y_p, where:

$$\rho = \rho_n + \rho_p, \tag{1}$$

and

$$Y_p = \frac{\rho_p}{\rho_p + \rho_n}. \tag{2}$$

Since we have assumed charge neutrality the charge fraction Y_p is fixed at the value Y_e, which denotes the number of electrons per baryons. For the present study we shall neglect neutrino contributions, Y_ν, and use the electron fraction instead of the total leptonic one. A similar approximation has been used in Refs [1, 2, 4].

We have assumed that nuclei are present in the system. All nuclei with $N = Z$ and with mass number $A \leqslant 18$ have been included in the calculations, as well as all nuclear species with $20 \leqslant A \leqslant 130$. By including this set of nuclei we are extending the formalism of [1–7], which includes only one nuclear mass. Free (drip) nucleons and nuclei are at thermal equilibrium. Nucleons in the drip phase are interacting via a two-body density dependent interaction of the Skyrme type (Sk^*) [14, 23, 24] and the nuclei are treated like elementary particles in Boltzmann's statistics. Equilibrium conditions are given by the balance equation [8, 32]:

$$\beta\mu_{A,Z} = -(A - Z)y_n - Zy_p + \beta B_{A,Z}, \tag{3}$$

where

$$y_q = \mu_q \beta, \tag{4}$$

β is the inverse temperature and $\mu_q (q = \mathrm{n, p})$ is the chemical potential for neutrons ($q = \mathrm{n}$) or protons ($q = \mathrm{p}$) in the drip phase. The balance condition (3) looks, in principle, different from the more conventional equilibrium conditions of [1–7], which are based on the minimization of the total free energy. The present approach, based on [8], and the conventional one of [1–7] give similar EOS, as it will be shown in Section 3. Since we are interested in the study of effects, upon the EOS, induced by the nuclear component of the system any other suitable method to compute the EOS can be used. For a review of these methods see [11].

The quantity $B_{A,Z}$ is the nuclear binding energy for a nucleus with mass number A and proton number Z; the nuclei included in the calculations fulfill the condition [8]:

$$B_{Z,A} = Zm_p + (A - Z)m_n - M_{A,Z} > 0. \tag{5}$$

The nuclear number density for a nucleus with mass number A and Z protons, $n_{A,Z}$, is defined by

$$n_{A,Z} = \left(\frac{2\pi m_{Z,A}}{h^2\beta}\right)^{3/2} e^{\beta\mu_{A,Z}}\Theta, \tag{6}$$

where Θ is the nuclear partition function [31]

$$\Theta = e^{-aT} \sum_l (2l + 1) e^{[-\hbar^2 l(l+1)]/2IT}. \tag{7}$$

Table I. *EOS for lepton fraction y_l, entropy per baryon s and total baryon density ρ. The value of the temperature (T) is shown, as obtained by using the method of Lattimer et al. (LRPL) [5] and the present one (TI) calculated with the temperature independent version of the Liquid Drop Model of [13]*

		T[MeV]					
		s = 1		s = 2		s = 3	
y_l	log (ρ)	LPRL	TI	LPRL	TI	LPRL	TI
0.2	−4.00			0.750	0.644	1.500	1.482
	−3.50			0.937	1.164	1.975	2.266
	−3.00	0.750	0.588	1.949	2.009	3.148	3.434
	−2.50	1.500	1.160	3.257	3.415	4.872	5.355
	−2.00	2.414	2.106	5.271	5.755	8.155	8.978
	−1.50	4.063	3.479	8.419	9.174	14.519	15.183
0.3	−4.00	0.652	0.571	1.335	1.353	1.780	1.904
	−3.50	0.912	0.805	1.707	1.827	2.236	2.566
	−3.00	1.283	1.117	2.397	2.610	3.359	3.620
	−2.50	1.877	1.668	3.616	3.922	4.989	5.410
	−2.00	2.879	2.590	5.574	5.796	8.084	8.354
	−1.50	4.330	3.718	8.499	9.373	13.812	12.805
0.4	−4.00	1.092	1.122	1.690	1.674	1.925	2.046
	−3.50	1.500	1.357	1.917	2.169	2.379	2.713
	−3.00	1.707	1.722	2.649	2.931	3.462	3.735
	−2.50	2.325	2.301	3.822	4.078	5.073	5.304
	−2.00	3.262	3.139	5.726	5.665	8.017	7.734
	−1.50	4.529	4.127	8.569	7.768	13.371	11.333

In Eqs (6)–(7) translational and rotational degrees of freedom of the nucleus are included. Intrinsic excitations are not included explicitly, except for the renormalization of the level density parameter [31].

Following the treatment of Lattimer *et al.* [5] we have defined the volume available to the drip matter by the ratio:

$$\frac{V_{\text{drip}}}{V_{\text{Total}}} = 1 - \sum_{Z,A} V_{Z,A} n_{Z,A}, \tag{8}$$

where the sum is taken over all allowed partitions of the mass number and nuclear charge included in the calculations. These equations account for the inclusion of more than one nuclear mass [5].

