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Single β^+ decay in ^{26}Mg: a comparison between the shell model and quasiparticle random phase approximation*

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Abstract. Single β-decay strength distributions, for β^+ transitions in ^{26}Mg, are calculated using effective two-body interactions in the framework of the shell model and quasiparticle random phase approximation (QRPA). It is found that the QRPA, if treated in a consistent way, yields centroids for the strength distributions and predictions for the total strength, which are in good agreement with the shell model results. The detailed structure of the strength distribution, however, is different since the shell model generates more states than the QRPA. This conclusion turns out to be independent of the effective nucleon–nucleon interaction used.

1. Introduction

The theoretical analysis of single β-decay transitions is of interest because, among other reasons, it could lead to a better understanding of more rare events like, for example, double β-decay processes, in which β^- and β^+ decays occur as virtual steps. Theoretical studies of double β-decay modes strongly rely upon several nuclear structure approximations [1–8], most of which are based on mean field and collective excitation models. It would be desirable to compare these approximations with full shell model treatments, in order to test their validity. Such a comparison is still unfeasible for heavy nuclei. Moreover, the few data available on double β-decay modes [9–12], do not allow for a systematic study of relevant nuclear structure effects, except for the well established suppression [1–8]. However, single β^- and β^+ transitions have been better studied, both theoretically [13, 14] and experimentally [15]. Therefore, one can draw some conclusions about the validity of models frequently used in double β-decay studies [1–8], like the quasiparticle random phase approximation (QRPA), by comparing their predictions with those of the shell model (SM)on single β^- and β^+ transitions. Such a comparison can only be meaningful if the two types of calculation are performed in the same model space and use the same two-body interaction.

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Since SM calculations of nuclear wavefunctions and decay rates for charge-exchange transitions cannot be performed for heavy nuclei, the results of approximation schemes, like QRPA, must be compared with those of an exact SM calculation for cases in which both calculations can be performed. We have chosen the case of β^+ transitions in ^{26}Mg for such a comparison. Another reason for the selection of the s–d shell is that for this model space several types of effective nucleon–nucleon (NN) interactions are available. Thus there are two-body matrix elements which have either been determined empirically, trying to reproduce a large amount of experimental data in an SM calculation [16], or been derived directly from a realistic NN interaction [17].

While β^- transitions are well reproduced, for the case of heavy nuclei and in the context of the QRPA approach [18], β^+ transitions are less well described. This is due to Pauli suppression, which means that the results to some extent are more sensitive to the nuclear structure model involved than the β^- transitions. Therefore a comparison between QRPA and SM predictions on β^+ transitions could be of some use in dealing with further analysis of single and double β-decay processes.

In order to test the accuracy of a certain approximation in a nuclear structure calculation, it is important to study the approximation for a realistic NN interaction and to demonstrate that the conclusions are independent of the detailed structure of this interaction. Therefore, we have performed SM and QRPA calculations by using two different interactions.

(i) The A-dependent interaction determined by Wildenthal [16] (hereafter denoted as WI). For this interaction SM calculations have been performed for all nuclei in the s–d shell adjusting all matrix elements of the effective Hamiltonian so that the energies of low-lying nuclear states are reproduced. In order to obtain a good fit for all nuclei throughout the entire s–d shell, a smooth A-dependence of the two-body matrix elements has been considered. We are using the matrix elements for $A = 26$.

(ii) The two-body interaction GA derived and tabulated in [17]. This effective interaction GA has been derived from a modern version of the one-boson-exchange potential fitted to describe NN scattering phaseshifts. The effects of configurations outside the s–d SM space have been taken into account using a non-perturbative approach.

A systematic comparison between QRPA and SM results for single β-decay transitions in the s–d shell, has been previously reported by Lauritzen [14]. However, there are differences in the manner in which our results and those of [14] were obtained. Thus, we performed our SM and QRPA calculations of β^+ strength functions starting from the same single-particle basis, using the same interaction and taking the corresponding SM corrected single-particle energies and occupation factors as input for the QRPA calculations. Moreover, we have performed state-dependent BCS calculations to determine the BCS occupation factors and quasiparticle energies by using pairing channels of the same interaction which is used to generate the excited states connected by the charge-exchange transitions.

