Effects due to temperature-dependent nuclear binding energies on the equation of state for hot nuclear matter

O. G. Benvenuto
Facultad de Ciencias Astronomica y Geofisicas, Universidad Nacional de La Plata, La Plata, Buenos Aires, Argentina

O. Civitarese and M. Reboiro
Departamento de Física, Universidad Nacional de La Plata, La Plata, Buenos Aires, Argentina
(Received 17 November 1992)

The influence of finite temperature nuclear effects upon the adiabatic index, for a system of nuclei, nucleons, and leptons, is discussed. It is found that the inclusion of temperature-dependent nuclear binding energies affects the behavior of the adiabats and of the adiabatic index, particularly, at low entropies.

PACS number(s): 21.90.+f

I. INTRODUCTION

The interest in the study of the equation of state (EOS) for hot and dense matter has been motivated by the need to set up limits on the stability of nuclear systems under extreme conditions [1–8] and, more recently, by the exploration of the physics of supernoave [9,10]. The existing literature is very rich and we shall not review it here [11]. Rather, we shall focus our attention on a very restricted aspect of the problem, namely, on the study of the influence of temperature-dependent nuclear structure effects upon adiabats and upon the adiabatic index Γ_S. The relevance of adiabats and Γ_S in the physics of supernovae has been emphasized by Bethe [11]. Numerical results, obtained by using different models, have been presented by the Illinois group [Lamb-Lattimer-Pethick-Ravenhall (LLPR)] and by Cooperstein [12], among others. A common feature of the LLPR and Cooperstein approaches is the assumed quadratic temperature dependence of the nuclear free energy. It is the aim of this paper to show that the treatment of nuclear intrinsic thermal excitations, starting from a realistic temperature dependence of the nuclear liquid drop [13], could affect the behavior of the adiabatic index. We have used the formalism which has been presented in a previous paper [14] and we have compared our results with the ones obtained by Cooperstein [12] for low entropy values ($S=1,2$) and with the results of LLPR for higher values of the entropy ($S=3$). We have found that the behavior of Γ_S, for the present case, qualitatively agrees with the results of Ref. [12], at low entropies, and that the approximations of Ref. [12] are indeed justified concerning the temperature dependence of the nuclear structure components of the calculations.

A brief review of the formalism is presented in Sec. II. Results for the EOS of the system and related parameters are shown and discussed in Sec. III. Conclusions are drawn in Sec. IV.

II. FORMALISM

We shall briefly describe, in this section, the equations which we have used to calculate adiabats,

$$T=T(n)|_S \ , \tag{1}$$

and the adiabatic index Γ_S,

$$\Gamma_S=\left.\frac{\partial \log p}{\partial \log n}\right|_S \ , \tag{2}$$

where p is the pressure and n is the density for a system with leptons, drip baryons, and nuclei at equilibrium at a fixed temperature. Charge neutrality is assumed; the fraction of leptons per baryons is given by Y_L, such that

$$n_e+n_\nu=Y_L n_B \ , \tag{3}$$

where n_e is the total electron/positron density

$$n_e=\frac{N(e^-)+N(e^+)}{V} \ , \tag{4}$$

n_ν is the neutrino density, n_B is the baryon density

$$n_B=\frac{N(\text{neutrons})+N(\text{protons})}{V} \ , \tag{5}$$

and V is the (fixed) volume. The leptonic components are described by a relativistic gas, with chemical potentials μ_e and μ_ν, which are determined from the condition of β equilibrium with the baryon drip phase:

$$\mu_e+\mu_{\text{protons}}=\mu_{\text{neutrons}}+\mu_\nu +(m_{\text{neutrons}}-m_{\text{protons}}-m_e)c^2 \ . \tag{6}$$

In the relativistic Fermi gas limit, the total electron and neutrino number densities are given by

$$n_e=\frac{2}{6\pi^2(\hbar c)^3}(\mu_e^3+\pi^2T^2\mu_e) \ , \tag{7}$$

$$n_\nu=\frac{1}{6\pi^2(\hbar c)^3}(\mu_\nu^3+\pi^2T^2\mu_\nu) \ . \tag{8}$$

The conservation of charge is given by the constraint $n_e=n_{\text{protons}}$, and the number of protons is the sum of protons in the drip phase (n_p^{drip}) and in nuclei ($Zn_{A,Z}$). Therefore we can write for the number of protons the relationship

$$n_p = \sum_{A,Z} Z n_{A,Z} + \left[1 - V_0 \sum_{A,Z} A n_{A,Z}\right] n_p^{\text{drip}} , \qquad (9)$$

where the factor in front of the drip term corresponds to the excluded volume due to nuclei. The sum indicated by A,Z goes over all nuclei on the stability line.