The above definitions have been used to construct balance equations [6]:

$$\sum_{A,Z} Z n_{A,Z} + \left(1 - \frac{V_{\text{nuclear}}}{V_{\text{total}}}\right)\rho_{\text{p}}^{\text{drip}} = Y_{\text{p}}\rho, \tag{9}$$

$$\sum_{A,Z} N n_{A,Z} + \left(1 - \frac{V_{\text{nuclear}}}{V_{\text{total}}}\right)\rho_{\text{n}}^{\text{drip}} = (1 - Y_{\text{p}})\rho, \tag{10}$$

which can be solved for fixed values of the temperature T and of the density ρ.

The solutions are functions of the chemical potential for neutrons and protons in the drip phase. Once the chemical potentials are determined, the solutions of eqs (9)–(10) are used to construct the total free energy of the system:

$$F = F^{(\text{nuclear})} + F^{(\text{drip})} + F^{(\text{electrons})}, \tag{11}$$

where:

$$F^{(\text{nuclear})} = \sum_{A,Z} n_{A,Z} f_{A,Z}, \tag{12}$$

and

$$F^{(\text{drip})} = E^{(\text{drip})} - TS^{(\text{drip})}. \tag{13}$$

The contribution to the free energy due to electrons, $F^{(\text{electrons})}$, is defined in Ref. [5] and the nuclear free energy, $F^{(\text{nuclear})}$, includes the translational and rotational contributions which appears in the definition of $n_{A,Z}$ of Eqs (6)–(7), as we have mentioned above, and an intrinsic free energy $f_{A,Z}$ [5, 6].

The intrinsic nuclear free energy term is parametrized in the way which has been proposed in [13], with an explicit temperature dependence of the coefficients corresponding to the liquid drop model expansion. The expression for the nuclear free energy per unit mass is the following:

$$f_{A,Z} = a_{\text{v}} + a_{\text{s}} A^{-1/3} + a_{\text{c}} A^{-2/3} + a_0 A^{-1} + f_{\text{Coul}} + f_{\text{asym}}. \tag{14}$$

The first four terms of the free energy are the volume, surface, curvature and constant-shift contributions, as discussed in [13], respectively, and

$$f_{\text{Coul}} = Z^2(c_1 A^{-4/3} + c_2 A^{-2}), \tag{15}$$

and

$$f_{\text{asym}} = c_{\text{asym}} \frac{(N - Z)^2}{A}, \tag{16}$$

are the Coulomb and asymmetry free energy terms. The Coulomb contribution corresponds to a single nucleus. The Coulomb lattice correction to this value [10] is described below.

The coefficients which appear in the definition of the nuclear free energy, namely: a_{v}, a_{s}, a_{c}, a_0, c_1, c_2 and c_{asym}, are functions of the nuclear temperature T. At lowest order in T these coefficients can be parametrized as quadratic functions of the temperature, with constant terms which are determined from the standard temperature independent Liquid Drop Model expansion. Following the notation of [13] we can write for them the expression:

$$a_k(T) = a_k(T = 0)(1 - \alpha_k T^2), \tag{17}$$

where with the index k we have denoted different contributions (volume, surface, curvature, constant shift, etc) to the above quoted nuclear free energy, eq. (14).

The parameters α_k are the critical values for inverse temperatures associated to volume, surface, curvature, Coulomb and asymmetry terms of the nuclear free energy. The adopted values can be found in [13]. The value of each parameter α_k determines the higher value of the temperature T which can be reached without violating the approximation. Limiting temperatures, extracted from these coefficients, are of the order of 11 MeV [13]. In the form proposed in [13], one has, for the nuclear binding energy, the following expression

$$E_b(A, Z, T) = a_v(T)A + a_s(T)^{2/3} + a_c(T)^{1/3} + a_0(T) + \frac{J(T)AI^2}{1 + \frac{9J(T)}{4Q(T)A^{1/3}}} + c_1(T)\frac{Z^2}{A^{1/3}} + c_2(T)\frac{Z^2}{A} + \delta\Delta(T) \tag{18}$$

the term $I = (N - Z)/A$; $\delta\Delta(T)$ is a parametrized temperature dependent pairing correction. The value of the parameters α_k are listed in the Table I of [13].

