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The nuclear level density parameter and nuclear structure effects at finite temperature

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Abstract. The temperature dependence of the nuclear level density parameter is extracted from data and compared with results obtained by using the statistical model of nuclear reactions. The analysis of the results suggests that the temperature dependence of the level density parameter can be interpreted in terms of the temperature dependence of the nucleon separation energy. It is found that the observed behaviour of the level density parameter, as a function of the temperature, is consistent with the known temperature dependence of the fission barrier.

1. Introduction

The study of the temperature dependence of the level density parameter [1] has been the subject of several publications [2–7]. Experimental information about it can be extracted from data on hot compound nucleus formation and decay [8–21] . From the theoretical side it is related to the description of hot nuclear systems [22, 42]. A very detailed account of the problem can be found in the review article of [43].

The value of the level density parameter can be determined, under statistical assumptions, by the relation between excitation energy and temperature [1]. For a system with A nucleons it varies from $a = A/15\ \mathrm{MeV}^{-1}$ and $a = A/10\ \mathrm{MeV}^{-1}$ for a Fermi gas and for fermions in a harmonic oscillator potential, respectively, to $a = A/8\ \mathrm{MeV}^{-1}$, which is the empirical value extracted from the average spacing observed in slow neutron resonances [1].

Nuclear structure effects upon the value of the level density parameter have been considered by the inclusion of shell corrections [22–31], pairing correlations [32, 33] and collective excitations [34]. Shell corrections to the nuclear free energy are temperature-dependent and they collapse at temperatures, T, of the order of 2 MeV [31]. The collapse of pairing correlations occurs at lower temperatures, $T \approx 0.5$ MeV [32, 33], and collective excitations collapse at $T \approx 1$ MeV [34]. It has been shown that the temperature dependence of nuclear correlations can affect the value of the level density parameter at low temperatures [7] . The results of [7] show that these nuclear correlation effects disappear at $T \approx 4$ MeV. Mean field effects dominate, at higher values of T, unless a collapse of the mean field is produced. The Fermi gas model predicts a value for the level density parameter which is smaller than the value corresponding to a correlated system. The decrease of the level density parameter, with increasing values of T, has been interpreted as a signature for the collapse of nuclear residual interactions [35].

Different parametrizations of the nuclear free-energy have been discussed [6, 36–41]. It has been argued that the observed temperature dependence of the nuclear level density

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parameter cannot be explained by considering only mean field effects [6, 7]. This statement is certainly valid for the low-temperature regime, where the effect of residual interactions is important. However, at high temperatures, a description of the level density parameter based on a single mean field does not account for the observed trend of the data [6].

Experimental evidence on the temperature dependence of the level density parameter has been reported recently [14–21]. The decrease of the level density parameter, for high temperatures, has been extracted from the measurement of the evaporation of light particles emitted from hot nuclei [14]. The intercalibration of nuclear thermometers of [15] has allowed for an empirical determination of the relation between emission and apparent temperatures. The influence of the level density upon precise lifetimes has been measured and interpreted in [16]. The analysis of nuclear de-excitation modes, in terms of an effective Fermi gas formula for the level density, has been performed in [17]. The correlation between the decrease of the height of the fission barriers, for the evaporation of particles from hot nuclei, and the changes in the level density has been analysed in [18]. Experimental results concerning the temperature dependence of the level density parameter have been reported in [19–21]. The general picture which emerges from these experimental results is that a reduction of the level density parameter $a(T)$ occurs as the temperature is increased [14–21].

The temperature dependence of the coefficients of the liquid drop model (LDM) expansion of the free energy has been studied long ago [41]. This parametrization reproduces the result of calculations based on microscopic theories at finite temperature [41]. Also, from the T dependence of the LDM [41, 42], the temperature dependence of the fission barrier has been extracted. The predicted fission barriers are dependent upon the value of the surface coefficient of the LDM [8, 42]. A similar effect is obtained in a LDM of clusters [44]. Therefore the possibility of a competition between different mean fields can also be considered, at least as a conjecture, in dealing with the temperature dependence of the level density parameter.

Limiting temperatures for the formation of a compound nucleus have been reported by Abe *et al* [45]. Emission temperatures have been extracted by Hagel *et al* [8] from the analysis of data on particle-evaporation reactions from highly excited compound nuclei. These values show that mean field predictions for limiting temperatures are usually much higher than the observed values [8–13, 45]. A large piece of experimental evidence on the above-introduced thermal effects can be found also in the analysis of the data shown in [14–22]. As said before, relevant theoretical studies have been reported in [6, 7].