Details of these calculations are described in sections 2 and 3. We have found that the QRPA results, for calculations corresponding to both WI and GA interactions, do indeed show that the strength and energy centroids for β^+ transitions agree fairly well with the SM results. These results and a comparison between the features exhibited by the WI and GA forces are presented and discussed in section 3. Finally, our conclusions are summarized in section 4.

2. Formalism

In this section we are going to describe the main steps which we have followed in the present work. First we shall briefly describe the SM approach which we have applied to treat the interactions WI and GA. Next we shall introduce the associated QRPA equations and the expression for the strength functions.

The first step of our SM study corresponds to a conventional SM calculation for the ground state of ^{26}Mg($J = 0, T = 1$) in a basis of SM configurations coupled to angular momentum J and isospin T. Denoting this state by Ψ_0 one can calculate single-particle occupation probabilities for the protons in state i

$$n_{\mathrm{occ}}(i,\mathrm{p}) = \langle \Psi_0 | c^\dagger_{i,\mathrm{p}} c_{i,\mathrm{p}} | \Psi_0 \rangle$$
$$v^{\mathrm{SM}}_{i,\mathrm{p}} = \sqrt{\frac{n_{\mathrm{occ}}(i,\mathrm{p})}{2j_i+1}} \tag{1}$$

where $c^\dagger_{i,\mathrm{p}}$ and $c_{i,\mathrm{p}}$ refer to the particle creation and annihilation operator for a proton (index p) in state i, and $2j_i+1$ stands for the degeneracy of the state i. Corresponding equations hold for the neutrons (index n). If now we define the transition operator for the β^+ strength by

$$O_{\beta+} = \sum_{i,j} \frac{\langle i||\sigma||j\rangle}{\sqrt{3}} [c^\dagger_{i,\mathrm{n}} c_{\bar{j},\mathrm{p}}]^{J=1} \tag{2}$$

one can calculate a state

$$|\Psi_1\rangle = O_{\beta+}|\Psi_0\rangle. \tag{3}$$

This state is a state of the daughter nucleus ^{26}Na, which carries the whole β^+ strength but which is not an eigenstate of the Hamiltonian. Now starting from the state Ψ_1 one can perform a Lanczos iteration scheme to evaluate eigenstates of the Hamiltonian which carry β^+ strength. This scheme turned out to be very efficient for the calculation of strength distributions [19, 20].

We now turn to the discussion of the QRPA approach. Nuclear structure factors entering in the calculation of single β-decay rates [6] are given by matrix elements of the one-body transition density

$$\rho_{\mathrm{pn}}(J^\pi,\omega_k) = \langle J^\pi,\omega_k||[c^\dagger_{\mathrm{p}} c_{\bar{\mathrm{n}}}]^J||0^+\rangle \tag{4}$$

and

$$\rho_{\mathrm{pn}}(J^\pi,\omega_k) = \langle J^\pi,\omega_k||[c^\dagger_{\mathrm{n}} c_{\bar{\mathrm{p}}}]^J||0^+\rangle \tag{5}$$

for β^- and β^+ transitions, respectively. After performing quasiparticle transformations [21, 22] and representing the nuclear wavefunction of the states $|J^\pi,\omega_k\rangle$ by linear combinations of unlike (proton–neutron) quasiparticle pair excitations, as they are given by the QRPA [18], these one-body transition densities can be written as follows

$$\rho_{\mathrm{pn}}(J^\pi,\omega_k) = \sqrt{2J+1}(u_{\mathrm{p}} v_{\mathrm{n}} X_{\mathrm{pn}}(J^\pi,\omega_k) + v_{\mathrm{p}} u_{\mathrm{n}} Y_{\mathrm{pn}}(J^\pi,\omega_k)) \tag{6}$$

