ELSEVIER

Nuclear Physics A 607 (1996) 152–162

NUCLEAR PHYSICS A

Contributions of unique first-forbidden transitions to two-neutrino double-β-decay half-lives

O. Civitarese [a,1], J. Suhonen [b]

[a] *Department of Physics, University of La Plata, C.C.67, (1900) La Plata, Argentina*

[b] *Department of Physics, University of Jyväskylä, P.O.B. 35, SF-40351, Jyväskylä, Finland*

Received 25 March 1996; revised 14 May 1996

Abstract

The two-neutrino double-β-decay transition of ^{76}Ge to the ground state of ^{76}Se is calculated using a realistic proton–neutron force and including unique first-forbidden single-β-decay transitions to virtual 2^- intermediate states. The corresponding nuclear matrix elements are computed by using the proton–neutron quasiparticle random-phase approximation (pn-QRPA). From the results of the present study it is concluded that the inclusion of virtual unique first-forbidden transitions, as possible "new" steps contributing to the half-life of the two-neutrino double-β-decay mode, can be disregarded.

1. Introduction

The theoretical description of nuclear double-β-decay processes is one of the open questions in the field of nuclear-structure theories and it has relevance for the search of new physics beyond the standard model [1]. A compilation of experimental and theoretical results can be found in [2]. For a review of some of the latest developments, both in theory and in experiments, the reader is referred to [3].

The description of nuclear matrix elements for the two-neutrino ($2\nu\beta\beta$) double-β-decay mode [4] has received much attention during the last decade. A compilation of different theoretical methods used to calculate these nuclear matrix elements can be found in [5].

[1] Fellow of the CONICET, Argentina.

PII S0375-9474(96)00225-4

Since most of the methods and results have been discussed extensively we would like to concentrate, in this section, on the approximations that are used in dealing with the calculation of nuclear matrix elements for the $2\nu\beta\beta$ decay mode, namely [6]:

(i) closure is avoided and a complete sum over virtual excitations of the intermediate nucleus, with mass, neutron and proton numbers A, $N-1$ and $Z+1$, should be used, instead;

(ii) the set of virtual intermediate excitations can be generated by the action of the Fermi (τ) and Gamow–Teller ($\sigma\tau$) operators on the initial and final ground states. The corresponding virtual transitions belong to the set of allowed single-β-decay transitions;

(iii) leptons are assumed to be in s-wave states and the corresponding phase-space factors are calculated accordingly, as dictated by the energy and momentum conservation rules as well as by the angular momentum and parity conservation rules.

The approximations (i)–(iii) are independent of the method selected to describe the initial, final and intermediate nuclear wave functions and they simply reflect the separable (lepton–hadron) structure of the weak-interaction Hamiltonian and the very schematic structure of the vertex functions calculated perturbatively at second order in the weak-interaction Hamiltonian [6,7].

The use of intermediate virtual excitations, other than the allowed ones mentioned in (ii), has been advocated by some authors [8,9] in spite of the fact [7] that the leptonic wave functions, for a single vertex, would include terms which are proportional to the product of the electron or neutrino momentum and the nuclear radius ($p_{e(\nu)}R$). In consequence, their contributions to the second-order process, which implies the product of two of such vertex functions, would be severely suppressed [10].

In order to verify that this suppression is indeed present in the nuclear double-β decay, and that the assumptions of [8,9] cannot be applied for this case, we have calculated the matrix elements of virtual unique first-forbidden (UFF) β-decay transitions for the decay sequence ^{76}Ge(0^+,ground state) $\rightarrow$ ^{76}As(2^-) $\rightarrow$ ^{76}Se(0^+,ground state). The set of $J^\pi = 2^-$ states in ^{76}As has been described in the QRPA approximation and in the proton–neutron two-quasiparticle basis [11,12]. The calculations have been performed by using a realistic interaction constructed from the renormalized G-matrix obtained with the Bonn one-boson-exchange potential (OBEP) [13] and also by using a schematic interaction of separable type. The details of the formalism are presented in Section 2.