In the present work we would like to discuss the temperature dependence of the level density parameter starting from the evaporation model [46]. In section 2 the cooling model for the evaporation of particles from a hot compound nucleus of [45, 46] is used to determine the value of the level density parameter with the assumption of temperature-dependent separation energies. The results obtained with this model are discussed in section 3 and compared with values extracted from data [8–13]. Conclusions are drawn in section 4.

2. Formalism

The starting point of the present analysis is the cooling model of [45, 46]. The cooling model is based on a formula due to Weisskopf [45]. It describes the energy balance associated with the statistical emission of a particle from a compound nucleus [46]. Following Weisskopf's argument the intrinsic excitation energy per particle of a nucleus at temperature T is distributed between the mean value of the kinetic energy of the emitted particle and the separation energy of the emitted particle relatively to the initial nucleus. Neglecting angular momentum effects it reads

$$\Delta E = E_{\mathrm{i}} - E_{\mathrm{f}} \approx 2T + B(T) \tag{1}$$

where E_{i} and E_{f} are the initial and final energies and $B(T)$ is the temperature-dependent separation energy for the emitted particle. The contact with the statistical model of nuclear excitations is established by defining the de-excitation energy in terms of the nuclear level density parameter [8, 16, 45, 46]

$$\Delta E = \frac{aT^2}{A} - \frac{T}{A} + \frac{E_0}{A}. \tag{2}$$

The inverse level density parameter $K(T)$ is defined as

$$K(T) = \frac{A}{a} \tag{3}$$

and in terms of the separation energy $B(T)$ it is written as

$$K(T) = \frac{T^2}{2T + B(T) + T/A - E_0/A}. \tag{4}$$

The temperature dependence of the nuclear binding energy has been calculated by Bonche *et al* [40]. The corresponding separation energy has been parametrized by Abe *et al* [45]. It can be written as

$$B(T) = B_0\left(1 - \left(\frac{T}{T_{\mathrm{lim}}}\right)^2\right) \tag{5}$$

for nucleons and

$$B_\alpha(T) = 4B(T) - b_\alpha \tag{6}$$

for α-particles. In the above equation $b_\alpha = 28.3$ MeV is the binding energy of a α-particle. The parameter T_{lim} is the limiting temperature and for values of T larger than this value the separation energy vanishes; B_0 is the ground-state separation energy. Using the above-mentioned parametrization for the separation energy the inverse level density parameter $K(T)$ shows a linear dependence on T at temperatures higher than the limiting temperature. At lower temperatures it shows a quadratic dependence on T. The comparison between this predicted temperature dependence of $K(T)$, due to the temperature dependence of the separation energy, and that observed will be discussed in section 3.

In order to introduce temperature-dependent nuclear structure effects, other than the temperature dependence of the nuclear separation energy, one can represent schematically the eigenvalue distribution of a nuclear Hamiltonian by the spectral density distribution

$$\rho(E) = \sum_{n=1}^{N} \delta(E - E_n) + \Theta(E - E_{\mathrm{c}})g(E) \tag{7}$$

where the first term of $\rho(E)$ represents discrete states and $g(E)$ is a continuous distribution function; the function $\Theta(x)$ is the step function defined as $\Theta(x) = 1$ for $x > 0$. This form for the Hamiltonian spectral distribution is inspired by the results of realistic calculations [49]. The first term of $\rho(E)$ corresponds to a set of correlated nuclear states with a discrete energy spacing up to a cut-off E_{c}; the energy of each state is represented by the value E_n; the function $g(E)$ can be modelled by a Gaussian or a binomial distribution [49].

This spectral density $\rho(E)$ is used to compute self-consistently the excitation energy at a given T and for a fixed number of nucleons. From the numerical relation between the excitation energy and the temperature T one can extract a value of $K(T)$. The results of realistic calculation of nuclear structure properties [49] can be used to select a suitable set of parameters for $\rho(E)$. We have performed T-dependent calculations of the scale between excitation energy and temperature, using this spectral density function. With this model one can describe, qualitatively, the main features of the inverse level density parameter $K(T)$. The result of these calculations can be compared with the available data, as we shall show in section 3.