and

$$\rho_{\rm pn}(J^\pi,\omega_k) = \sqrt{2J+1}(u_{\rm n}v_{\rm p}X_{\rm pn}(J^\pi,\omega_k) + v_{\rm n}u_{\rm p}Y_{\rm pn}(J^\pi,\omega_k)) \tag{7}$$

for β^- and β^+ decays, respectively. Here J^π represents angular momentum and parity ($J^\pi = 1^+$) while ω_k the energy of the excited states of the final nucleus which are involved in the transition from the ground state of the initial nucleus $|0^+\rangle$. BCS occupation factors are denoted by u and v. $X_{\rm pn}(J^\pi,\omega_k)$ ($Y_{\rm pn}(J^\pi,\omega_k)$) are forward (backward)-going amplitudes corresponding to the QRPA structure of the final states.

The QRPA treatment of a given Hamiltonian H, defined as the sum of a single particle, $H_{\rm sp}$ and of a two-body, V, terms is performed in two steps, namely: (a) by solving the state-dependent BCS equations [23] for the pairing sector of H; and (b) by transforming the remaining part of H to the resulting quasiparticle basis and diagonalizing it in a two quasiparticle basis.

The matrix elements of the two-body interaction, V, are usually given in an angular momentum–isospin coupled representation. The transformation of these matrix elements to a particle–hole and particle–particle representation is given by the well-known expressions

$$\langle j_1({\rm p})j_2({\rm n}) : J|V|j_3({\rm p})j_4({\rm n}) : J\rangle = \frac{1}{2}\sum_{T=0,1}\langle j_1 j_2 : JT|V|j_3 j_4 : JT\rangle \times \sqrt{(1+\delta_{j_1 j_2})(1+\delta_{j_3 j_4})} \tag{8}$$

for the proton–neutron particle–particle configurations and

$$\langle j_1({\rm p})j_2^{-1}({\rm n}) : J|V|j_3({\rm p})j_4^{-1}({\rm n}) : J\rangle = -\sum_{J'}(2J'+1)\times \begin{Bmatrix} j_1 & j_4 & J' \\ j_3 & j_2 & J \end{Bmatrix} \times \langle j_1({\rm p})j_4({\rm n}) : J'|V|j_3({\rm p})j_2({\rm n}) : J'\rangle \tag{9}$$

for the proton–neutron particle–hole matrix elements, respectively. The particle–particle isospin-dependent matrix elements $\langle j_1 j_2 : JT|V|j_1' j_2' : JT\rangle$ are antisymmetrized and normalized.

The starting values for the bare single-particle energies, $\epsilon_i^{(0)}$, which are given with respect to the ^{16}O core of the s–d SM calculation, are shifted by a state-dependent correction ${\rm Tr}(V\rho)_i$, where ρ is the one-body density associated with H; we have expressed corrected single-particle energies, ϵ_i, by

$$\epsilon_i = \epsilon_i^{(0)} + \frac{1}{2(2j_i+1)}\sum_{j,J,T}\langle ij : JT|V|ij : JT\rangle\frac{(2J+1)}{(2j_j+1)}n_{\rm occ}(j)(1+\delta_{ij}) \tag{10}$$

where $n_{\rm occ}(j)$ are the SM occupation probabilities defined in equation (1).

With these corrected single-particle energies one can then solve state-dependent BCS equations [23] in order to determine BCS occupation factors u_i and v_i and quasiparticle energies E_i. This is done by taking pairing ($T = 1$) channels of the SM interaction V. Therefore the state-dependent BCS equations are given by

$$\Delta_i = \sum_k \sqrt{\frac{2j_k+1}{2j_i+1}}u_k v_k V|kk : 0,1\rangle \tag{11a}$$

$$E_i = \sqrt{(\epsilon_i - \lambda)^2 + \Delta_i^2} \tag{11b}$$

$$2u_i v_i = \frac{\Delta_i}{E_i} \tag{11c}$$

$$u_i^2 - v_i^2 = \frac{\epsilon_i - \lambda}{E_i} \tag{11d}$$

with the matrix element of the NN interaction in equation (11*a*) referring to antisymmetrized and normalized matrix elements introduced earlier. The Fermi energies λ are adjusted independently for protons (λ_p) and neutrons (λ_n) such that the particle numbers for the two kinds of nucleons agree with those of the nucleus under consideration.