Another parametrization of the nuclear binding energy, at finite temperature, can be found in [22]. Following Davidson *et al.* [22], it reads

$$E_b(A, Z, T) = \alpha(T)A + \beta(T)A^{2/3} + \left(\gamma(T) - \frac{\eta(T)}{A^{1/3}}\right) \times \left(\frac{4t_\zeta^2 + 4|t_\zeta|}{A}\right) + 0.8076\frac{Z^2R(0)}{A^{1/3}R(T)} \times \left(1 - \frac{0.7636}{Z^{2/3}} - \frac{2.29R(0)^2}{(R(T)A^{1/3})^2}\right) + \delta(T)\frac{f(A, Z)}{A^{3/4}} \tag{19}$$

where $t_\zeta = (Z - N)/2$ and $f(A, Z) = -1$, 0 or 1 for even-even, even-odd and odd-odd nuclei, respectively, $R(T)$ is a temperature dependent Coulomb radius ($R(T) = 1.07(1 + 0.01T)$); as it is said above, the value of the coefficients ($\gamma(T)$, $\eta(T)$, $\beta(T)$, $\alpha(T)$ and $\delta(T)$) are given in [22].

A comparison between the temperature dependence described by the above expressions, eqs (18)–(19), is shown in Figs 1 and 2. The E_b of eq. (19), shows a strong temperature dependence, as compared to the one of eq. (18), particularly, for the case of the symmetry coefficient (see Fig. 1). The loss of binding at relatively small values of the temperature seems to be a regular feature of the results calculated from eq. (19).

The contribution to the total free energy due to nucleons in the drip phase has been calculated by following the method of [2]. For this part of the calculations we have used the Skyrme interaction Sk^* including thermal effects, as it has been discussed in [14, 23, 24].

The Coulomb free energy includes lattice corrections. Following the method of [10] it is computed in the Wigner–Seitz cell. As a function of the nuclear specific volume u the lattice correction reads:

$$f_{\text{Coul}}^{(\text{lattice})}(u) = f_{\text{Coul}}[1 - (3/2)u^{1/3} + (1/2)u]. \tag{20}$$

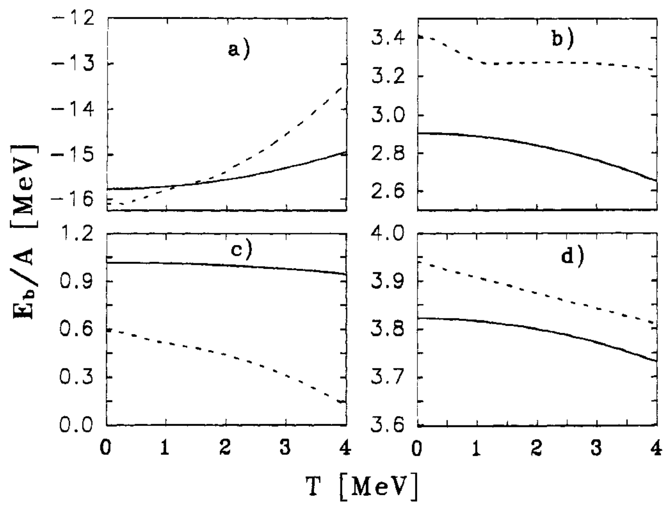

Fig. 1. Temperature dependence of the volume (a), surface (b), symmetry (c) and Coulomb (d) terms of the nuclear binding energy per nucleon, E_b/A, taken from Guet *et al.* [13] (solid line) and from Davidson *et al.* [22] (dashed lines, for the case of ^{208}Pb.

Once the total free energy of the system has been defined, with the above introduced expressions, the *EOS* can be computed.

Following the work of [8], the equation of balance (3) and the constraints (9) and (10) are solved for a given temperature T, for a fixed lepton to baryon fraction Y_l and for a total baryon density ρ; this system of equations is highly non-linear and it combines the exponential dependence of $n_{A,Z}$ upon y_q (q = n and p) with the linear dependence upon ρ_q^{drip} (q = p, n). Moreover, since $n_{A,Z}$ depends on the nuclear chemical potential $\mu_{A,Z}$ the system of equations is also dependent upon the adopted parametrization of the nuclear free energy.

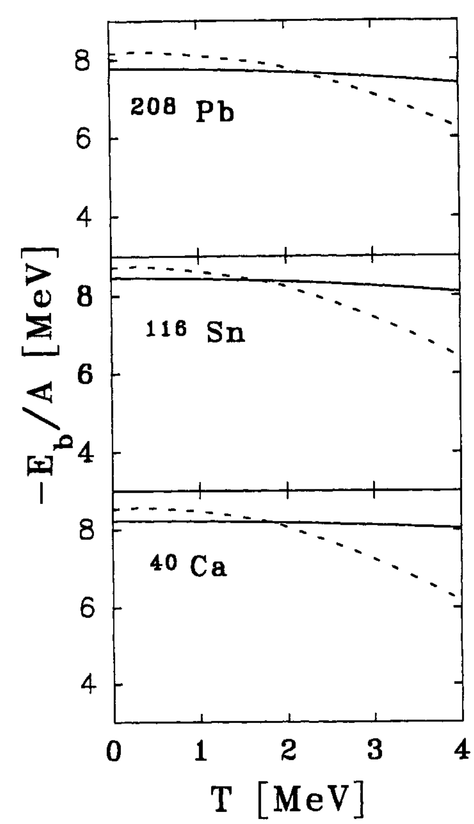

Fig. 2. Temperature dependent nuclear binding energy, for different nuclei, calculated with the parameters of [13], eq. (18) solid lines) and[22], eq. (19), (dashed lines), respectively.