The conservation of the baryonic number is enforced by a similar relation: namely,

$$n_B = \sum_{A,Z} A n_{A,Z} + \left[1 - V_0 \sum_{A,Z} A n_{A,Z}\right] (n_p^{\text{drip}} + n_n^{\text{drip}}) , \qquad (10)$$

which includes the neutron drip component, as well. The transition of neutrons and protons from nuclei to a drip phase is controlled by the balance equation

$$\beta\mu_{A,Z} = \beta B_{A,Z} - Z y_p - (A - Z) y_n \qquad (11)$$

for each set of allowed values of N, Z, A ($N + Z = A$), where $B_{A,Z}$ is the binding energy and $y_q (q = p,n) = \beta(V_q - \mu_q)$. The quantities V_q and μ_q are the one-body potential and the Fermi energy, respectively, for neutrons ($q = n$) and protons ($q = p$) in the drip phase and they are obtained from the temperature-dependent plane-wave limit of the Skyrme interactions (Sk*) [15]. The parameter β is the inverse temperature ($\beta = 1/T$), with the temperature T given in units of MeV.

For the nuclear number density $n_{A,Z}$, corresponding to a nucleus with A nucleons and Z protons, we have used the expression

$$n_{A,Z} = \lambda^{3/2} e^{\beta\mu_{A,Z}} \sum_J (2J+1) e^{\beta E(J,T)} , \qquad (12)$$

where

$$E(J,T) = \frac{\hbar^2 J(J+1)}{2\mathcal{Y}} + E^*(T) \qquad (13)$$

is the rotational plus intrinsic energy of the nucleus at angular momentum J and temperature T. The factor $\lambda^{3/2}$ is the translational factor for a nucleus with mass $M_{A,Z}$ and $E^*(T)$ is the internal excitation energy. All quantities entering in the definition of $E(J,T)$ can be parametrized in terms of well-known relationships between the level-density parameter, the spin cutoff factor, and the moment of inertia, $\mathcal{Y}$ at fixed temperature [16].

Details concerning the method which we have adopted to solve the above introduced system of coupled equations can be found in our previous paper. For the sake of completeness, let us briefly comment on the main steps of the procedure, which are as follows.

(i) The total (global) baryon number density is externally fixed. It is written in terms of baryons in nuclei and baryons in a drip phase.

(ii) The total lepton number density is externally fixed, assuming charge neutrality of the system. It includes electrons, positrons, neutrinos, and antineutrinos.

(iii) The available volume, for baryons in the drip phase, is obtained by including suitable "excluded volume factors" which arise after taking into account the volume occupied by nuclei.

(iv) The dynamics of the drip phase is described by a mean field which is built up by a two-body interaction of the Skyrme Sk* type.

(v) Nuclear partition functions and nuclear mass distributions are calculated by using a temperature-dependent version of liquid drip expansion [13].

(vi) Leptons are described in the relativistic limit.

(vii) Changes of nuclear properties due to the presence of a vapor phase are neglected.

Therefore, we shall discuss, in presenting the results, the validity of the present approach concerning the range of densities and temperatures where it can be applied.

With the above-mentioned approximations we have computed the solution of balance equations of the usual Saha type [17] for a class of nuclei along the stability line.

As mentioned before, intrinsic nuclear free energies are described by a temperature-dependent liquid drop expansion [13], and we have avoided the use of the Fermi gas limit, which is not a good approximation for the low-temperature–low-density regime which we have selected for the present calculations. With these elements we have computed adiabats from the definition of the total free energy:

$$F = F_{\text{nuclear}} + F_{\text{drip}}(\text{baryons}) + F_{\text{leptons}} \qquad (14)$$

and the entropy

$$S = -\left.\frac{\partial F}{\partial T}\right|_V . \qquad (15)$$

The corresponding EOS is readily obtained from the definition of the pressure:

$$p = n^2 \frac{\partial(F/n)}{\partial n} , \qquad (16)$$

where n is the externally fixed global matter density. With this value for p the adiabatic index Γ_S can be computed.