The analysis of the results, which are presented and discussed in Section 3, was performed by extracting the information about virtual single-β-decay transitions $2^- \rightarrow 0^+$ leading to the ground state of ^{76}Se. The ft values for these transitions and the strength distribution associated with the β^- and β^+ branches of the feeding of the virtual intermediate states are presented in Section 3, where also the energy distribution of the UFF transitions is analyzed. The matrix elements of the $2\nu\beta\beta$ decay mode are calculated at dominant order in the UFF nuclear form factors and the order of magnitude of a typical UFF contribution to the total decay rate is presented and discussed in Section 3. Conclusions are drawn in Section 4.

2. Formalism

The method currently adopted for the calculation of $2\nu\beta\beta$ observables, within the pn-QRPA approach, consists of the following steps [11–13]:

(a) Definition of virtual excitations of the intermediate nucleus with $(N-1, Z+1)$ starting from the proton–neutron (pn) excitations of the initial (N, Z) and final $(N-2, Z+2)$ nuclei. These excited states are needed to construct virtual, otherwise energetically forbidden, single-β-decay transitions which connect the initial and final states of the $2\nu\beta\beta$ transitions.

(b) The $2\nu\beta\beta$ matrix elements, calculated in this way, contain the leading-order QRPA contributions, characterized by the forward- and backward-going pn-amplitudes.

(c) Due to the separable character of the weak Hamiltonian, the leptonic contributions always appear with the same multipole order as the hadronic contributions.

In the following we shall discuss briefly the formalism corresponding to the above steps.

2.1. The weak-interaction Hamiltonian

To begin with, let us define the separable (leptonic–hadronic) weak Hamiltonian written in terms of the multipole-order expansion given in textbooks [7]. The terms that contribute to the UFF transitions with $\Delta J = 2$ read

$$H_{\mathrm{weak}} = g\sqrt{\frac{(2J_i+1)}{2\pi^3 5(2J_f+1)}} \sum_{\kappa_e,\kappa_\nu,m_e,m_\nu,M} \langle J_i M_i 2M | J_f M_f \rangle \times \langle j_e - m_e j_\nu - m_\nu | 2M \rangle (-)^{j_e+m_e} a^*_{k_e m_e}(\hat{p}_e) b_{k_\nu m_\nu}(\hat{p}_\nu) G_2(\kappa_e, \kappa_\nu) \,. \tag{1}$$

The quantity G_2 is the product of the leptonic and nuclear form factors and it is given by

$$G_2(\kappa_e, \kappa_\nu) = \alpha_{\kappa_e} \sqrt{5} M_2(k_e, k_\nu) \,, \tag{2}$$

where α_{κ_e} are the electron Coulomb amplitudes [7]. The explicit expression for the quantity $M_2(k_e, k_\nu)$ is written as [7]

$$M_2(k_e, k_\nu) = -2\sqrt{\tfrac{5}{3}} (p_e R)^{k_e - 1} (p_\nu R)^{k_\nu - 1} F^{(0)}_{211} \,. \tag{3}$$

In these expressions the nuclear (hadronic) and leptonic contributions are explicitly indicated by the reduced matrix elements $F^{(0)}_{211}$ and by the electron (neutrino) momentum p_e (p_ν), R being the nuclear radius. The coefficients $a_{k_e m_e}$ and $b_{k_\nu m_\nu}$ are vector spherical harmonics depending on the directions of $\boldsymbol{p}_e$ and $\boldsymbol{p}_\nu$, respectively. The quantities k_e and k_ν are the electron and neutrino quantum numbers in the standard notation [14], and $k_{e(\nu)} = |\kappa_{e(\nu)}|$. The expression for $M_2(k_e, k_\nu)$ has been written at the dominant order in the UFF form factors, as defined in [7]. Higher-order corrections are proportional to higher powers of the electron or neutrino momentum and they can be neglected in the case of $2\nu\beta\beta$ modes.