As we have mentioned before, the use in the present work of the method of El Eid and Hillebrandt [8], to compute the EOS, does not necessarily imply that this method is superior to any other one reported in the available literature. It has been used as a suitable framework for the present discussion on nuclear temperature dependent effects. For the sake of completeness we have also computed the EOS using other method, the one of [5]. The comparison of the results for the EOS calculated with the method used by Lattimer *et al.* [5] (LPRL); which is based on the minimization of the total free energy; and the present one, obtained from the solution of the equations given in [8, 32], is shown in Tables I and II. From the results which are shown in these tables it is seen that both methods give a similar EOS.

3. Results and discussion

The total pressure, P, of the system can be expressed in terms of the total free energy, F, and it is given by:

$$P = \rho \frac{\partial F}{\partial \rho} - F. \tag{21}$$

The contribution to the total pressure due to the nuclear component of the system can be obtained by subtracting, from the r.h.s of the above equation, the pressure due to the electron phase.

The parameters of the two body interaction which we have used to compute the free energy of nucleons in the drip phase are given in [2].

In order to investigate effects due to the temperature dependence of the nuclear free energy upon the EOS of the system, we have calculated it for the following cases:

(*a*) with the temperature independent parametrization of the nuclear free energy (TI) of [13], i.e: using eq. (18) with constant (temperature independent) coefficients $a_k(T = 0)$, c.f. eq. (17);

(*b*) with the temperature dependent (TD) one, i.e: using eq. (18) with the coefficients given by eq. (17).
and

(*c*) with the parameters introduced in eq. (19).

The EOS obtained by using the present method (TI case) and the one of LPRL are shown in Tables I and II. As it is said above, the EOS is similar, for both set of results. These results give some confidence about the use of present method, concerning this part of the calculations, and allows one to discuss on the temperature dependence of the nuclear components of the problem. The results shown in Table III correspond to the calculated nuclear mass abundance, from the *EOS* computed with the approximations (a), (b) and (c). At first view, the results of Table III do not differ significantly, except at relatively high temperature, since heavy masses are suppressed more rapidly in the temperature dependent cases.

The total pressure P is shown in Fig. 3, for the three cases (a), (b) and (c). The isotherms are clearly independent of the parametrization used to compute the nuclear contribution. It is evident from the curves shown in this figure that the adopted temperature dependence of the nuclear free energy does not affect the results. This is expected since the total pressure is dominated by the contribution of the relativistic electron gas [5, 6]. This feature is also consistent with the fact that the temperature associated to the collapse of the nuclear bulk-term of the binding energy, T_{bulk}, is of the order of 17 MeV [13]; a value which is larger than the maximum value of T (6 MeV) allowed in our calculations.

The results which are displayed in Fig. 4 correspond to the nuclear contributions to the pressure. The results obtained by using the TD expression (case b) for the nuclear free energy yield a lower nuclear pressure, for a given T, as compared to the TI case (case a). For the case of the binding energy of [22], curves c), the nuclear pressure is even lower and it shows a peculiar behaviour at temperatures of the order of $T = 4$ MeV to $T = 6$ MeV. In fact, it shows instabilities at very low densities which are clearly non-physical.

Another interesting feature of the EOS is the predicted value of the adiabatic index Γ [5, 6] at low entropy. From the EOS, both in the LPRL and TI approximations, (see

Table II. *Adiabatic index Γ obtained from the EOS. The meaning of the variables y_1, ρ, s and of the approximations (LRPL) and (TI) is given in the caption of Table I*

		Γ					
		$s = 1$		$s = 2$		$s = 3$	
y_1	log (ρ)	LPRL	TI	LPRL	TI	LPRL	TI
0.2	−4.00			1.290	1.354	1.370	1.372
	−3.50			1.375	1.381	1.375	1.372
	−3.00	1.355	1.352	1.370	1.377	1.390	1.372
	−2.50	1.340	1.337	1.365	1.348	1.400	1.368
	−2.00	1.320	1.300	1.340	1.287	1.390	1.301
	−1.50	1.365	1.281	1.300	1.223	1.240	1.111
0.3	−4.00	1.325	1.337	1.310	1.312	1.333	1.321
	−3.50	1.310	1.315	1.325	1.326	1.350	1.337
	−3.00	1.315	1.318	1.334	1.337	1.360	1.339
	−2.50	1.320	1.324	1.333	1.321	1.365	1.333
	−2.00	1.310	1.311	1.320	1.292	1.355	1.298
	−1.50	1.275	1.296	1.295	1.261	1.200	1.204
0.4	−4.00	1.305	1.310	1.315	1.313	1.310	1.319
	−3.50	1.310	1.316	1.320	1.323	1.340	1.328
	−3.00	1.315	1.322	1.333	1.326	1.345	1.325
	−2.50	1.320	1.325	1.333	1.317	1.345	1.318
	−2.00	1.315	1.322	1.315	1.304	1.340	1.305
	−1.50	1.290	1.316	1.270	1.295	1.100	1.264