III. RESULTS AND DISCUSSION

The equations which have been presented in the previous section have been solved for temperatures (T) up to 16 MeV and for global densities up to the nuclear matter saturation density. The lepton fractional coefficient has been fixed at the value $Y_l = 0.3$. This value is representative of conditions required by supernova models [11].

Nuclear masses $A \leq 18$ (with $N = Z$), $20 \leq A \leq 130$, and ^{4}He nuclei have been included in the calculation of nuclear partition functions. The parameters entering into the definition of the nuclear level density have been taken from Ref. [16]. The parameters of the Skyrme interaction Sk*, used to describe the baryon drip phase, have been taken from Ref. [15]. In calculating nuclear partition functions we have used two different approximations: namely, (a) by assuming that all nuclear states above the cold yrast line of a given nucleus contribute to the density of states and, (b) by assuming that nuclear ground states are also dependent upon the externally fixed value of the temperature. Approximation (a) will then be referred to as a case with temperature-independent nuclear binding energies, while approximation and (b) will instead be referred to as a case with temperature-dependent nuclear binding energies. In case (a) the standard liquid drop model parametrization of nuclear ground-state energies has been adopted [18] while in case (b) we have adopted the parametrization which has been proposed in Ref. [13]. The meaning of all other elements of the present calculations is standard and it has been explained in the previous work [14]. We have restricted our calculations to low entropy values ($S \leq 3$), following the findings of Cooperstein [12] and Lattimer *et al.* [5]. As we have said before we have focused our attention on the following questions: (a) the validity of the quadratic expansion of the nuclear free energy, as a function of the temperature, as used by Cooperstein; (b) the influence upon the equation of state of temperature-dependent nuclear structure effects, such as the explicit consideration of the temperature-dependent liquid drop model parametrization of Ref. [13]; (c) the influence of nuclear mass distributions upon the equation of state. Since we have not considered changes on nuclear properties induced by the presence of a nucleon vapor we shall restrict to relatively low values of the global density, below the value of the density corresponding to nuclear saturation. In this respect the present calculation has the same drawbacks as that of Cooperstein [12], but we would like to emphasize that we are interested in qualitative changes of the equation of state which can be attributed to temperature-dependent nuclear structure effects.

Results corresponding to the adiabatic index Γ_S are shown in Fig. 1. The comparison of the results which are displayed in this figure shows the following features: (i) the low-entropy–low-density sector of the curves shows a reduction of Γ_S below the relativistic leptonic value ($\Gamma_S = \frac{4}{3}$); (ii) the Cooperstein approach [12] also leads to a drastic increase above this value for densities below the nuclear saturation density. Lattimer *et al.* have reported [5] a similar trend but for higher entropy values. Present results, both for the temperature-independent and for the temperature-dependent nuclear binding energies, are somehow inbetween and they show a tendency to increase the value of the adiabatic index but at larger densities. Since the present approach cannot be extended to densities larger than the nuclear saturation density we cannot really assume that an infall ($\Gamma_S < \frac{4}{3}$) will be followed by a rebound ($\Gamma_S > \frac{4}{3}$). At least we can safely say that, for the low-entropy regimen, which is familiar to the supernovae scenario of Bethe [11], temperature-dependent nuclear structure effects influence the behavior of the adiabatic index Γ_S. Particularly, the results which are displayed in Fig. 1 show that the approximations of Cooperstein [12], concerning the temperature dependence of the nuclear free energy, are valid. The inclusion of temperature-dependent nuclear binding energies helps in softening the somehow drastic behavior of Γ_S reported in Ref. [12]. A

