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Double beta decay and the proton-neutron residual interaction

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Abstract

The validity of the pn-QRPA and -RQRPA descriptions of double beta decay transition amplitudes is analyzed by using an exactly solvable model. It is shown that the collapse of the QRPA is physically meaningful and that it is associated with the appearance of a state with zero energy in the spectrum. It is shown that in the RQRPA this particular feature is not present and that this approach leads to finite but otherwise spurious results for the double beta decay transition amplitudes near the point of collapse.

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The neutrinoless double beta decay ($\beta\beta_{0\nu}$) violates lepton number conservation and requires the existence of massive Majorana neutrinos [1]. Due to this fact the detection of this decay has attracted considerable experimental effort [2]. The two neutrino mode of the double beta decay ($\beta\beta_{2\nu}$), on the other hand, can be described as a second order process in the standard electroweak theory, its decay rate being independent of any new physics beyond the standard model. Both to predict and to analyze the data on double beta decay require a precise calculation of the various nuclear matrix elements needed to compute the corresponding half-lives.

The quasiparticle random phase approximation (QRPA), including a particle-particle channel in the residual interaction, can reproduce the experimentally determined two-neutrino double-beta decay ($\beta\beta_{2\nu}$) half-lives. This is so because the transition amplitudes are strongly suppressed for certain values of the force parameters. This result was tested by comparing the results of the QRPA against an exactly solvable model [3,4] and including realistic residual interactions [5,6].

However, the predictive power of the QRPA is strongly diminished because the ground state to ground state $\beta\beta_{2\nu}$ transition amplitudes are extremely dependent upon the structure of the adopted proton-neutron interaction. For some critical values of the model parameters the QRPA collapses, i.e. the energy of the first excited QRPA state vanishes. It makes the theory nearly useless for some particular cases, the worst of which is the $\beta\beta_{2\nu}$ decay of ^{100}Mo [3,4,7-9].

A renormalized version of the QRPA (RQRPA) [10,11], which includes some corrections beyond the

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quasiboson approximation, has been reformulated recently [12] and applied to the $\beta\beta_{2\nu}$ decay [13]. Contrary to the QRPA in the RQRPA there is no collapse for any set of values of the force's parameters. It was presented as a reliable tool and was applied to the $\beta\beta_{2\nu}$ decay of ^{100}Mo [13]. Similar results were found with the inclusion of proton-neutron pairing interactions [14].

There are some concerns about the formulation of the RQRPA. The most important one is that it makes use of the exact commutation relations between phonons, adding one-quasiparticle scattering terms to the ordinary quasi-boson approximation, but it does not include similar terms in the hamiltonian and in the transition operators. In the present letter it will be shown that the collapse of the QRPA correlates with the presence of a state with zero energy which is not found in the RQRPA spectrum. This shortcoming is not only a failure of the RQRPA but it is shared by other higher order approximations. Spurious excitations are exactly separated in the RPA but higher order approximations can introduce spurious components which finally dominate the results [15].

The model hamiltonian [16–18] consists of a single particle term, a pairing term for protons and neutrons and a schematic charge-dependent residual interaction including particle-hole and particle-particle channels. It has been shown that this interaction, when treated in the framework of the QRPA, produces similar results as those obtained by using a G-matrix constructed from the OBEP Bonn potential in reproducing single- and double-beta decay matrix elements [18–20]. In this letter we will consider the single shell limit ($j_p = j_n = j$) and monopole term ($J = 0$) of the residual interaction. As we shall show later on this model, which is not intended to accurately reproduce actual data, does indeed display the qualitative features of a realistic pn-QRPA calculation. Excitation energies, single- and double-beta decay transition amplitudes and ground state correlations, in this model, depend on the particle-particle strength parameter in the same way as they do in more elaborated calculations with many single particle levels and with more realistic interactions. The present case corresponds to beta decay transitions of the Fermi type.