After some trivial replacements, the form factors $G_L(\kappa_e, \kappa_\nu)$ can be written as

$$G_L(\kappa_e, \kappa_\nu) = -\frac{\sqrt{5}}{3} F_{211}^{(0)} \alpha_{\kappa_e} p_\nu R\,, \tag{4}$$

for $\kappa_e = \pm 1, \kappa_\nu = \pm 2$, and

$$G_L(\kappa_e, \kappa_\nu) = -\frac{\sqrt{5}}{3} F_{211}^{(0)} \alpha_{\kappa_e} p_e R\,, \tag{5}$$

for $\kappa_e = \pm 2, \kappa_\nu = \pm 1$, respectively. The UFF selection rules restrict the terms appearing in Eq. (1) to those with $\kappa_e + \kappa_\nu = \Delta J + 1$. The nuclear form factors $F_{211}^{(0)}$ are basically the reduced matrix elements of the UFF multipole-moment operator $M(j_A, \kappa = 1, \lambda = 2)$ [15]. The definition of this axial moment is given below.

Inserting these expressions in the $2\nu\beta\beta$ decay-rate formula [6]

$$d\lambda = 2\pi\delta \left(E_0 - \sum_f E_f \right) \left| \sum_m \frac{\langle f|H_{\text{weak}}|m\rangle\langle m|H_{\text{weak}}|i\rangle}{(E_i - E_m - p_\nu - E_e)} \right|^2 , \tag{6}$$

and assuming that the lepton energies are replaced by their average values, one gets the expression for the decay rate of the $2\nu\beta\beta$ mode mediated by UFF virtual transitions. Each term of the sum in Eq. (6) will be proportional to $(p_1 R)(p_2 R)$, where $p_1(p_2)$ are the electron or neutrino momenta, for the first (second) $2\nu\beta\beta$ vertex, and thus the total decay rate will be proportional to $(pR)^4$. Unless the nuclear matrix elements of the UFF are much larger than the allowed Gamow–Teller (GT) ones, the dependence on higher powers of (pR) (typically a quantity of the order of 10^{-2} for the $2\nu\beta\beta$ decay) will make their contribution to the $2\nu\beta\beta$ half-life inconsistent with data by factors of the order of 10^6 or even larger.

However, since for the presently discussed $2\nu\beta\beta$ transition the ground state of the double-odd nucleus ^{76}As is a 2^- state, one might be tempted to compute the yield for virtual UFF decays going to this and other 2^- states, in spite of the dominance of the allowed virtual GT transitions.

2.2. *The QRPA treatment of virtual $J^\pi = 2^-$ pn-excitations*

The pn-QRPA method [11–13] is applied to expand the total Hamiltonian,

$$H = H_0 + H_{\text{int}}\,, \tag{7}$$

in the correlated pn-phonon basis

$$\Gamma^\dagger(2_k^- \mu) = \sum_{pn} [X_{pn}(2^- k) A^\dagger(pn, 2^- \mu) - Y_{pn}(2^- k) A(pn, 2^- \tilde{\mu})]\,. \tag{8}$$

Here H_0 is the unperturbed proton and neutron single-quasiparticle Hamiltonian

$$H_0 = \sum_\nu E_\nu \alpha_\nu^\dagger \alpha_\nu\,, \tag{9}$$

and H_{int} is the interaction term constructed from the realistic G-matrix [13].

The operators A that appear in these definitions are proton–neutron quasiparticle-pair operators coupled to angular momentum and parity $J^\pi = 2^-$,

$$A^\dagger(pn, 2^-\mu) = [\alpha_p^\dagger \alpha_n^\dagger]_{2^-\mu},$$
$$A(pn, 2^-\bar{\mu}) = (-1)^{2-\mu}(A^\dagger(pn, 2^-, -\mu))^\dagger. \tag{10}$$

The amplitudes $X_{pn}(2^-k)$ and $Y_{pn}(2^-k)$ and the corresponding eigenvalues ω_k can then be determined by solving the pn-QRPA equation of motion [13]

$$[H, \Gamma^\dagger(2_k^-\mu)] = \omega_k \Gamma^\dagger(2_k^-\mu) \tag{11}$$

in its matrix form.