Finally, QRPA matrix equations [1, 2, 4] can be solved by defining the block-matrices:

$$A_{\mathrm{pn,p'n'}} = \delta(\mathrm{pp'})\delta(\mathrm{nn'})[\tilde{E}_\mathrm{p} + \tilde{E}_\mathrm{n}] + g_{\mathrm{pp}} G(\mathrm{pn,p'n'},J) \times (u_\mathrm{p}u_{\mathrm{p'}}u_\mathrm{n}u_{\mathrm{n'}} + v_\mathrm{p}v_{\mathrm{p'}}v_\mathrm{n}v_{\mathrm{n'}}) + g_{\mathrm{ph}} F(\mathrm{pn,p'n'},J) \times (u_\mathrm{p}u_{\mathrm{p'}}v_\mathrm{n}v_{\mathrm{n'}} + v_\mathrm{p}v_{\mathrm{p'}}u_\mathrm{n}u_{\mathrm{n'}}) \tag{12a}$$

$$B_{\mathrm{pn,p'n'}} = g_{\mathrm{pp}} G(\mathrm{pn,p'n'},J) \times (u_\mathrm{p}v_{\mathrm{p'}}u_\mathrm{n}v_{\mathrm{n'}} + v_\mathrm{p}u_{\mathrm{p'}}v_\mathrm{n}u_{\mathrm{n'}}) - g_{\mathrm{ph}} F(\mathrm{pn,p'n'},J) \times (u_\mathrm{p}v_{\mathrm{p'}}v_\mathrm{n}u_{\mathrm{n'}} + v_\mathrm{p}u_{\mathrm{p'}}u_\mathrm{n}v_{\mathrm{n'}}) \tag{12b}$$

where $G(\mathrm{pn,p'n'})$ and $F(\mathrm{pn,p'n'})$ are shorthand notations for the particle–particle and particle–hole proton–neutron matrix elements, equations (8)–(9), respectively, and g_{pp} (g_{ph}) are renormalization factors corresponding to these channels. The quasiparticle energies $\tilde{E}_p$ and $\tilde{E}_n$ for the protons and neutrons have to be readjusted in such a way that the Fermi energies for protons and neutrons correspond to the same energy. For neutron–particle proton–hole states this can be achieved by evaluating

$$\begin{aligned} \tilde{E}_\mathrm{p} &= E_\mathrm{p} - \lambda_\mathrm{p} + \lambda_\mathrm{n} \\ \tilde{E}_\mathrm{n} &= E_\mathrm{n} \end{aligned} \tag{13}$$

with quasiparticle energies E and Fermi energies λ as obtained from the solution of the BCS equation (11). General properties of the QRPA approach can be found in textbooks [21, 22]. We have applied this technique for the case of unlike quasiparticle pairs $A^\dagger_{\mathrm{pn}}(JM) = [a^\dagger_\mathrm{p} a^\dagger_\mathrm{n}]^{JM}$ and in this basis we have defined phonon creation operators

$$\Gamma^\dagger_{\omega_k}(JM) = \sum_{\mathrm{pn}} [X_{\mathrm{pn}}(J^\pi,\omega_k) A^\dagger_{\mathrm{pn}}(JM) - Y_{\mathrm{pn}}(J^\pi,\omega_k) A_{\mathrm{pn}}(\overline{JM})]. \tag{14}$$

With these matrix elements we can thus solve the corresponding QRPA eigenvalue problem and obtain the wavefunctions and eigenvalues associated with excited J^π states in the final nucleus.