Table III. *Nuclear (most abundant) mass A, as a function of the total baryon density (ρ), for lepton fraction $y_l = 0.3$ and for the isotherms denoted by the temperature (T). The results shown in the third and fourth columns correspond to the temperature independent parametrization of the nuclear free energies (TI) and to the temperature dependent one (TD) of [13], respectively. The results shown in the last column correspond to the EOS calculated with the temperature dependent parametrization of [22]*

	A			
log (ρ)	*T*[MeV]		TI	TD
−3	1.	93	89	93
	2.	71	63	75
	3.	25	25	23
	4.	24	4	4
	5.	4	4	4
−2	1.	101	101	95
	2.	85	79	83
	3.	57	39	76
	4.	29	29	74
	5.	29	29	20
−1.	2.	100	91	98
	3.	73	63	75
	4.	33	33	74
	5.	33	33	21

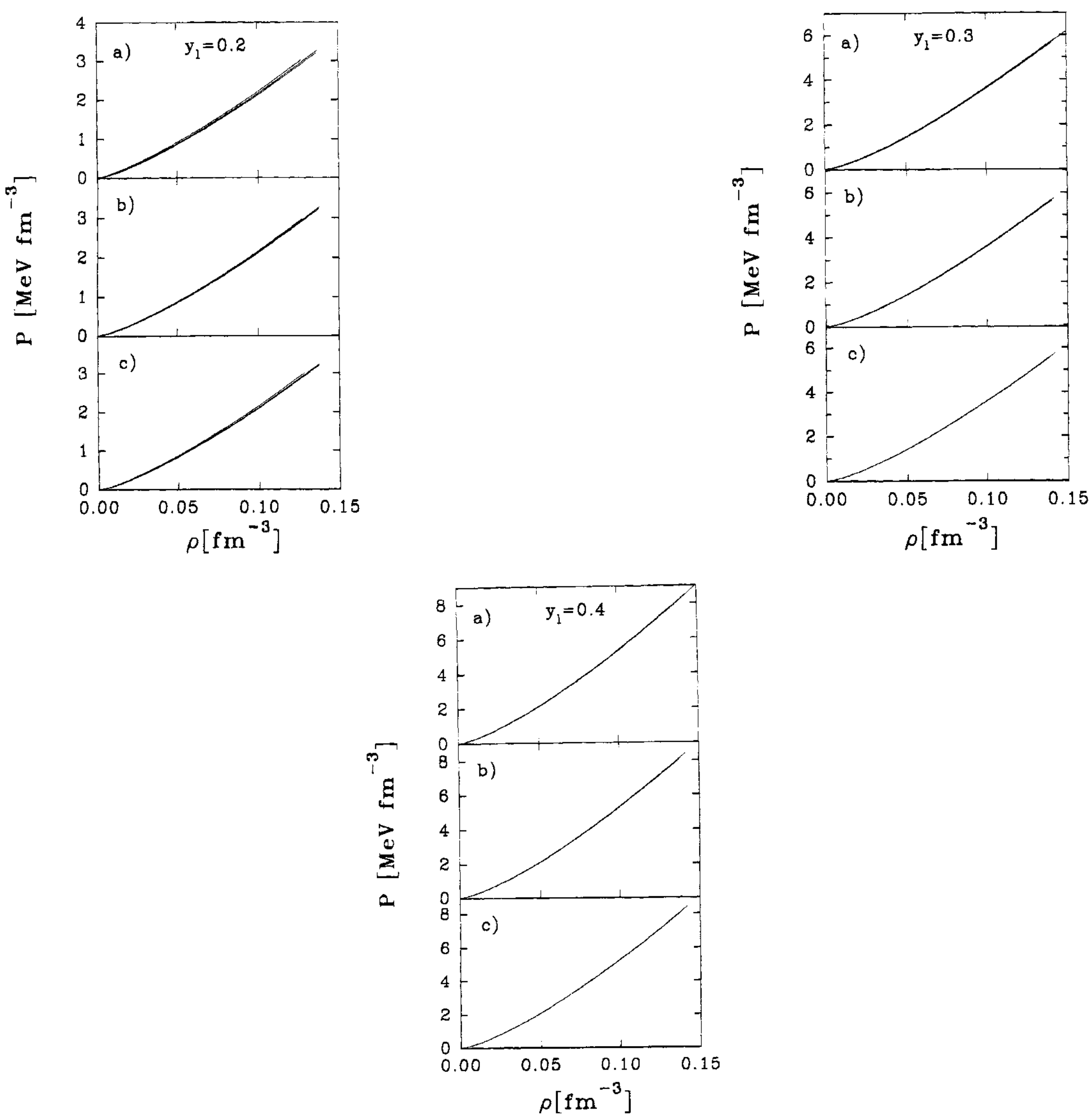

Fig. 3. Total pressure, *P*, as a function of the density, *ρ*, for different values of the temperature *T*. The curves shown for each case read from bottom to top: *T* = 2, 4 and 6 MeV, respectively. Cases (a) and (b) correspond to temperature independent (TI) and temperature dependent (TD) parametrizations of the nuclear binding energy of [13] and (c) to the parametrization given in [22]. The results are shown for different values of the lepton fraction y_l.