2.3. The β-decay operators

The operators $\hat{\beta}^\pm$, for UFF transitions, are defined as the particle–hole components of the multipole moments $M(j_A, \kappa = 1, \lambda = 2)$ [15]. Indicating the total angular momentum, the orbital and spin components of these multipole moments, obtained from the multipole expansion of the weak axial-vector current, by $M(211)$ in the notation of [15], one has

$$M(211;\mu)^{(\pm)} = -g_A \sum_k ir(k)[Y_1(r(k))\sigma(k)]_{2\mu}\tau(k)^{(\pm)}. \tag{12}$$

Hereafter, for brevity, we shall denote these multipole operators as $\hat{\beta}^\pm_{2\mu}$. For open-shell systems they are written in terms of quasi-proton and quasi-neutron pair-creation and pair-annihilation operators

$$\hat{\beta}^-_{2\mu} = \sum_{pn} M(pn)[u_p v_n A^\dagger(pn, 2^-\mu) + v_p u_n A(pn, 2^-\bar{\mu})], \tag{13}$$

with

$$M(pn) = \frac{1}{\sqrt{5}}(p||\hat{\beta}^-_2||n) \tag{14}$$

being the reduced matrix element of the multipole operator of Eq. (12). The action of the isospin operator τ^- upon quasiparticles is already indicated by the proton (p) and neutron (n) one-quasiparticle labels and, as usual, BCS factors are denoted by u and v.

The reduced matrix elements,

$$\beta^-_m = (2^-_m||\hat{\beta}^-_2||0^+_i), \tag{15}$$

can be expressed in terms of the pn-QRPA amplitudes, as in the case of allowed GT transitions [13], by writing the pn-two-quasiparticle operators $A^\dagger$ and A of Eq. (10) in terms of the pn-QRPA phonon operators $\Gamma^\dagger$ and Γ of Eq. (8). The result is the same as for the GT transitions, except for the replacement of the reduced matrix elements of the spin operator by the reduced matrix elements $M(pn)$ of Eq. (14).

In the same way, virtual β^+ transitions leading to the intermediate one-phonon states of ^{76}As can be written in terms of the UFF amplitudes as

$$\beta^+_m(0^+) = \frac{1}{\sqrt{5}} \sum_{m'} (0^+_f || \hat{\beta}^- || 2^-_{m'}) \langle 2^-_{m'} | 2^-_m \rangle . \tag{16}$$

The quantity $\langle 2^-_{m'} | 2^-_m \rangle$ is the overlap of the two different pn-QRPA representations for the set of 2^- states (one based on the initial, the other on the final ground state) [12]. The above expressions have been computed to leading order in the pn-QRPA amplitudes.

Up to this point the formalism that is presented is completely analogous to the one of Refs. [16–18], except for the use of a realistic interaction in addition to a schematic one.

Finally, let us introduce the expression that is used to calculate the ft-value for the UFF transitions; following Ref. [15] one has

$$ft_{1/2}(2^- \rightarrow 0^+) = \frac{D g_V^2}{4\pi M(2^- \rightarrow 0^+)} , \tag{17}$$

where

$$M(2^- \rightarrow 0^+) = \frac{f_C}{5} |\langle 0^+ || \hat{\beta}^-_2 || 2^- \rangle|^2 . \tag{18}$$

In the above expression, $D = 6250$ s, g_V is the vector coupling constant ($g_A/g_V = -1.23$), and the factor f_C represents the conversion of the reduced matrix elements of Eq. (14) to Cartesian coordinates (as mentioned before this is the conversion which we have assumed in introducing the form factors $F^{(0)}_{211}$ of Eq. (3)). The matrix elements to be used in the calculation of the ft-value of Eq. (17) are given in natural units [7].