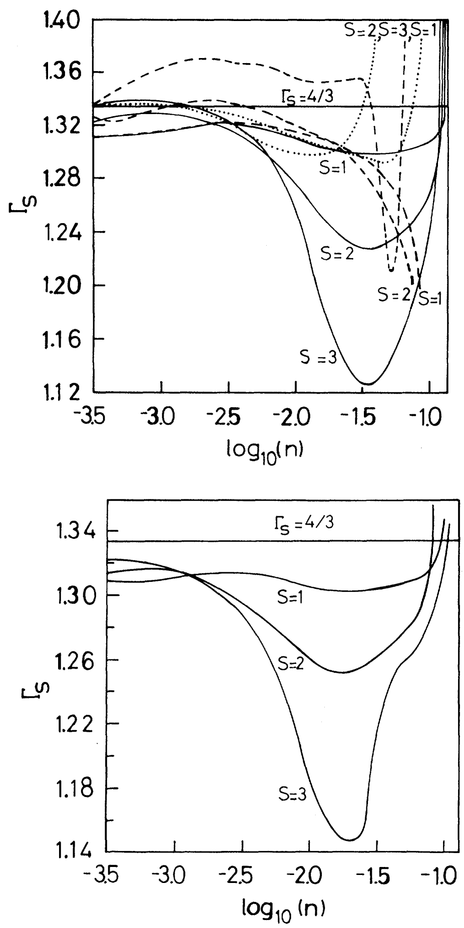


FIG. 1. (a) Adiabatic index Γ_S as a function of the global density n (fm^{-3}). Entropy per baryon values (S) are indicated on each curve. Dotted and dashed lines correspond to results taken from Cooperstein [12] and Lattimer *et al.* [5], respectively. Present results, for the calculations performed with constant, i.e., temperature-independent, nuclear binding energies, are indicated by solid lines. (b) Adiabatic index Γ_S as a function of the global density n for different entropies (S). These results have been obtained by using temperature-dependent nuclear binding energies in the nuclear partition function.

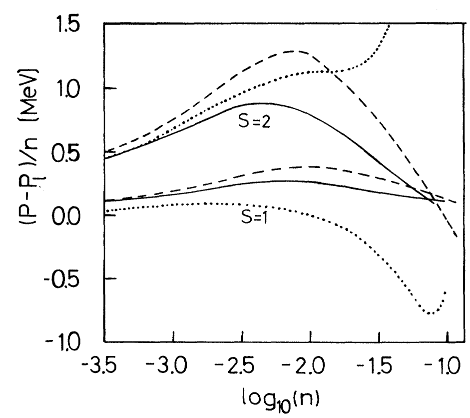
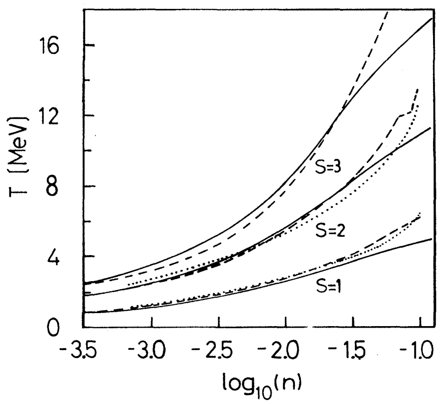


FIG. 2. Pressure per baryon (total pressure minus lepton pressure), in units of MeV, as a function of the global density n for different entropies (S). Solid and dashed lines correspond to present results obtained by using temperature-dependent and temperature-independent nuclear binding energies, respectively. Results taken from Cooperstein [12] are indicated with dotted lines.



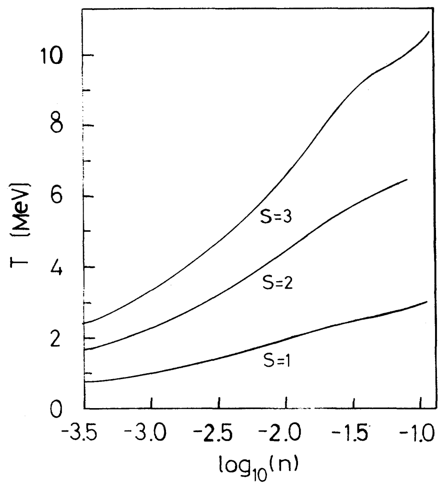


FIG. 3. (a) Temperature (T) vs density (n) at constant entropy per baryon (S). Cooperstein [12] and Lattimer *et al.* [5] results are indicated by dotted and dashed lines, respectively. Solid lines correspond to the present results obtained with temperature-independent nuclear binding energies. (b) Temperature vs density curves at constant entropy per baryon. Solid lines correspond to present results obtained by using temperature-dependent nuclear binding energies in the nuclear partition function.