The schematic hamiltonian reads

$$H = \sum_p e_p a_p^\dagger a_p - G_p S_p^\dagger S_p + \sum_n e_n a_n^\dagger a_n \\ - G_n S_n^\dagger S_n + 2\chi \beta^- \cdot \beta^+ - 2\kappa P^- \cdot P^+, \qquad (1)$$

with

$$S_p^\dagger = \sum_p a_p^\dagger a_{\bar p}^\dagger/2, \quad S_n^\dagger = \sum_n a_n^\dagger a_{\bar n}^\dagger/2,$$

$$\beta^- = \sum_{i,j} \langle i|\tau^-|j\rangle a_i^\dagger a_j, \quad P^- = \sum_{i,j} \langle i|\tau^-|j\rangle a_i^\dagger a_{\bar j}^\dagger, \qquad (2)$$

$a_p^\dagger = a_{j_p m_p}^\dagger$ being the particle creation operator, $a_{\bar p}^\dagger = (-1)^{j_p - m_p} a_{j_p - m_p}^\dagger$ its time reversal and τ^- the isospin lowering operator ($\tau^-|n\rangle = |p\rangle$). The parameters χ and κ play the role of the renormalization factors g_{ph} and g_{pp} introduced in the literature [3–6].

The hamiltonian (1) can be expressed in terms of generators of an SO(5) algebra [21–23]. It can be reduced to an SO(5) and isospin scalar [24] if its parameters are selected as

$$e_p = e_n, \quad \chi = 0, \quad G_p = G_n = 4\kappa. \qquad (3)$$

This SO(5) scalar limit was used to study the "pairing plus monopole model" many years ago [25,26]. If $\chi \neq 0$ the isospin symmetry is broken in the particle-hole channel and for $4\kappa \neq G_p(G_n)$ this symmetry is broken in the particle-particle channel. The Hilbert space is constructed using the eigenstates of the isoscalar hamiltonian, which are labeled by the number of particles N, the isospin T and its projection T_z. If $G \neq 4\kappa$ the eigenstates have definite N and T_z but not good isospin since the Hamiltonian mixes states with different isospin T.

We have selected $N_n > N_p$ and a large j to mimic the realistic situation in medium and heavy nuclei. In this letter we will use the following parameters:

$$j = 19/2, \quad N = 20, \quad 1 \le T_z \le 5,$$
$$e_p = 0.7 \text{ MeV}, \quad e_n = 0.0 \text{ MeV},$$
$$G_p = G_n = 0.2 \text{ MeV}, \quad \chi = 0.025 \text{ MeV}. \qquad (4)$$

κ will be used as a running parameter.

Lowest energy 0^+ states of different nuclei are shown in Fig. 1, for $G = 4\kappa$, in an energy vs. Z plot. States are labeled by (T, T_z). It is clear that a simple model with both like particles and pn-pairing interactions can reproduce the qualitative form of the mass

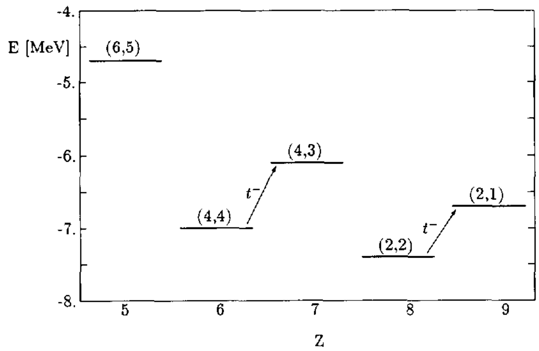

Fig. 1. Lowest energy states of the different nuclei, labeled by (T, T_Z), for $G = 4\kappa$. Allowed Fermi transitions are denoted by arrows.

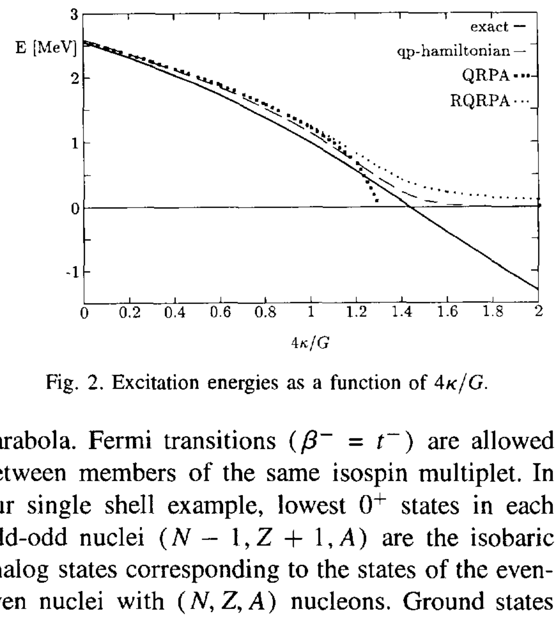

Fig. 2. Excitation energies as a function of $4\kappa/G$.

parabola. Fermi transitions ($\beta^- = t^-$) are allowed between members of the same isospin multiplet. In our single shell example, lowest 0^+ states in each odd-odd nuclei $(N-1, Z+1, A)$ are the isobaric analog states corresponding to the states of the even-even nuclei with (N, Z, A) nucleons. Ground states of the initial (N, Z, A) and final $(N-2, Z+2, A)$ even-even nuclei have different isospin and the Fermi-double-beta decay is forbidden in the isoscalar limit.