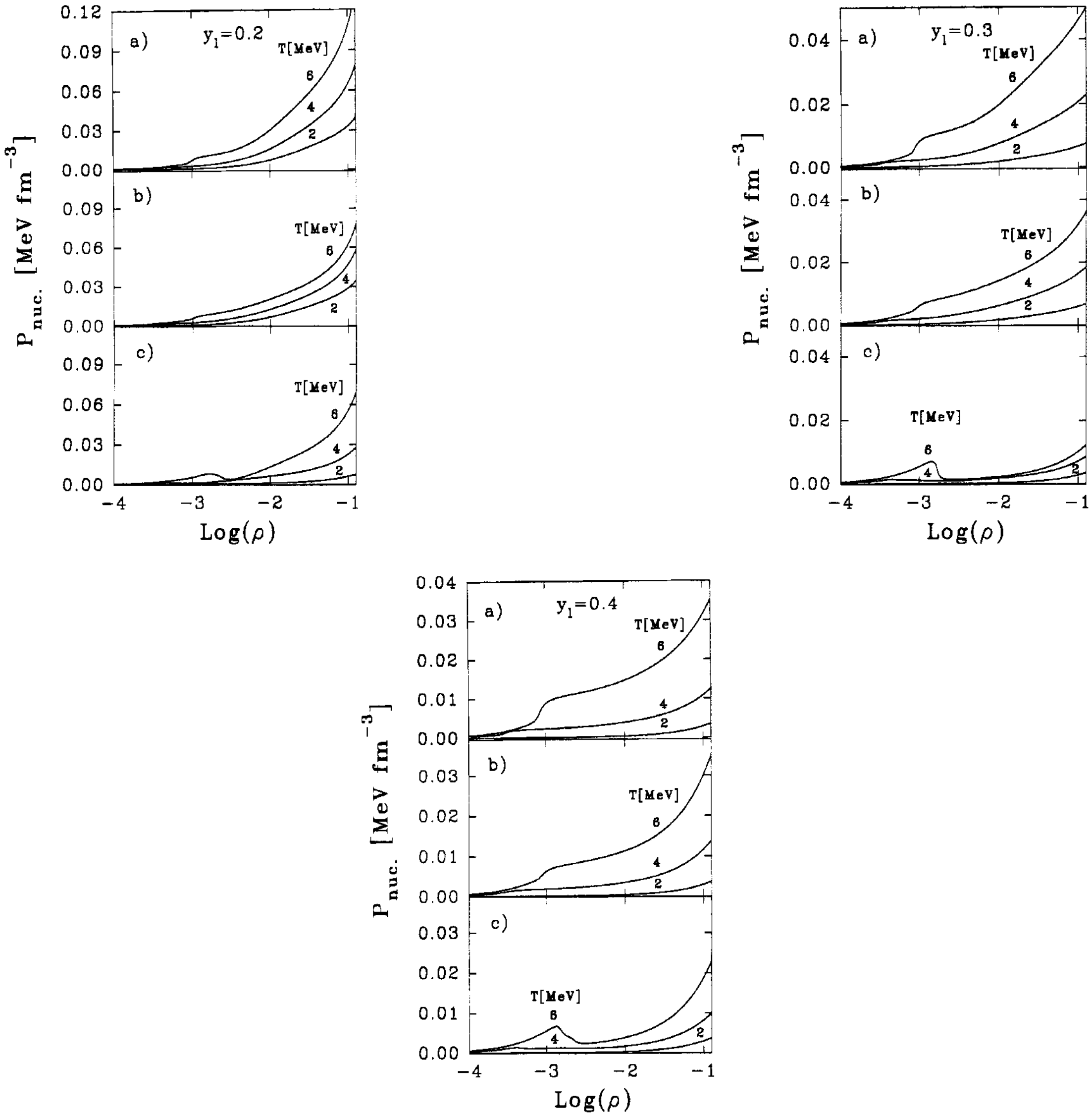

Fig. 4. Pressure of the nuclear phase, P_{nuc}, as a function of the density for various values of the temperature T. Cases (a) and (b) correspond to the (TI) and (TD) parametrizations given in [13] and case (c) corresponds to parametrization of the nuclear binding energy of [22].

