Schematic study of perturbative effects on the two-neutrino double beta decay to excited states*

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Abstract. Matrix elements for the two-neutrino double beta decay of ^{136}Xe to the ground and excited states of ^{136}Ba are calculated using a schematic model. The separable proton–neutron interaction includes, in addition to the particle–hole channel, a particle–particle channel. It is found that this force yields nearly the same results as realistic interactions for leading-order QRPA contributions. Perturbative effects associated with the coupling between quasiparticles and phonons, both of the charge-conserving and charge-non-conserving type, are studied and found to be dominated by the coupling to charge-non-conserving 1^+ proton–neutron phonons.

The study of the nuclear double beta decay continues to attract attention. The issues concerning the detectability of the neutrinoless mode [1], the suppression of the ground-state-to-ground-state (gs → gs) two-neutrino mode ($2\nu\beta\beta$) [2] and the decay to excited states (gs → 2_1^+) [3, 4] are currently open. The calculation of the relevant nuclear matrix elements has been discussed, for example, in [5]. In this article we aim to extend the discussion of [4] to introduce perturbative corrections to nuclear matrix elements for the gs → 2_1^+ $2\nu\beta\beta$ decay mode. We have done this in the context of a separable interaction in order to get an idea about the features which could possibly emerge in a more lengthy calculation using realistic interactions. In view of the smallness of the nuclear matrix elements involved [4] we should like to see if the coupling between quasiparticles and charge-conserving quadrupole phonons and charge-exchanging dipole phonons can affect the decay rate for gs → 2_1^+ $2\nu\beta\beta$ transitions. This research is required for the analysis of current experiments [6]. In this article we discuss transitions from the ground state of ^{136}Xe. The details of the formalism will be reported elsewhere [7].

In the present calculations we use for the proton–neutron channel the schematic two-body interaction:

$$V(\chi, \kappa) = 2\chi : \beta_1 \beta_1^\dagger : -2\kappa : P_1 P_1^\dagger : \tag{1}$$

as given by Kuz'min and Soloviev [8] and more recently by Muto *et al* [9]. This is an extension of the model introduced by Halbleib and Sorensen [10]. The operators β_1 and P_1 are written in the quasiparticle representation and stand for the particle–hole and particle–particle channels of the dipole charge-exchange operator $\sigma\tau$; the coupling constant χ can be fixed by adjusting the excitation energy of the giant Gamow–Teller resonance (GTGR),

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and the coupling constant κ is a free parameter in this model. The structure of the pn-QRPA forward and backward matrices $A(p'n', pn)$ and $B(p'n', pn)$, defined by the Hamiltonian which includes the single-quasiparticle term and the interaction (1), is rather similar to that obtained with a non-separable interaction [11]. We have diagonalized this Hamiltonian in the quasiproton–quasineutron (pn) basis, avoiding numerical complications which arise when using dispersion relations [9, 10].

Once the QRPA wavefunctions and eigenvalues for proton–neutron configurations are determined, the remaining part of the Hamiltonian can be treated perturbatively. We have adopted the methods of the nuclear field theory (NFT) of nuclear excitations [12] to derive vertex functions for the description of the coupling between quasiparticles and phonons. In the present case, since we are dealing with pn 1^+ phonon excitations, the NFT vertex functions corresponding to the scattering of a neutron by a 1^+ phonon are different from those corresponding to the scattering of a proton [7].

In addition to the intermediate 1^+ states, we have computed the wavefunction for the first excited quadrupole state of the final nucleus. For this channel a separable interaction of the quadrupole–quadrupole type [13] was adopted and treated in the QRPA approach for proton–proton and neutron–neutron configurations. The observed energy of the first excited quadrupole state (2_1^+) was used to determine the strength of the quadrupole interaction. It also determines the value of the matrix element for electromagnetic transitions in the final nucleus and the vertex functions for the coupling of the quadrupole phonon with quasiparticles [14]. The corresponding equations can be found in [7].