The results shown by a full line, in Fig. 2, represent the excitation energy E_{exc} of the lowest 0^+ state in the odd-odd intermediate nucleus $(T_z = 3)$ with respect to the parent even-even nucleus $(T_z = 4)$, as a function of $4\kappa/G$. It is clear that when $4\kappa/G \approx 1.4$ proton-neutron correlation dominates over proton-proton and neutron-neutron pairing correlations and the excitation energy goes to zero. This behavior is not a surprise and is similar to that found in the case of the pairing plus quadrupole Hamiltonian [27]. In this case, if the quadrupole-quadrupole interaction is strong enough, the system becomes permanently deformed. This was shown long time ago using the "pairing plus monopole" model [25,26] which is a solvable two-level model obeying the algebra of SO(5). In this letter we have applied the same ideas to the proton-neutron problem.

It must be stressed that, in realistic cases, odd-odd nuclei in the lower energy sector of the mass parabola cannot have negative excitation energies, as compared to the even-even nuclei with the same mass, because it will be in contradiction with the main evidence for pairing effects in medium and heavy nuclei. It would also suppress completely the double beta decay because the single beta decay would be allowed between even-even and odd-odd nuclei. For the cases where the hamiltonian (1) predicts negative excitation energies, i.e. the above mentioned overbinding of odd-odd nuclei, quadrupole-quadrupole interactions and permanent deformations have to be considered.

Now we turn to the QRPA, the RQRPA and their validity. After performing the Bogolyubov transformations, separately, for protons and neutrons, we have obtained the qp-hamiltonian

$$
\begin{aligned}
H = {} & (\epsilon_p - \lambda_p) N_p + (\epsilon_n - \lambda_n) N_n \\
& + \lambda_1 A^\dagger A + \lambda_2 (A^\dagger A^\dagger + AA) \\
& - \lambda_3 (A^\dagger B + B^\dagger A) - \lambda_4 (A^\dagger B^\dagger + BA) \\
& + \lambda_5 B^\dagger B + \lambda_6 (B^\dagger B^\dagger + BB)
\end{aligned} \tag{5}
$$

with $\epsilon_p = \epsilon_n = G\Omega/2$ being the quasiparticle energies and λ_p, λ_n the chemical potentials. Introducing the quasiparticle creation operators $\alpha_p^\dagger, \alpha_n^\dagger$ [15] and $\Omega = (2j+1)/2$, we have defined

$$
A^\dagger = \left[\alpha_p^\dagger \otimes \alpha_n^\dagger\right]^{J=0}_{M=0}, \quad B^\dagger = \left[\alpha_p^\dagger \otimes \alpha_{\tilde{n}}\right]^{J=0}_{M=0},
$$

$$
N_i = \sum_{m_i} \alpha^\dagger_{im_i} \alpha_{im_i} \quad i = \mathrm{p, n},
$$

$$
\lambda_1 = 4\Omega \left[\chi(u_p^2 v_n^2 + v_p^2 u_n^2) - \kappa(u_p^2 u_n^2 + v_p^2 v_n^2)\right] ,
$$

$$
\lambda_2 = 4\Omega(\chi + \kappa) u_p v_p u_n v_n ,
$$

$$
\lambda_3 = 4\Omega(\chi + \kappa) u_n v_n (u_p^2 - v_p^2) ,
$$

$$
\lambda_4 = 4\Omega(\chi + \kappa) u_p v_p (u_n^2 - v_n^2) ,
$$

$$
\lambda_5 = 4\Omega \left[\chi(u_p^2 u_n^2 + v_p^2 v_n^2) - \kappa(u_p^2 v_n^2 + v_p^2 u_n^2)\right] ,
$$

$$\lambda_6 = -\lambda_2. \tag{6}$$

The operators $A, A^\dagger$ together with their counterparts for identical particles and $B, B^\dagger, N_p, N_n$ are the generators of the SO(5) algebra [21].

The linearized version of hamiltonian (5) is obtained by keeping only the first line of Eq. (5), i.e. by neglecting the scattering terms proportional to B and $B^\dagger$. Its solutions were discussed in a previous paper [28].