3. Results and discussion

3.1. Description of $J^\pi = 2^-$ states in ^{76}As

For the present calculation of the set of 2^- states in ^{76}As we have adopted the single-particle basis, pairing parameters and the effective interaction used for the case of the allowed 1^+ virtual excitations described in [12]. The single-particle basis includes 10 proton and 10 neutron single-particle states, the core being at $N = Z = 20$. The 24 active neutrons and 12 active protons are described as independent quasiparticle states with quasiparticle energies and BCS occupation numbers calculated with the renormalized monopole component of the G-matrix, with the corresponding neutron and proton couplings [12] $g_{\text{pair}} = 1.18(1.04)$ and $g_{\text{pair}} = 1.19(1.08)$ for the initial and final nucleus, respectively.

In the resulting pn-quasiparticle basis, consisting of 34 configurations, we have diagonalized the QRPA matrix with the particle–hole and particle–particle coupling constants of the 2^- channel being fixed at $g_{ph} = g_{pp} = 1$, respectively. The meaning of these renormalization factors is given in [13].

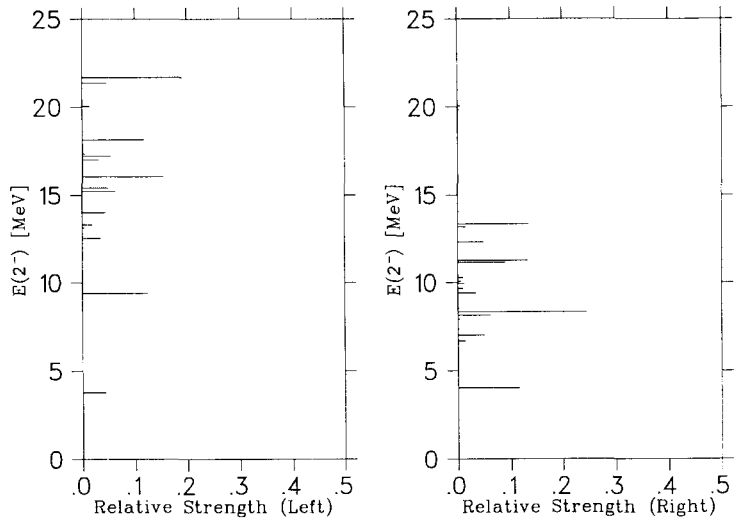

Fig. 1. Energies and relative strengths, for the virtual UFF single-β-decay transitions to $J^{\pi} = 2^{-}$ states in ^{76}As. The set of states indicated by "Right" and "Left" corresponds to pn-QRPA states based on the ground state of ^{76}Ge and ^{76}Se, respectively.

As a main difference from the calculation of the spectra of the intermediate 1^{+} states, one immediately notices that changes of the particle–particle coupling constant g_{pp} do not influence the structure of the 2^{-} states and that in consequence one should not expect to find any dependence of the calculated $2\nu\beta\beta$ matrix elements on g_{pp}, like the cancellation found for Gamow–Teller excitations using the same realistic interaction [19].

The two sets of solutions for the pn-QRPA 2^{-} states, based on the initial (^{76}Ge) and final (^{76}Se) ground states, are shown in Fig. 1. In this figure the energy and the contribution to the total strength for UFF single-β-decay transitions are shown for each state belonging to these two sets of solutions. A strong fragmentation of the intensity is observed, as well as the practically vanishing yield for g.s. $\rightarrow 2^{-}$ transitions below 9 MeV for the first set of states (based on the initial ground state) and above 13 MeV for the second set of states (based on the final ground state). It means that only a few states can eventually contribute to virtual two-step transitions from ^{76}Ge to ^{76}Se. This, again, differs from the observed behavior of the allowed Gamow–Teller transitions where a large number of possible two-step contributions has been obtained in the calculations [12].