To estimate the order of magnitude of the perturbative corrections to the $2\nu\beta\beta$ observables produced by particle-vibration coupling processes, we have computed the decay of ^{136}Xe to the first excited quadrupole state of ^{136}Ba. We have included perturbative corrections to the matrix elements connecting virtual 1^+ states of ^{136}Cs with a final 2_1^+ state. The processes which are accounted for are represented by the diagrams shown in figure 1. In addition to the QRPA leading-order terms, given by diagrams D123 and D321, we have included charge-non-conserving (D213 and D312) and charge-conserving (D231 and D132) scattering processes, respectively.

The single-particle states which we have adopted are taken from the Coulomb-corrected Woods–Saxon potential [15]. The basis includes 12 single-particle states, for both protons and neutrons, above the core $A = 40$ ($N = Z = 20$). A monopole-pairing interaction, with strengths $G_n = G_p = 22/A$ MeV, was used to compute BCS occupation factors and gap values. The initial ($Z = 54$) and final ($Z = 56$) proton gaps, obtained with this interaction, are of the order of 1.0 and 1.2 MeV, respectively. The resulting neutron gap, for $N = 80$, is of the order of 1.0 MeV. These values reproduce the data quite well.

The 1^+ spectrum of the intermediate odd–odd nucleus can be calculated starting from the ^{136}Xe ground state. In this calculation, the coupling constant χ was fixed at the value 0.15 MeV and with it the energy of the giant Gamow–Teller (GTGR) excitation in ^{136}Cs is 16.70 MeV. The GTGR state exhausts about 90% of the total strength for β^- transitions. The same coupling constant gives a state at 7.8 MeV which carries 74% of the total strength for β^+ transitions. We have verified that the Gamow–Teller sum rule is satisfied by the resulting complete set of pn-QRPA states. The same calculation was repeated for 1^+ excitations based on the ground state of ^{136}Ba. The results show the same features. Following the method of [4, 11], the overlap between the two representations of the intermediate virtual excitations of ^{136}Cs was computed. With the corresponding wavefunctions we have calculated the matrix element for the gs $\rightarrow$ gs $2\nu\beta\beta$ transition. The value for this matrix element, for $\kappa = 0.0$, is very much the same as that obtained with a realistic force [4].

The structure of the 2_1^+ excitation of ^{136}Ba was computed for an excitation energy of
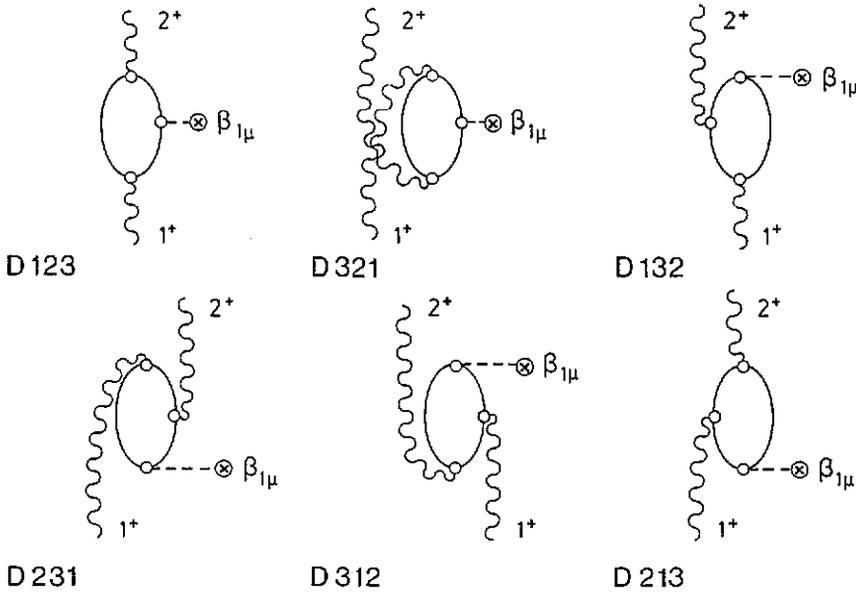
Figure 1. Diagrams representing the single beta decay of a proton–neutron 1^+ excitation into a quadrupole one-phonon state. Phonons are represented by wavy lines, the beta-decay operator is denoted by $\beta_{1\mu}$ and the time ordering of the vertices is denoted at the bottom of each diagram. Connecting solid lines represent quasiparticles.