The temperature dependence of the nuclear free energy, corresponding to the liquid drop model [41], can be written

$$F = F_{\rm sym} + F_{\rm as} + F_{\rm Coul} \tag{8}$$

where the symmetric, asymmetric and Coulomb terms are given by

$$\begin{aligned} F_{\rm sym}(A) &= a_{\rm v}A + a_{\rm s}A^{2/3} + a_{\rm c}A^{1/3} + a_0 \\ F_{\rm as}(A, I) &= \frac{JAI^2}{[1 + (9J/4Q)A^{-1/3}]} \\ F_{\rm Coul}(A, Z) &= c_1 Z^2 A^{-1/3} + c_2 Z^2 A^{-1} \\ I &= (N - Z)/A\,. \end{aligned} \tag{9}$$

The coefficients $a_{\rm i}(T)$ are taken from [41] and are given by

$$a_{\rm i}(T) = a_{\rm i}(T = 0)(1 - x_{\rm i}T^2)\,. \tag{10}$$

A similar dependence is adopted for the coefficients $c_{\rm i}(T)$. J and Q are the volume asymmetry energy and surface stiffness coefficient, respectively. The numerical values of the parameters of (8) are given in [41]. Fission barriers, for the nuclei which have been calculated in [41], are characterized by end-point temperatures of the order of $T = 5.3$ and 4.0 MeV for the cases of ^{208}Pb and ^{240}Pu, respectively. A similar approach has been developed in [42]. The temperature dependence of the parameters of the liquid drop model used in [42] is slightly different from the one used in [41] and the corresponding end-point temperature is of the order of $T = 8$ MeV. In both cases, [41, 42] , the height of the fission barrier has been calculated with the fissibility parameter

$$\begin{aligned} \chi(T) &= Z^2(1 + 5.2 \times 10^{-3}\ {\rm MeV}^{-2}\ T^2)/(AF_\chi) \\ F_\chi &= 50.88(1 - \kappa I^2) \\ \kappa &= 1.7826 \end{aligned} \tag{11}$$

leading to

$$B_{\rm f}(T) = 0.83 n_{\rm s}(19 - 0.12\ {\rm MeV}^{-2}\ T^2)(1 - \chi(T))^3\ {\rm MeV} \tag{12}$$

for the fission barrier $B_{\rm f}(T)$. The multiplicative factor $n_s = 0.9444(1 - \kappa I^2)$ is taken from [42]. Using this equation, the end-point temperature can change from $T = 8$ to 6 MeV by changing the factor in front of the T^2-term from 0.12 to 0.21. This factor is associated with the T-dependence of the surface tension and of the nuclear surface. This effect has been suggested in [42]. A similar change of the end-point temperature, for the fission barrier, can be obtained from a modified liquid drop model which includes clusters of nucleons as elementary degrees of freedom [44]. This modified liquid drop model has been extracted from a fit based on the $SU(4)$ symmetry group. The resulting expression for the energy reproduces the observed binding energies [44]. The fact that different T-dependent parametrizations of the LDM can be reflected upon the values of $K(T)$ is evident. The possibility to distinguish between effects due to different mean fields is, of course, dependent on the data and we shall use it as a conjecture in our analysis.

3. Results and discussion

For the present analysis we have selected a set of data corresponding to different compound nucleus reactions for the system ^{14}N on ^{154}Sm at bombarding energies $E_b = 261$ and 490 MeV [8], at $E_b = 600$ MeV for the system ^{20}Ne on ^{165} Ho [10] and $E_b = 650$ MeV for ^{60}Ni on ^{100}Mo [13], respectively. We have also extracted information from more recent measurements, which have been reported in [14–21]. The data points have been obtained from coincidence measurements between emitted particles and residual nuclei and from the velocity distribution of residues. The data displayed in figure 1 show a pronounced change at $T \approx 3.5$ MeV. At low temperatures the value of the inverse level density parameter is of the order of 8 MeV while at high temperatures it is of the order of 13 MeV. The value at low T agrees with the empirical value extracted from slow neutron resonances [1] while the value at high T is approximately the value predicted by the fermi gas model [1]. However, the set of points extracted from [10] disagrees with other measurements and shows the opposite trend. The points extracted from [8] show a saturation at T of the order of 5 MeV while the data from [13] change more rapidly around $T = 4$ MeV. These points have been obtained from the measurement of different particle-channels and can be interpreted as evidence for thermalization [13].