similar conclusion can be extracted from the results which are shown in Fig. 2, where the contribution to the total pressure due to baryons is displayed, as a function of the global density. One can see from these curves that the effect on the pressure due to baryons becomes important at densities below and near the nuclear saturation density, once the temperature dependence of the nuclear binding energy is considered. Otherwise, the pressure seems to be dominated by the leptons. Differences between Cooperstein [12] and the present results can also be due to the inclusion of neutrinos in the leptonic phase. The most significant result of the present approach is shown in Fig. 3. While the relationship between temperature and density at constant entropy looks very much the same for different approximations, particularly at low densities, the values which are obtained by using temperature-dependent nuclear binding energies yield to lower temperatures, at fixed density and entropy. It means that the system can indeed acquire larger densities at relatively low temperatures [$T \leq 6$ MeV ($S=2$), $T \leq 3$ MeV ($S=1$)] by a conventional mechanism based on the transition of baryons from a nuclear to a drip phase. This conventional mechanism could be the consideration of excited configurations of the "cold" liquid drop, as in the present case.

IV. CONCLUSIONS

We have presented an application of a previously reported formalism [14] to treat temperature-dependent nuclear structure effects in the context of the equation of state for hot nuclear matter. We have reported some results concerning the value of the adiabatic index Γ_S for such a system. We have found that most of the previously discussed approximations [12] seem indeed justified, at least qualitatively. The solution of Saha's equations for a system with leptons, drip baryons, and nuclei which are treated including thermal excitations [13], leads to an EOS with interesting features. Particularly, the behavior of Γ_S shows a trend which is similar to the one required by currently adopted supernovae models [11]. Also, the present EOS results, for the temperature-dependent liquid drop, leads to cooler adiabats at high densities.

ACKNOWLEDGMENTS

This work has been partially supported by the Consejo Nacional de Investigaciones Cientificas y Tecnicas (CONICET) and the Comision de Investigaciones Cientificas de la Pcia de Bs.As (CIC). O.G.B. is a member of the CIC and O.C. and M.R. are members of the CONICET.

[1] J. M. Lattimer, Annu. Rev. Nucl. Part. Sci. **31**, 337 (1981).
[2] J. M. Lattimer and D. G. Ravenhall, Astrophys. J. **223**, 314 (1978).
[3] D. Q. Lamb, J. M. Lattimer, C. J. Pethick, and D. G. Ravenhall, Phys. Rev. Lett. **41**, 1623 (1978).
[4] D. Q. Lamb, J. M. Lattimer, C. J. Pethick, and D. G. Ravenhall, Nucl. Phys. **A360**, 459 (1981).
[5] J. M. Lattimer, C. J. Pethick, D. G. Ravenhall and D. Q. Lamb, Nucl. Phys. **A432**, 646 (1985).
[6] J. M. Lattimer and F. Douglas Swesty, Nucl. Phys. **A535**, 331 (1991).
[7] P. Bonche and D. Vautherin, Nucl. Phys. **A372**, 496 (1981).
[8] H. A. Bethe, G. B. Brown, J. Cooperstein, and J. R. Wilson, Nucl. Phys. **A403**, 625 (1983).
[9] H. A. Bethe, G. E. Brown, J. Applegate, and J. M. Lattimer, Nucl. Phys. **A324**, 487 (1979).
[10] G. Baym, H. A. Bethe, and G. E. Brown, Nucl. Phys. **A375**, 481 (1982).
[11] H. A. Bethe, Rev. Mod. Phys. **62**, 801 (1991) and refer-

ences therein.
[12] J. Cooperstein, Nucl. Phys. **A438**, 722 (1985).
[13] C. Guet, E. Strumberger, and M. Brack, Phys. Lett. B **205**, 427 (1988).
[14] O. G. Benvenuto, O. Civitarese, and M. Reboiro (unpublished).
[15] D. Vautherin and D. M. Brink, Phys. Rev. C **5**, 626 (1972).
[16] A. Gilbert and A. G. W. Cameron, Can. J. Phys. **43**, 1446 (1965).
[17] D. D. Clayton, *Principles of Stellar Evolution and Nucleosyntesis* (University of Chicago Press, Chicago, 1983), pp. 356–360.
[18] Aa Bohr and B. Mottelson, *Nuclear Structure* (Benjamin, New York, 1969), Vol. I, Chap. 2.