We can now introduce the definitions which are needed to compute unperturbed and QRPA strength functions for single β-decay transitions. The unperturbed strength functions in the BCS approximation are given by

$$S_{\beta^-}(\mathrm{BCS}) = \sum_{\mathrm{pn}} |\langle \mathrm{p}||\sigma||\mathrm{n}\rangle|^2 u_\mathrm{p}^2 v_\mathrm{n}^2 \tag{15}$$

$$S_{\beta^+}(\mathrm{BCS}) = \sum_{\mathrm{pn}} |\langle \mathrm{p}||\sigma||\mathrm{n}\rangle|^2 v_\mathrm{p}^2 u_\mathrm{n}^2. \tag{16}$$

The QRPA strength functions are defined by folding the one-body transition density, equations (4)–(5), with the QRPA wavefunctions:

$$S_{\beta-}(\mathrm{QRPA}) = \sum_{\omega_k}\left\{\sum_{\mathrm{pn}}\langle \mathrm{p}||\sigma||\mathrm{n}\rangle(u_{\mathrm{p}}v_{\mathrm{n}}X_{\mathrm{pn}}(J^{\pi},\omega_k) + u_{\mathrm{n}}v_{\mathrm{p}}Y_{\mathrm{pn}}(J^{\pi},\omega_k))\right\}^2$$
$$S_{\beta+}(\mathrm{QRPA}) = \sum_{\omega_k}\left\{\sum_{\mathrm{pn}}\langle \mathrm{p}||\sigma||\mathrm{n}\rangle(u_{\mathrm{n}}v_{\mathrm{p}}X_{\mathrm{pn}}(\omega_k) + u_{\mathrm{p}}v_{\mathrm{n}}Y_{\mathrm{pn}}(\omega_k))\right\}^2. \tag{17}$$

From (15)–(17) we can compute the strength sum (Gamow–Teller sum rule) and it can be expressed as

$$S_{\beta-} - S_{\beta+} = 3(N-Z) \tag{18}$$

which is preserved by the QRPA.

3. Results and discussion

Results for the occupation factors of the single-particle states calculated in the SM approach (see equation (1)) and the corrected single-particle energies calculated with the occupation probabilities (equation (10)) are listed in table 1. These numbers show a significant dependence on the interaction used. For the protons as well as for the neutrons one observes a large occupation probability and a large binding energy for the $d_{3/2}$ state if the interaction GA is used. The $d_{3/2}$ state turns out to be even more bound than the $1s_{1/2}$ state. This might be an artifact of the interaction GA. Actually this interaction has been derived microscopically for SM calculations in the beginning of the s–d shell. It yields very reasonable results for those nuclei [17] but has not yet been used to evaluate nuclear states in the middle of the shell. For our present investigation, however, it is quite useful to test the agreement of QRPA and the SM using two interactions (GA and WI), which display different features.

Table 1. Shell model. Corrected single particle energies, ϵ_i (in MeV) (see equation (10)), and SM occupation factors v_i^{SM}, (see equation (1)), for proton and neutron states in the s–d shell. Results indicated by GA and WI correspond to calculations performed with the GA [17] and WI [16] interactions, respectively.

	Neutrons		Protons	
State	ϵ_i (MeV)	v_i^{SM}	ϵ_i (MeV)	v_i^{SM}
Interaction WI				
$d_{5/2}$	−10.15	0.891	−6.78	0.728
$d_{3/2}$	−4.53	0.415	−1.33	0.351
$1s_{1/2}$	−7.21	0.523	−4.47	0.406
Interaction GA				
$d_{5/2}$	−12.78	0.778	−10.32	0.675
$d_{3/2}$	−9.89	0.686	−7.22	0.466
$1s_{1/2}$	−8.50	0.489	−6.81	0.449

As outlined earlier we have solved state-dependent BCS equations (equations (11)) in order to determine BCS occupation factors and quasiparticle energies from the same interaction which enters the SM calculation. Our method of obtaining the BCS factors is different from the procedure followed by Lauritzen [14]. Thus in the calculation of [14] effective gap parameters were determined from the observed odd-even mass differences and then the values of these parameters were used to calculate quasiparticle energies and occupation factors. In our method we try to treat pairing effects consistently, both in the SM and in the QRPA calculation. The values obtained for the occupation factors $v_p(v_n)$, for the quasiparticle energies $E_p(E_n)$ and for the BCS gaps Δ are listed in table 2.