The temperature dependence of the the inverse level density parameter, extracted from the cooling model [45–47] with a temperature-dependent separation energy, (5) and (6), is shown in figures 2 and 3, for the evaporation of neutrons and α-particles, respectively.

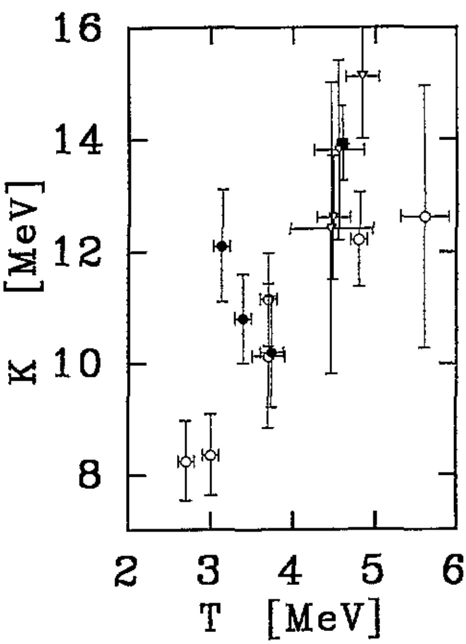

Figure 1. Temperature dependence of the inverse level density parameter, $K(T)$, extracted from data. The experimental values are denoted by open circles [8], full circles [10], triangles [13] and a full square [20], respectively.

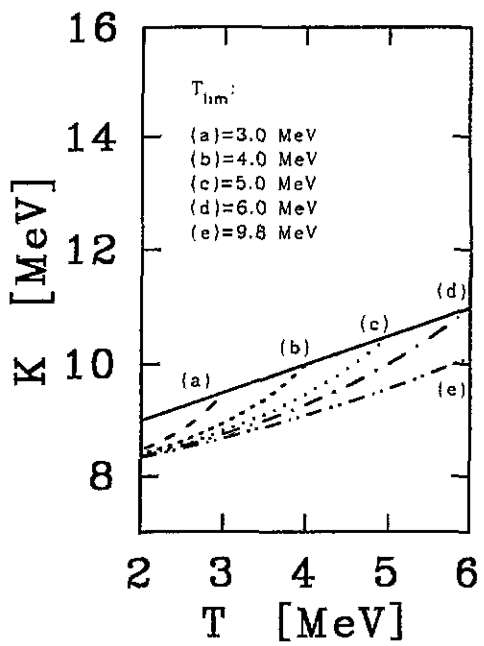

Figure 2. Temperature dependence of the inverse level density parameter extracted from the cooling model [45] with a temperature-dependent nucleon separation energy. The results shown in the figure correspond to different values of the limiting temperature, T_{lim}. The value $T = 9.8$ MeV is the limiting temperature extracted from the LDM [41].

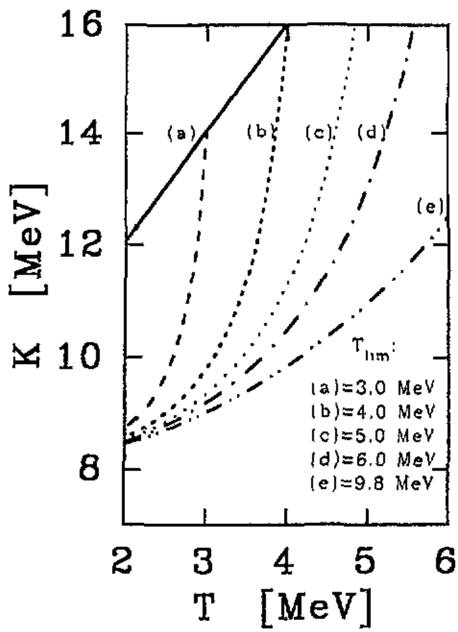

Figure 3. Temperature dependence of the inverse level density parameter corresponding to the cooling model with a temperature-dependent separation energy for α-particles. The values of T_{lim} are listed using the notation of figure 2.

The curves are shown for different values of the limiting temperature T_{lim}. The predicted temperature dependence agrees with data. However, the transition temperature, i.e. the value of T for which a linear dependence of $K(T)$ with T is obtained, is much lower for α-particles than for neutrons. Results obtained by using constant, temperature-independent, separation energies show a linear dependence of $K(T)$ with T at low temperatures which is not consistent with the data.