Finding the eigenvalues and eigenvectors of hamiltonian (5) requires the same algebraic techniques involved in solving the original hamiltonian. But the complexity of the problem increases severely, due to the fact that neither the quasiparticle number nor the quasiparticle isospin projection (or equivalently the number of proton and neutron quasiparticles) are good quantum numbers. It implies that the dimension of the basis increases in two orders of magnitude. Technical details and a number of different examples are given elsewhere [24].

The dashed line in Fig. 2 shows the dependence of the excitation energy $E_{\rm qp}$ for hamiltonian (5). It reproduces the exact results reasonably up to $4\kappa/G \approx 1.4$. From there on it goes to zero instead of taking negative values.

The QRPA matrix is a 2×2 one, with sub-matrices $\mathcal{A}_{\rm QRPA} = 2\epsilon + \lambda_1$ and $\mathcal{B}_{\rm QRPA} = 2\lambda_2$. The eigen-energy is $E_{\rm QRPA} = [(2\epsilon + \lambda_1)^2 - 4\lambda_2^2]^{1/2}$. It is shown as a large-dot line in Fig. 2. It becomes an imaginary number if $2\lambda_2 > 2\epsilon + \lambda_1$. It means that for this limit the zero-boson component of ground state ceased to be dominant [28]. The collapse occurs very near the point where the exact excitation energies become negative. It is a very important point. The overestimation of the proton-neutron correlations in the QRPA mimics the more complicated physics found in the exact case. However it gives a clear signal about drastic changes in the correlations, in this region of the parameters governing the residual pn-interaction.

In the renormalized QRPA the structure of the ground state is included explicitly [11]. The RQRPA energy $E_{\rm RQRPA}$ is always real. Its value must be obtained by solving simultaneously a set of non-linear equations [13,28]. It is shown as a small-dot curve in Fig. 2. This figure strongly resemble Fig. 1 of Ref. [13] and Fig. 2 of Ref. [8], where the energy of the first excited state is plotted against the particle-

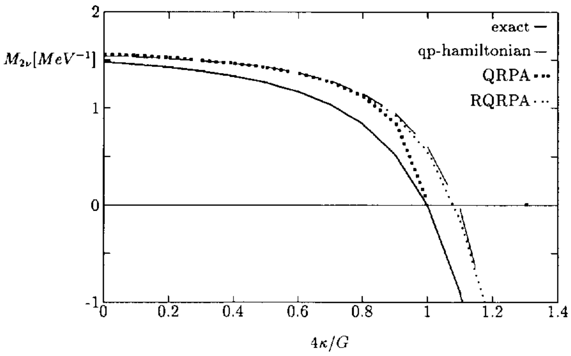

Fig. 3. $\beta\beta_{2\nu}$ transition amplitudes $M_{2\nu}$ vs. $4\kappa/G$.

particle strength parameter $g_{\rm pp}$. The curves for the QRPA and the RQRPA are quite similar to those shown here. The new feature discussed in this letter is that the exact excitation energies are closer to the QRPA energies rather than to the renormalized ones.

The amplitudes $M_{2\nu}$ are evaluated as

$$M_{2\nu} = \sum_\lambda \frac{\langle 0_f|\beta^-|0_\lambda\rangle\langle 0_\lambda|\beta^-|0_i\rangle}{E_\lambda - E_i + \Delta} \tag{7}$$

We have selected $\Delta = 0.5$ MeV. The values of $M_{2\nu}$, corresponding to the exact solution, are shown in Fig. 3 (full line) as a function of $4\kappa/G$. For the other approximations we have diagonalized the hamiltonian (5) for the initial ($N_p = 6, N_n = 14$, ground state $|0_i\rangle$) and final ($N_p = 8, N_n = 12$, ground state $|0_f\rangle$) ground-states of the participant even-even nuclei. The overlap between the two descriptions of the intermediate states belonging to the double-odd nucleus [5,6] was included in the above formula (Eq. (7)). The sum runs over the allowed intermediate states corresponding to this model space [24].

In all the cases the curve corresponding to $M_{2\nu}$ is very similar to that found in realistic calculations [3,5,6,13], including its cancellation near the collapse of the QRPA description. The RQRPA extends this curve far beyond the value of κ at which the QRPA collapses. But the validity of this result can be questioned because the RQRPA missed the vanishing of the excitation energy, as we have discussed before.