Table 2. BCS calculations. Quasiparticle energies E_j (MeV), gap parameters Δ_j (MeV) and BCS factors occupation factors v_j for the proton and neutron states in the s–d shell (see equation (11)). Results labelled with WI and GA correspond to the WI and GA interactions, respectively. The Fermi energies obtained for WI are −8.240 MeV for the neutrons (−6.433 for protons) while the corresponding values for GA are −11.159 MeV (−10.170).

	Neutrons			*Protons*		
State	E_i (MeV)	Δ_i (MeV)	v_i^{BCS}	E_i (MeV)	Δ_i (MeV)	v_i^{BCS}
Interaction WI						
$d_{5/2}$	2.493	1.604	0.940	1.976	1.945	0.767
$d_{3/2}$	4.137	1.831	0.227	5.609	2.340	0.215
$1s_{1/2}$	2.042	1.766	0.499	2.762	1.943	0.380
Interaction GA						
$d_{5/2}$	3.217	2.780	0.867	2.641	2.636	0.727
$d_{3/2}$	3.540	3.304	0.566	4.482	3.377	0.414
$1s_{1/2}$	3.353	2.044	0.322	3.902	1.989	0.264

Comparing the occupation factors obtained in the BCS approach with those resulting from the SM calculation, one finds that the depletion of the lowest shell ($d_{5/2}$) is slightly smaller in the BCS than in the SM approach. This indicates that the SM describes correlations in addition to the BCS, which enhance the depletion of the low-lying states and in general smear out the occupation probability around the Fermi surface. The general agreement between the SM and the QRPA, however, is fairly good. If one considers the dependence of the results on the interaction, similar features are observed to those discussed earlier for the SM. In addition the occupation factors obtained in the BCS approach using the interaction GA show quite a strong population for the $d_{3/2}$ state. This can be traced back to diagonal matrix elements between nucleons in $d_{5/2}$ and $d_{3/2}$ states, which are more attractive for GA than for the WI interaction. Furthermore one finds that the pairing gaps obtained are slightly larger for the interaction GA indicating stronger pairing components for this interaction.

QRPA strength distributions, for β^+ transitions in ^{26}Mg, are shown in figure 1. In performing these calculations we have fixed the renormalization factors g_{pp} and g_{ph} at the value: $g_{pp} = g_{ph} = 1.0$, which means that we have used the SM interaction without any renormalization. In the same figure QRPA results are also compared with results obtained with the quasiparticle Tamm–Dancoff approximation (QTDA). The s–d shell allows for seven proton–neutron two-quasiparticle configurations with angular momentum $J = 1$. As one can see from figure 1, the β^+ strength is concentrated

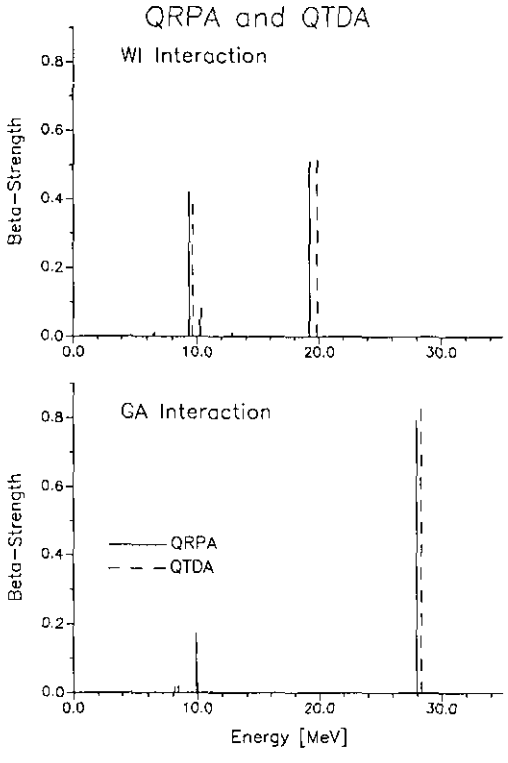

Figure 1. QRPA β^+ strength distribution $S_{\beta+}$, in fractions of the total strength, for transitions in ^{26}Mg. Results in full lines correspond to QRPA while those obtained in QTDA are presented by broken lines. The upper part of the figure shows results obtained for the interaction WI [16], and the lower part has been obtained assuming the GA [17] interaction. Force: $g_{pp} = g_{ph} = 1.0$.

essentially in two states which are dominated by the superposition of the proton–$d_{5/2}$, neutron–$d_{3/2}$ and the proton–$d_{3/2}$, neutron–$d_{5/2}$ two-quasiparticle configurations.