A schematic representation of nuclear correlation effects can be constructed from a spectral distribution with low lying (discrete) excitations and with a high-energy continuum, represented by a Gaussian distribution. This schematic representation reproduces the features of a realistic calculation [48, 49] of the nuclear excitation spectra. The low-energy sector of the spectral distribution $\rho(E)$, equation (7), describes collective excitations in highly correlated states while the high energy part of it corresponds to uncorrelated particle and hole excitations. The temperature dependence of $K(T)$ predicted by this model is shown in figure 4. The different curves have been obtained by using different parameters for the spectral distribution, as discussed above. The resulting values of $K(T)$ would correspond to a situation where the increase of T produces the collapse of nuclear correlations at relatively low values of the temperature. The thermal collapse of residual nuclear interactions produces the bending of $K(T)$ at high T, an effect which is displayed by some of the data of figure 1. This model shows schematically the influence of finite-size effects upon $K(T)$ [50–52].

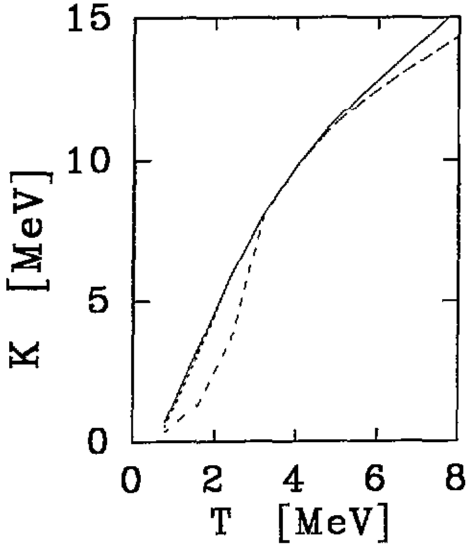

Figure 4. Inverse level density parameter extracted from the spectral density $\rho(E)$ given in (7). Full, broken and long-dashed lines correspond to different values of the cut-off E_c = 40, 65 and 165 MeV, respectively.

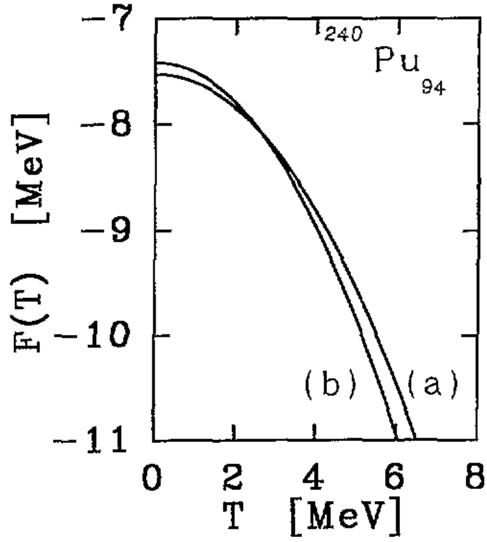

Figure 5. Nuclear free energy per nucleon, $F(T)$, for the nucleus ^{240}Pu as a function of the temperature T. (a) corresponds to the standard LDM [41]; (b) corresponds to the LDM of Cauvin *et al* [44] adjusted to reproduce the fission barrier given by Sauer *et al* [42].

In order to understand the significance, if any, of the data-points around $T = 3$–4 MeV, we have computed the free energy from temperature-dependent LDM parametrizations [41, 44]. The results shown in figure 5 have been obtained by using the set of LDM parameters listed in [41, 44]. The curves shown in figure 5 represent two different configurations, i.e. two different mean fields. The transition temperature shown in this figure is of the order of 3.5 MeV. This value is similar to the one associated to a change of the T-dependence of $K(T)$. If such a connection between data and theory can be established then cluster effects might play a role in the temperature dependence of $K(T)$. It would be a signature of temperature-dependent effects which cannot be explained by the collapse of residual interactions. A similar conclusion has been drawn in the study of the temperature dependence of the fission barrier, where cluster effects have been mentioned in connection with the change of the value of surface contributions to the fission barrier at finite T [42]. As an example, the temperature dependence of the predicted fission barriers using the standard and modified LDM are shown in figure 6. The agreement between the surface-modified LDM (curve 6(b)) and the LDM including clusters [44] (curve 6(c)) is significant. Finally, the empirical values of the inverse level density parameter $K(T)$ are compared with values obtained by using different approximations. The results are shown in figure 7. The theoretical values shown in this figure are extreme values extracted from T-independent descriptions of nuclear correlations. In the same figure 7 we have included theoretical values extracted from temperature-dependent Hartree–Fock calculations performed by using effective interactions [7]. It is evident, from the results shown in this figure, that nuclear structure effects at finite T are important and cannot be neglected.