0.819 MeV. The corresponding $B(E2)$ value is of the order of 14 W.u. in comparison with the experimental value of about 19 W.u. The theoretical value has been obtained using standard effective neutron (0.355) and proton (1.355) charges [14]. The corresponding coupling constant for quadrupole interactions, χ_2, in the final nucleus is of the order of 0.23×10^{-2} MeV fm^{-4}. With the corresponding quadrupole wavefunction we have computed the leading-order QRPA matrix element for gs $\rightarrow 2_1^+$ $2\nu\beta\beta$ transition, and again found a good agreement with the previous calculations carried out with realistic interactions [4].

As a next step, we have computed matrix elements and perturbative corrections for different values of the coupling constant κ. The results are shown in figure 2 . The main difference between the calculations performed with realistic interactions [4] and the present ones is that the value of κ which produces the collapse of the pn-QRPA ($\kappa = 0.08$ MeV) is far from the value ($\kappa = 0.04$ MeV) at which the suppression of the matrix element for the gs $\rightarrow$ gs $2\nu\beta\beta$ transition takes place. For all the cases shown in figure 2, the GT sum rule is obeyed.

The change introduced by particle–particle interactions in the wavefunctions for virtual 1^+ excitations, mainly reflects the strength distribution for β^+ transitions based on the final ground state. The particle–particle interaction splits this strength distribution to a couple of states at excitation energies of about 6–7 MeV. This is a feature already found in realistic calculations [4]. For the sake of completeness, and to assess the reliability of the wavefunctions computed using the present model, we have also calculated the $2\nu\beta\beta$ transitions to final states consisting of two quadrupole phonons. The resulting matrix elements are shown in figure 2 as a function of κ.

The perturbative corrections to the gs $\rightarrow 2_1^+$ $2\nu\beta\beta$ matrix element are listed in table 1. It is found that the largest contribution comes from the process represented by D231. This is due to the change in the wavefunction for 1^+ excitations induced by the attractive particle–
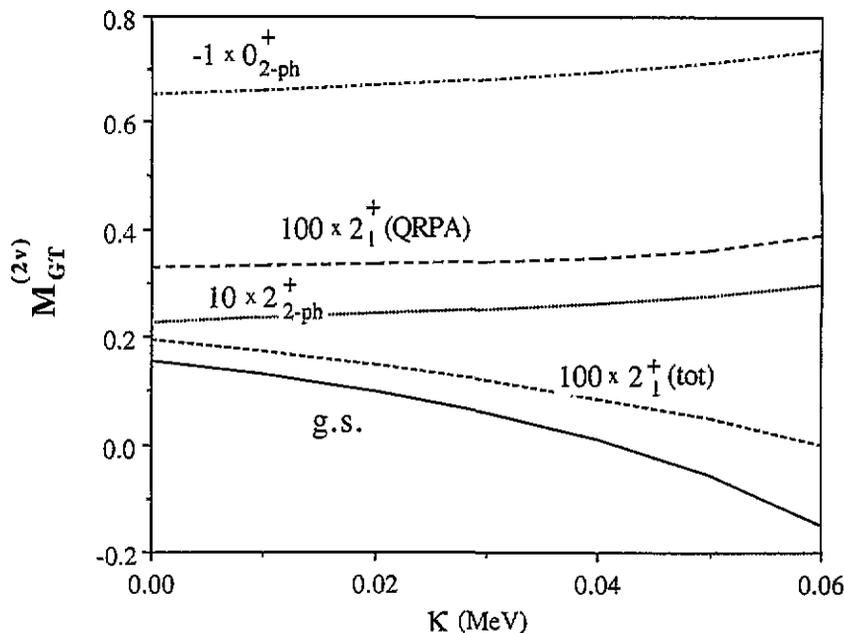
Figure 2. Dependence of the $2\nu\beta\beta$ matrix elements $M_{GT}^{(2\nu)}$ upon the coupling constant κ of the particle–particle channel of the proton–neutron interaction. The curves correspond to gs $\to$ gs transitions (gs), gs $\to$ 2_1^+ transitions without $(2_1^+\,(\text{QRPA}))$ and with $(2_1^+\,(\text{tot}))$ perturbative corrections, and transitions to members of the two-phonon multiplet $(0_{2\text{-ph}}^+$ and $2_{2\text{-ph}}^+)$, respectively. The matrix elements are scaled by the electron rest-mass (see [4]) and κ is given in MeV.