To conclude, we have presented an exactly solvable model which resembles the main features of the realistic models used to describe the structure of the nuclei involved in double beta decay processes. We have compared the exact values for the excitation energies and double beta decay transition amplitudes with those obtained with the approximate qp-hamiltonian, the QRPA and renormalized RQRPA ones. We have shown that the collapse of the QRPA correlates with the presence of a state with zero excitation energy. This state is present in the exact solution of the model. It is found that the RQRPA solutions do not show the presence of such state. As a direct consequence of this fact this approximation gives finite but spurious results for the transition matrix elements, which unfortunately are not supported by the exact results. The presence of spurious states, introduced by the renormalization procedure, could be responsible for such an odd behavior. The effect of these spurious contributions upon the transition amplitudes and the mixing of orders, in the sense of the order classification in powers of $1/\Omega$, found in the RQRPA wave functions are discussed in detail somewhere else [24].

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References

[1] J.D. Vergados, Phys. Rep. 111 (1986) 1;
A. Faessler, Prog. Part. Nucl. Phys. 21 (1988) 183;
T. Tomoda, Rep. Prog. Phys. 54 (1991) 53.
[2] M.K. Moe, Int. J. Mod. Phys. E 2 (1993) 507;
S.R. Elliot, M.K. Moe, M.A. Nelson and M.A. Vient, Nucl. Phys. B (Proc. Suppl.) 31 (1993) 68.
[3] P. Vogel and M.R. Zirnbauer, Phys. Rev. Lett. 57 (1986) 3148.
[4] J. Engel, P. Vogel and M.R. Zirnbauer, Phys. Rev. C 37 (1988) 731.
[5] O. Civitarese, A. Faessler and T. Tomoda, Phys. Lett. B 194 (1987) 11.
[6] K. Muto, E. Bender and H.V. Klapdor, Z. Phys. A 334 (1989).
[7] O. Civitarese, A. Faessler, J. Suhonen and X.R. Wu, Nucl. Phys. A 524 (1991) 404.
[8] O. Civitarese, A. Faessler, J. Suhonen and X.R. Wu, J. Phys. G: Nucl. Part. Phys. 17 (1991) 943.
[9] A. Griffiths and P. Vogel, Phys. Rev. C 46 (1992) 181.
[10] K. Hara, Progr. Theor. Phys. 32 (1964) 88;
K. Ikeda, T. Udagawa and Y. Yamaura, Prog. Theor. Phys. 33 (1965) 22.
[11] D.J. Rowe, Phys. Rev. 175 (1968) 1283; Rev. Mod. Phys. 40 (1968) 153;
J.C. Parick and D.J. Rowe, Phys. Rev. 175 (1968) 1293;
D.J. Rowe, Nucl. Phys. A 107 (1968) 99.
[12] F. Catara, N. Dinh Dang and M. Sambataro, Nucl. Phys. A 579 (1994) 1.
[13] J. Toivanen and J. Suhonen, Phys. Rev. Lett. 75 (1995) 410.
[14] J. Schwieger, F. Simkovic and Amand Faessler, Nucl. Phys. A 600 (1996) 179.
[15] D.J. Rowe, Nuclear Collective Motion (Methuen and Co. Ltd., London, 1970).
[16] V.A. Kuz'min and V.G. Soloviev, Nucl. Phys. A 486 (1988) 118.
[17] K. Muto, E. Bender, T. Oda and H.V. Klapdor-Kleingrothaus, Z. Phys. A 341 (1992) 407.
[18] O. Civitarese and J. Suhonen, J. Phys. G 20 (1994) 1441.
[19] O. Civitarese and J. Suhonen, Nucl. Phys. A 578 (1994) 62.
[20] O. Civitarese, J. Suhonen and A. Faessler, Nucl. Phys. A 591 (1995) 195.
[21] J.C. Parikh, Nucl. Phys. 63 (1965) 214.
[22] K.T. Hecht, Nucl. Phys. 63 (1965) 177.
[23] A. Klein and E.R. Marshalek, Rev. Mod. Phys. 63 (1991) 375.
[24] J.G. Hirsch, P.O. Hess and O. Civitarese, to be published.
[25] D. Agassi, Nucl. Phys. A 116 (1968) 49.
[26] D. Schütte and K. Bleuler, Nucl. Phys. A 119 (1968) 221.
[27] L.S. Kisslinger and R.A. Sorensen, Rev. Mod. Phys. 35 (1963) 853.
[28] J.G. Hirsch, P.O. Hess and O. Civitarese, Phys. Rev. C 54 (1996) 1976.