In order to discuss the physical validity of the present calculation we have computed the ft-values for single-β-decay UFF transitions. In particular, we have calculated the ft-value for the $2^{-} \rightarrow$ g.s. transition from the ground state of ^{76}As to the ground state of ^{76}Se, since this value can be compared with the available data [20].

The results which are shown in Table 1 reflect the sensitivity of the theoretical ft-values upon the correlations induced by the proton–neutron force. In this respect, and

Table 1
log ft value for the UFF transition in $A = 76$. The experimental value is taken from Ref. [20]. The theoretical values have been obtained with the present model, case (a), and by using a separable force [18] with different values of the coupling constant $\chi(2^-)$, case (b). Each set of results shown in the fourth column corresponds to $\chi(2^-) = 0.10$, 0.15 and 0.30 MeV·fm^{-2}, from top to bottom, respectively. The nuclear matrix elements $M(211)$ are given in natural units. The experimental [21] and theoretical values of the energy of the UFF resonance, E(UFF), are shown in the table. The energies are given in units of MeV

^{76}As $(2_1^-) \rightarrow {}^{76}$Se $(0^+_{g.s.})$	Experiment	Case (a)	Case (b)
log ft $(2_1^- \rightarrow 0^+_{g.s.})$	8.66	7.37	7.67
			7.82
			8.13
$\lvert M(211)\rvert \times 10^3$	1.64	7.23	5.08
			4.29
			3.04
E(UFF)	21.5	20.98	21.41
			22.34
			26.36

concerning the results obtained by using the separable force of [18], by varying the corresponding coupling constant within physically acceptable limits (0.10 MeV·fm^{-2} $\leqslant \chi(2^-) \leqslant$ 0.30 MeV·fm^{-2}) one can get a reasonable agreement with data. For the case of the realistic interaction the calculated ft-value disagrees with data by about one order of magnitude; however, it should be stressed that rather than performing a fit to the data we wanted to show that the included correlations were the adequate ones. The comparison of the results obtained with both the realistic interaction of [12] and the schematic one of [18], given in Table 1, points to the correct structure of the predicted set of low-lying 2^- excitations.

The quality of the high-lying virtual excitations is hard to assess, since little is known about the properties of high-lying 2^- states [21]. The experimental value of the energy of the UFF giant resonance [21] E(UFF), given in Table 1, has been tentatively assigned to a 2^- state. However, since only the $\Delta l = 1$ character of the state has been determined experimentally it can also be a 1^- excitation [21]. Since the experimental value of E(UFF) shown in Table 1 corresponds to heavier nuclei, we take it only as an indication about the trend of the calculated values.

The final matrix elements for the $2\nu\beta\beta$ transition, mediated by GT and UFF virtual transitions, are shown in Tables 2 and 3, respectively. For completeness, and in order to show the similarities between realistic and schematic [22] results concerning the properties of both the GT and UFF excitations, we have included in Table 2 the results obtained by using a schematic interaction with the relevant parameters given in the captions of the tables. The log ft values and the corresponding nuclear matrix elements as well as the excitation energy of the UFF resonance are similar in both the realistic- and schematic-model calculations and the same trend is observed for the case of the allowed GT transitions. The contributions to the $2\nu\beta\beta$ matrix element due to the complete set of virtual excitations of ^{76}As and the one produced by including only the first-excited 1^+

Table 2
$2\nu\beta\beta$ nuclear matrix elements mediated by allowed virtual transitions. The theoretical matrix elements, obtained with the realistic interaction, are given for two values of g_{pp}. The results shown in the fourth and fifth columns have been obtained by using a separable force [22] with $\chi(1^+) = 0.15$ MeV and $\kappa = 0.0$ and 0.10 MeV, respectively. The value of the experimental matrix element has been extracted from the data given in [23]. The theoretical values for $2\nu\beta\beta$ matrix elements, including only the first-excited 1^+ state of ^{76}As, are shown at the bottom of the table