Table 1. Contributions to the $2\nu\beta\beta$ gs $\to$ 2_1^+ matrix element. The results are listed for each of the processes shown in figure 1 as a function of the coupling constant κ. The last column gives the sum of all the perturbative corrections D132, D231, D213 and D312.

κ (MeV)	D123 + D321[a]	D132[a]	D231[a]	D213[a]	D312[a]	Sum[a]
0.00	0.328	−0.040	−0.049	−0.023	−0.024	−0.136
0.01	0.331	−0.040	−0.060	−0.028	−0.030	−0.158
0.02	0.335	−0.040	−0.076	−0.033	−0.041	−0.190
0.03	0.339	−0.038	−0.096	−0.043	−0.045	−0.222
0.04	0.347	−0.034	−0.123	−0.050	−0.057	−0.264
0.05	0.360	−0.023	−0.161	−0.055	−0.073	−0.312

[a] The numbers in the column are to be multiplied by 10^{-2}. The units for the matrix elements are $(m_e c^2)^{-3}$.

particle channel of the two-body interaction (1). The contributions from all the diagrams included in the calculation are free from instabilities caused by small energy denominators. The fact that the perturbative corrections to the GT matrix element for gs $\to$ 2_1^+ are mainly given by the coupling of quasiparticles to charge-non-conserving phonons can be attributed to the increase in backward-going amplitudes of the proton–neutron phonons induced by particle–particle interactions. However, the inclusion of perturbative corrections does not suppress the matrix element for gs $\to$ 2_1^+ transitions drastically. In the present example, the corrected GT matrix element for the gs $\to$ 2_1^+ transition is about half the value given by

· the leading-order QRPA contribution.

Here is a brief resumé of the above-mentioned and other features, found in the present analysis.

(i) Particle-number violation effects are small for the pn excitations. The largest deviation from the correct particle number is found for the 'giant' state which carries most of the β^+ strength; the total GT sum rule is also well preserved in this case.

(ii) The contributions to the matrix elements for the gs $\rightarrow$ gs and gs $\rightarrow 2_1^+$ $2\nu\beta\beta$ transitions are distributed over a large number of virtual excitations of the intermediate nucleus. It is simply impossible to isolate a single dominant transition. Thus, any sort of expansion around a fixed excitation energy becomes infeasible.

(iii) Since in this model the QRPA breaks down at a relatively large value of κ, one can indeed explore the changes in the wavefunctions which are responsible for the suppression of some of the relevant matrix elements. These changes are found to be related to the increase in the amplitude for pn pairs occupying high-lying intruder states. The effect of these pairs is unimportant for small values of κ. The growing importance of these new configurations mainly reflects the splitting of the β^+ strength.

In conclusion, we have investigated some important $2\nu\beta\beta$ matrix elements in a schematic nuclear model. All the interactions have been written in a separable form and the proton–neutron force includes a particle–particle channel in addition to the traditional particle–hole channel. The suppression of the matrix element for the gs $\rightarrow$ gs transition is reproduced by the model. The matrix elements for the gs $\rightarrow 2_1^+$ $2\nu\beta\beta$ transitions are computed as a function κ with and without perturbative corrections. The perturbative corrections, due mainly to the coupling of quasiparticles to charge-non-conserving phonons, may reduce the matrix elements for the gs $\rightarrow 2_1^+$ transition by a factor of 2, but do not suppress them completely, leaving the qualitative results of the previous calculations [4, 16, 17] valid.

Concerning the decay to members of the two-phonon multiplet, the schematic model reproduces the results obtained with a realistic interaction and yields a large matrix element for the transition to the final two-phonon 0^+ state. The collectivity of the final one-phonon quadrupole excitation is also well reproduced by the separable quadrupole–quadrupole interaction. The simplicity of the model, which none the less reproduces both qualitatively and quantitatively the results already obtained with realistic interactions, gives a good insight into the effects induced by proton–neutron particle–particle interactions. It confirms the hypothesis that the main reason for the suppression of matrix elements for the $2\nu\beta\beta$ mode is attributable to the fragmentation of the β^+ virtual branch of the decay.

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