The energies for the QRPA states which carry the β^+ strength are to a large extent determined by the residual interaction contained in the QTDA matrices $A_{pn,p'n'}$ of equation (12*a*). Note that the sum of the quasiparticle energies $\tilde{E}$ for the two dominant configurations discussed earlier is 4.31 and 6.29 MeV for the interaction WI (5.19 and 6.71 MeV for GA) while the QRPA energies are at 9.27 and 18.63 MeV (9.95 and 27.94 MeV for GA). The stronger interaction GA leads to a larger splitting and a shift to higher energies for the β^+ strength.

The small differences between QRPA and QTDA results are to be attributed to the fact that we are dealing with a light nucleus and that the effect of backward moving QRPA amplitudes on the calculated energies should be expected to be of minor importance. This is not a fault of the QRPA approach, as it has been suggested in [14], but rather it is due to the small dimension of the proton–neutron configuration space used in the calculations.

Accumulated strength functions:

$$S(\beta^+, E) = \frac{\sum_{\omega_k \leq E} S(\beta^+, \omega_k)}{\sum_{\omega_k} S(\beta^+, \omega_k)} \tag{19}$$

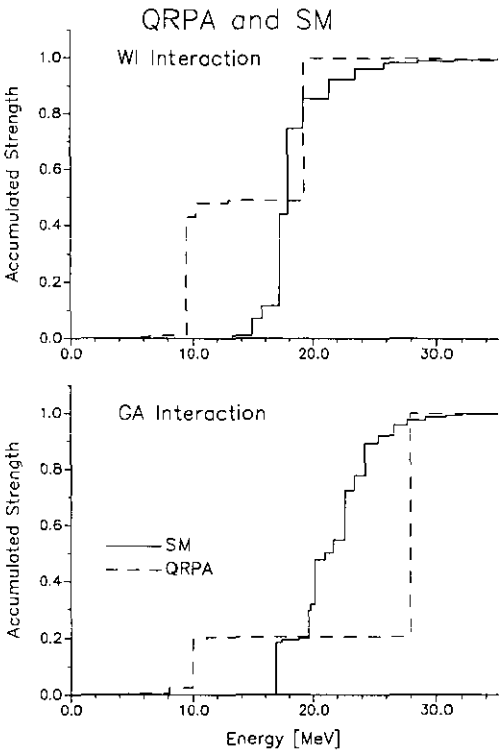

Figure 2. Accumulated strength function, $S(\beta^+, E)$ (see equation (19)), as a function of the energy, for SM and QRPA treatments of the WI [16] (upper part) and GA [17] interactions. The strength function obtained in the SM approach are presented by the full line, while the QRPA results are shown by the broken line.

are shown in figure 2, where they are compared with the SM results. From the results displayed in figure 2 we can observe that the β^+ strength is distributed over many more states in the SM than in the QRPA approach. This is to be expected from the fact that the SM considers many more configurations than those contained in the QRPA. Already from this figure, however, one can observe that general features for the calculated strength distribution obtained in the SM are reproduced by the QRPA. To study this on a more quantitative level we define energy centroids by:

$$\langle E \rangle = \frac{\sum_{\omega_k} \omega_k S(\beta^+, \omega_k)}{\sum_{\omega_k} S(\beta^+, \omega_k)}. \tag{20}$$

Results for these centroids obtained from the SM, the QRPA, the QTDA and for two-quasiparticle states without residual interaction are listed in table 3. One observes that the inclusion of QTDA effects increases the value for the centroids quite drastically compared with the unperturbed values and brings those values in the neighbourhood of the values calculated by the SM. As already discussed, the energies are not very much affected by including the QRPA correlations compared with the QTDA.