Table II) it is found that the predicted value of Γ, at entropy per baryon $s = 1$, 2 and 3 is of the order of 1.3 and remains nearly constant over the densities which we have considered. This smooth behavior differs from the huge changes (it varies from 1.18 to 1.33 and then it goes back to 1.2, for the three values of s) shown by the value of Γ obtained by using eq. (19), as we have noted in our calculations.

Figure 5 shows temperature versus density domains along adiabats, for different lepton fractions. The use of eq. (19) in the EOS yields lower values of the temperature, for densities around the normal nuclear matter density. The difference between the values obtained with eq. (18) and those corresponding to eq. (19) is of the order of (or larger than) 1 MeV for $s = 2$, 3.

It is then evident, from the results which are presented above, that for a realistic temperature dependence of the nuclear binding energy, like the one of [13], one should expect to find very small effects upon the EOS, except for the already discussed suppression of heavy mass nuclei from the distribution of most abundant nuclear masses and for a tendency to produce cooler adiabats. Both effects can be related to the fact that small, but non-negligible, changes in the binding energy of the hot nuclei have been obtained in the self-consistent treatment of Guet *et al.* [13]. The parametrization proposed by Davidson *et al.* [22] produces more drastic changes on the EOS, but the physical meaning of these changes is somehow dubious, as shown by the results of Fig. 4. In order to relate the drastic changes of the EOS corresponding to the approximation c) with the parameters of eq. (19) we would like to comment on the behavior of the bulk-term of the binding energy. The results are shown in Fig. 6. The derivative of the bulk of the binding energy, eq. (19), respect to the temperature has a bump at about $T \approx 0.5$ MeV but it is linearly dependent on T for the case of [6] and also for [13].

The appearance of this bump can be due to the fact that the coefficients of eq. (19) have been determined from a fit to the observed low-energy spectra of a huge number of nuclei [22]. It is known that finite nuclei show a rich variety of features strongly dependent on the nuclear interactions, which are reflected, particularly, on the low energy spectrum. Among these features the appearance of a low-lying excited state strongly affects the thermal behaviour of the nucleus. The thermal occupation of low-lying states determines a departure in the temperature dependence of the average (statistical) values of the characteristic functions, like the energy and the entropy. The shape of the specific heat corresponding to eq. (19), shown in Fig. 6, is a clear signal of finite size effects [33] which are always present when partition functions or level densities are constructed

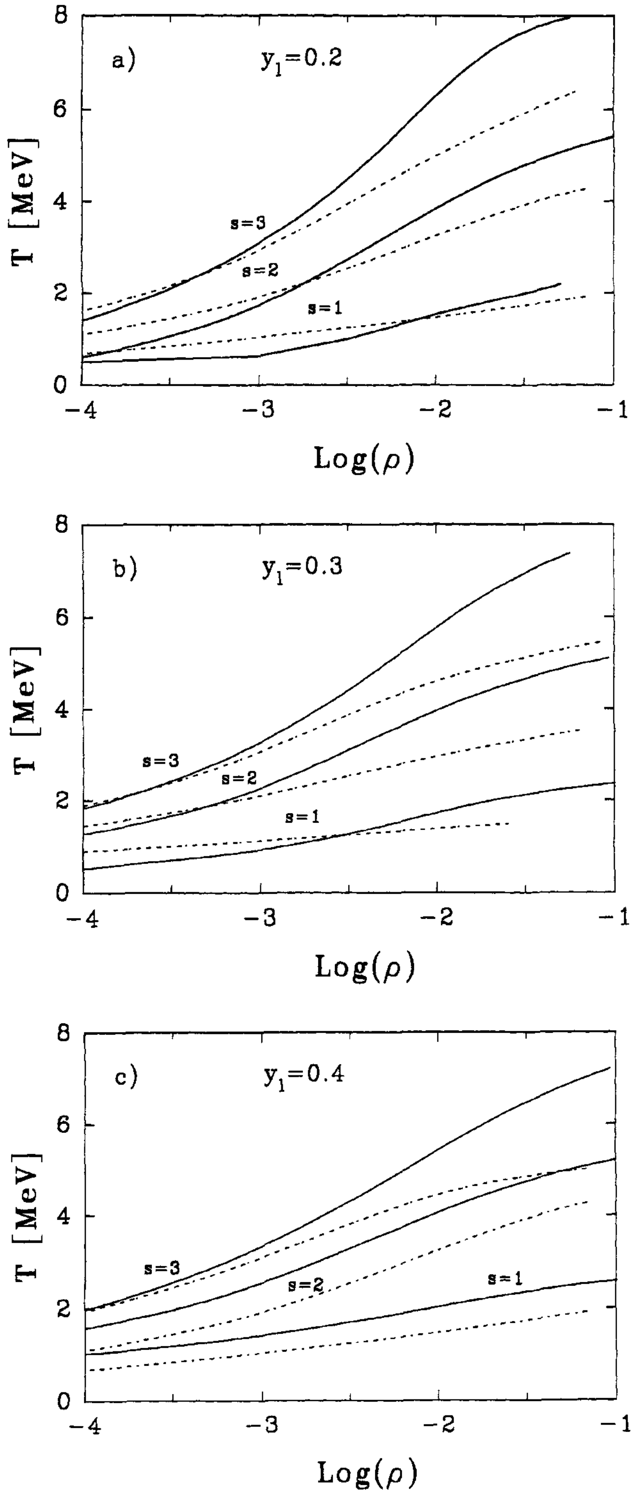

Fig. 5. The temperature (T) *vs.* density (ρ) plot at constant entropy (s). The results shown with solid and dashed lines correspond to the EOS calculated with the parameters given in [13] and [22], respectively.