Experiment	$g_{pp} = 0$	$g_{pp} = 1.0$	$\kappa = 0$	$\kappa = 0.10$
0.09	0.391	0.028	0.423	0.063
	0.053	0.010	0.056	0.061

Table 3
$2\nu\beta\beta$ nuclear matrix elements mediated by unique first-forbidden virtual transitions. The theoretical matrix elements obtained with the realistic interaction are given in the first and second columns, respectively. The results shown in the third column correspond to the separable force of [18] with the values of $\chi(2^-)$ given in Table 1. The theoretical values for $2\nu\beta\beta$ matrix elements, including only the first-excited 2^- state of ^{76}As, are shown below the values obtained by including the complete set of 2^- virtual excitations

$g_{pp} = 0$	$g_{pp} = 1.0$	Schematic force
0.402	0.409	0.286
		0.205
		0.100
0.295	0.385	0.186
		0.127
		0.056

(or 2^-) state of ^{76}As are shown in Tables 2 and 3. The dominance of a single two-step transition is clear for the case of the UFF transitions while the destructive interference between several virtual excitations is characteristic for the case of GT transitions.

In order to discuss the influence of this single two-step dominance of the UFF mode upon the $2\nu\beta\beta$ matrix element we have computed it assuming that all the virtual β^- and β^+ strengths, as resulting from the sum rule associated with both virtual branches of the decay path, go through a single 2^- state lying at the energy of the experimentally observed first-excited 2^- state. The corresponding strengths, for the β^- and β^+ virtual transitions in the realistic calculation, are 123 fm^2 and 48 fm^2, respectively. These values are essentially independent of g_{pp}. The resulting $2\nu\beta\beta$ matrix element (after scaling) is of the order of 4.17. This value is ten times larger than the results shown in Table 3 but still several orders of magnitude smaller than the value which would be needed to compensate for the severe retardation introduced by the factor 10^6 (resulting from the fourth power of the quantity (pR) included in the shape factor of the transition). It is now clearly seen that, even if the unrealistically large strength predicted by the single two-step transition involving only one 2^- state is used to compute the $2\nu\beta\beta$ half-life, the contribution to the $2\nu\beta\beta$ half-life by UFF virtual transitions is orders of magnitude smaller than the contribution coming from allowed GT virtual transitions. At the same time this would mean strong disagreement with the measured $2\nu\beta\beta$ half-life [2].

Comparison of the final GT and UFF matrix elements, without taking into account the leptonic phase-space factor, might tempt one to wrong conclusions about the importance of UFF virtual transitions in dealing with $2\nu\beta\beta$ decays. The fact that the allowed GT matrix element is nearly suppressed for realistic values of g_{pp}, as shown in Table 2, does not guarantee that contributions coming from first-forbidden transitions of Table 3 will be significant. What is important is the way in which these matrix elements are entering the $2\nu\beta\beta$ half-life expression. Actually, what decides the order of magnitude of the contribution is the dependence of the leptonic vertex functions upon powers of (pR). The ratio between the phase-space factors, for allowed and UFF $2\nu\beta\beta$ transitions, is of the order of 10^6. To compete with the double GT contribution the matrix elements of UFF virtual transitions should be at least one thousand times larger than the typical UFF matrix elements extracted from experiments. We all know that this can not be the case and the results of the present calculations do indeed show it.

The theoretical double GT matrix elements can approximately be cancelled by increasing the strength of the particle–particle coupling constant, as shown in [12], but from the experimental data [23] one can see that it should be small but finite. Even by assuming that the $2\nu\beta\beta$ transition rate is due to other transitions, i.e. if the allowed double GT matrix element is completely suppressed, one does not obtain the correct order of magnitude by extending to UFF transitions, due to the conflict mentioned in the previous paragraph.

Finally, we would like to comment on the effect of center-of-mass corrections (or, in other words, center-of-mass spuriosities) which should be included in dealing with radial-dependent operators. The corrections will, of course, be proportional to the inverse power of the nuclear mass (neglecting neutron–proton asymmetries). It means that, for the present case, the calculated matrix elements (of single-β-decay operators) will be slightly reduced, as compared to the uncorrected values [15]. However, it leaves the above-presented conclusions unchanged.