In contrast, the calculated total transition strength cannot be affected by performing the QTDA diagonalization. Therefore the QTDA result for the β^+ strength is identical to the one obtained in the BCS approximation (equation (16)). Note, that

Table 3. Results for β^+ transitions. The total transition strength S (see equations (16) and (17)) and energy centroids $\langle E \rangle$ (MeV) (see equations (20)) for β^+ transitions of ^{26}Mg calculated in the BCS, the QTDA, the QRPA and the SM are listed. The results were obtained for the interactions WI and GA considering the full strength of the interaction: $g_{pp} = g_{ph} = 1.0$.

	WI		GA	
Method	S	$\langle E \rangle$ (MeV)	S	$\langle E \rangle$ (MeV)
BCS	6.741	4.11	5.618	5.40
QTDA	6.741	14.88	5.618	25.12
QRPA	4.739	14.42	4.535	24.20
SM	3.407	18.28	4.162	21.73

in the most naive SM, which assumes that protons and neutrons are only occupying the $d_{5/2}$ shell, the β^+ strength for ^{26}Mg is given as 6.4. This means that the inclusion of BCS correlations alone yields a slight enhancement for the interaction WI but a decrease for GA. The QRPA correlations yield a sizable reduction for the total β^+ strength compared with the unperturbed BCS values, independent of the interaction used. This improves the agreement with the SM results considerably (see table 3). The SM result, however, is even more reduced, because the SM contains even more correlations than those taken into account in the QRPA (see also the discussion of the occupation factors at the beginning of this section).

To conclude the discussion of our results, let us comment on the dependence of QRPA results on the variation of the renormalization parameter g_{pp}. As it has been extensively discussed in [1, 2, 7, 8, 24, 25], QRPA results are very sensitive to the variation of g_{pp} in the vicinity of the unrenormalized value $g_{pp} = 1.0$. The β^+ strength to low-lying states has been found to decrease with an increase of g_{pp} [13], when this decay is calculated as a virtual channel. The same feature also persists in our model calculation. However, reducing g_{pp} by 10% leads to a reduction in the total β^+ strength by around 4% for both interactions used in the present investigation. A similar feature is also observed for the energy centroid of the β^+ distribution. In this case the reduction in the particle–particle interaction by 10% would also give a reduction in the calculated energy centroid of around 4% for both interactions used in our investigation. Note that neglecting the particle–particle interaction completely (by putting g_{pp}=0.0) would lead to an energy centroid in a QRPA calculation with the WI interaction of only 8.37 MeV.

4. Conclusions

The distribution of strength for the β^+ transition ^{26}Mg to ^{26}Na is evaluated in the QRPA considering the model space of the s–d shell. For this small model space one can also perform exact SM calculations and estimate the accuracy of the QRPA approach by a comparison of the results with those obtained in the SM calculation. For such a comparison we used the same Hamiltonian in both the QRPA and the SM calculation. One finds that the energy centroid of the calculated β^+ distribution is affected in particular by the residual interaction taken into account in the QTDA approximation. Assuming the empirical interaction fitted by Wildenthal [16] the energy centroid is shifted by 10 MeV whereas the use of another realistic interaction [17] even yields

19 MeV of repulsion. The ground-state correlations taken into account by the QRPA have only a small effect on the energy centroid but reduce the total strength by around 30%.

The QRPA method, when consistently applied to describe correlations induced by a residual two-body interaction, gives results which show similar features to those obtained in the SM, in spite of the severe truncation of the configuration space which is inherent in the QRPA model. It gives a reasonable description of the strength functions and the agreement between the calculated QRPA and SM energy centroids for the transitions also gives some confidence in the QRPA method. It should be pointed out that this comparison has been made for a light nucleus, because SM calculations can only be made for such systems. One may expect, however, that the relative importance of the collective degrees of freedom, which are considered in the QRPA approach, may be larger for heavier nuclei. Therefore the predictive power of the QRPA might even be better for such systems than exhibited in our present comparison.

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