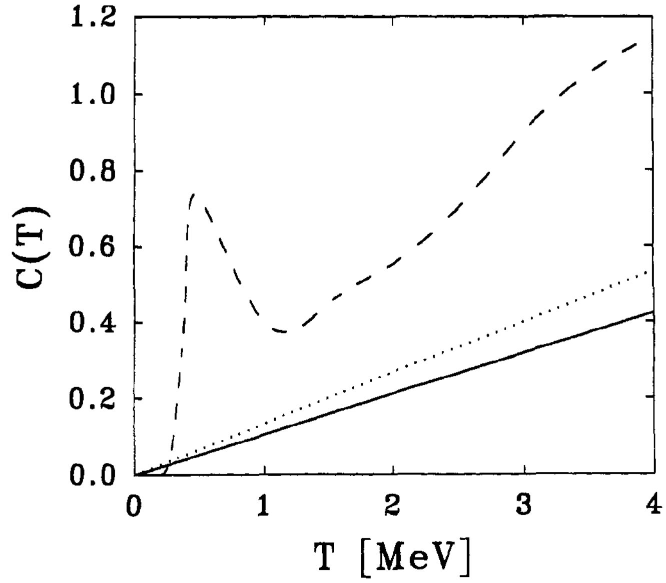

Fig. 6. Specific heat, C, for homogeneous nuclear matter (volume contribution), as a function of the temperature, T, calculated with the parameters given by Davidson *et al.* [22] (dashed lines), Lattimer *et al.* [6] (dotted lines) and Guet *et al.* [13] (solid lines).

with a finite (and small) number of levels. This is something certainly not present in the case of homogeneous nuclear matter or in the large scale fit of mean-field properties at finite T and it can explain the difference between the fit of [22] and the values of [6] and [13].

The results, which we have been presented, can be summarized in the following:

(*i*) The total pressure of the system is rather independent of the thermal dependence of the parameters which are included in the definition of the nuclear free energy and it is dominated by the electron gas. However, at a much smaller scale, some changes are observed in the nuclear pressure when temperature dependent nuclear binding energies are used to construct the EOS;

(*ii*) The nuclear pressure for the TD cases tends to be lower than the pressure corresponding to the TI case;

(*iii*) The low density region of the EOS shows an abundance of ^{4}He for both the TI and TD approximations. At larger densities the mass distribution is more dependent upon density for the temperature independent case than for the temperature dependent one.

The inclusion of TD nuclear free energies induces a suppression of heavy nuclei, as compared with the results obtained by using TI nuclear free energies. Therefore, a smaller number of nuclear states will be available to store entropy. This effect can be of some relevance in describing the infall epoch of type II supernovae [7, 11]. As said above the changes which can be produced in the EOS by introducing temperature dependent nuclear structure effects are very small. This conclusion is supported by the results obtained by using a realistic expression of the nuclear binding energy at finite temperatures [13]. More drastic changes can be produced on the EOS if the parametrization proposed in [22] is used. However, some doubts can be cast about the validity of such a parametrization, since it may be strongly limited by finite size effects. [33].

4. Conclusions

In this work we have reported some results concerning the temperature dependence of the nuclear free energy and its effects upon the equation of state for a system of nuclei and nucleons in a drip phase.

We have solved a system of coupled equations to determine the chemical potentials under balance conditions. The nuclear structure component of the calculations has been approximated by a temperature dependent version of the Liquid Drop Model and by another one proposed in [22].

The EOS of the system is not affected by the temperature dependence of the nuclear component. However, some changes due to the adopted temperature dependence of the nuclear free energy are observed, mainly the decrease of the nuclear pressure at fixed temperature. In addition some effects are observed in the mass abundance which show the suppression of heavy nuclei when temperature dependent nuclear binding energies are used to compute the EOS. For the temperatures and densities considered in the present work these are minor effects, indeed. The parametrization of [22] leads to cooler adiabats than the one corresponding to the EOS computed with a more standard set of parameters for the binding energy [13] but the results may be strongly affected by finite size effects (i.e: the finite number of nuclear levels included in the fit of [22] and its extrapolation to finite temperatures). It can change the nuclear contribution to the EOS significantly, but not in the sense advanced in [22] where the bump show in Fig. 6 was interpreted as a signal for a phase transition.

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