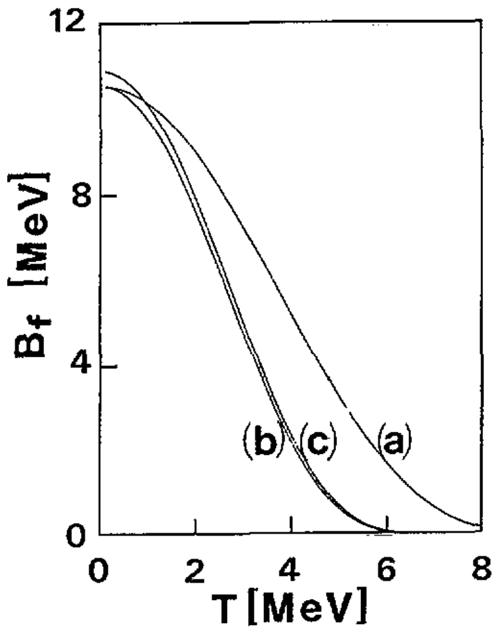

Figure 6. Temperature-dependent fission barriers, B_f, obtained with the parameters of [42], as a function of T. (a) Corresponds to the standard LDM [41], (b) to the LDM with a surface term with $x_s = 11.06 \times 10^{-3}$ MeV^{-2} and (c) to the LDM with clusters of [44], respectively.

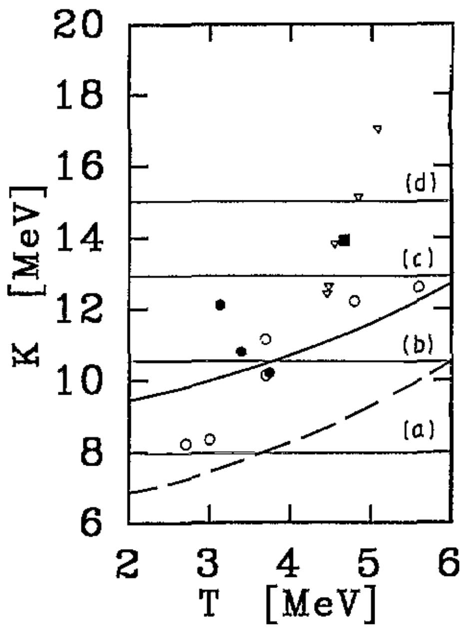

Figure 7. The inverse level density parameter $K(T)$ for different extreme cases: (a) extracted from slow neutron resonances [1], (b) calculated from a T-independent LDM [41], (c) saturation value at high T adopted in [8] and (d) the Fermi gas model, respectively. Data are shown as in figure 1, without the error bars. Full and broken lines correspond to the theoretical values of [7].

4. Conclusions

In this work we have reported some results concerning the temperature dependence of the nuclear level density parameter. The analysis of the data shows that they cannot be interpreted by using a constant scale factor between excitation energy and temperature. The results for the inverse level density parameter, obtained in the cooling model with a temperature-dependent particle-separation energy, are consistent with the data. The shift to lower temperatures of the end-point of the fission barrier, which can be obtained by using a liquid drop model version which includes clusters of nucleons [44], is also consistent with the evidence about limiting temperatures extracted from data. The picture which emerges from the present analysis can be summarized in the following:

(i) A consistent description of the observed temperature dependence of the level density parameter requires the use of temperature-dependent separation energies.

(ii) The temperature dependence of the level density parameter, at relatively low temperature, can be explained by the collapse of residual interactions.

(iii) The temperature dependence of the level density parameter, at relatively high temperature, can be explained by changes of the mean field [6, 7]. As a conjecture, cluster effects can be introduced to explain it in analogy with the predicted T-dependence of the fission barrier [42].

(iv) The data show that at temperatures of the order of $T = 3$–4 MeV a change in the temperature dependence of the level density parameter is observed.

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