4. Conclusions

In this work we have analyzed the influence of virtual UFF transitions upon the nuclear matrix element mediating the $2\nu\beta\beta$ transition from the ground state of ^{76}Ge to the ground state of ^{76}Se. The results of the pn-QRPA calculations, which are able to reproduce experimental data on real UFF transitions, show that virtual UFF transitions can not contribute to the $2\nu\beta\beta$ process because their matrix elements are too small to compensate for the strong suppression of the decay rate by the corresponding leptonic phase-space factors. Although these features have previously been advanced, based essentially on qualitative arguments, we have shown that they naturally emerge from a quantitative analysis, like the one presented here.

Acknowledgements

This work has been supported partly by the J.S. Guggenheim Memorial Foundation, the Jyväskylä Accelerator Laboratory and the Academy of Finland (contract no. 2856). In addition, O.C. thanks with pleasure the hospitality extended to him at the Department of Physics of the University of Jyväskylä.

References

[1] A. Faessler, Prog. Part. Nucl. Phys. 21 (1988) 183.
[2] V.I. Tretyak and Yu.G. Zdesenko, Atomic and Nucl. Data Tables 61 (1995) 43.
[3] M.K. Moe and P. Vogel, Ann. Rev. Nucl. Part. Sci. 44 (1994) 237.
[4] M. Doi, T. Kotani and E. Takasugi, Prog. Theor. Phys. Suppl. 83 (1985) 1.
[5] Proc. Int. Workshop on Double Beta Decay and Related Topics, Trento, Italy, 1995, ed. H.V. Klapdor-Kleingrothaus and S. Stoica (World Scientific, Singapore).
[6] F. Boehm and P. Vogel, Physics of Massive Neutrinos (Cambridge University Press, Cambridge, 1987).
[7] H.F. Schopper, Weak Interactions and Nuclear Beta Decay (North-Holland, Amsterdam, 1966).
[8] F. Simkovic, private communication.
[9] C. Barbero, F. Krmpotic and A. Mariano, Phys. Lett. B 345 (1995) 192.
[10] W.C. Haxton and G.J. Stephenson Jr., Prog. Part. Nucl. Phys. 12 (1984) 409.
[11] J. Suhonen, Nucl. Phys. A 563 (1993) 205.
[12] J. Suhonen and O. Civitarese, Phys. Lett. B 308 (1993) 212; Nucl. Phys. A 575 (1994) 251.
[13] J. Suhonen, T. Taigel and A. Faessler, Nucl. Phys. A 486 (1988) 91.
[14] R.G. Newton, Scattering Theory of Waves and Particles (McGraw-Hill, New York, 1966).
[15] A. Bohr and B.R. Mottelson, Nuclear Structure, Vol. I (Benjamin, New York, 1969).
[16] J.A. Halbleib and R.A. Sorensen, Nucl. Phys. A 98 (1967) 542.
[17] O. Civitarese, F. Krmpotic and O.A. Rosso, Nucl. Phys. A 453 (1986) 45.
[18] O. Civitarese and O.A. Rosso, Z. Phys. A 327 (1987) 51.
[19] O. Civitarese, A. Faessler and T. Tomoda, Phys. Lett. B 194 (1987) 11.
[20] B. Singh and D.A. Viggars, Nucl. Data Sheets 42 (1984) 233.
[21] D.J. Horen et al., Phys. Lett. B 95 (1980) 27; Phys. Lett. B 99 (1981) 383.
[22] V.A. Kuz'min and V.G. Soloviev, Nucl. Phys. A 486 (1988) 118; J. Phys. G 10 (1984) 1507.
[23] F.T. Avignone III et al., Phys. Lett B 256 (1991) 559;
A. Balysh et al., Phys. Lett. B 322 (1994) 176.