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# Nuclear structure calculation of the two-neutrino $\beta\beta$ decay transition $^{100}Mo \rightarrow {}^{100}Ru$

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**Abstract.** The half-life for the $2\nu\beta\beta$ decay transition in $^{100}Mo$ is calculated by using a conventional proton–neutron quasiparticle random phase approximation method and a recently proposed particle number projected quasiparticle random phase approximation formalism. The calculations of the relevant matrix elements have been performed by using a realistic effective two-body interaction constructed from the Bonn one-boson exchange potential. Suppression of the $2\nu\beta\beta$ decay matrix element is found both in the unprojected and projected models. The collapse of the quasiparticle random phase approximation formalism, induced by renormalized particle–particle interactions, is found to play an important role in dealing with the numerical stability of the results. The particle number projected results are found to be more stable than those corresponding to the unprojected formalism although the particle number projection does not suffice for a complete elimination of spurious ground-state correlations near the collapse of the quasiparticle random phase approximation.

## 1. Introduction

Theoretical and experimental analyses of $2\nu\beta\beta$ decay transitions have been receiving considerable attention [1–14], particularly in view of the consequences of these studies upon currently adopted concepts of the theory of weak interactions as well as for the information which can be extracted about the nature of the participant neutrinos [15]. Since all the theoretical concepts which are involved in the physics of the nuclear double beta decay have been reviewed recently [15, 16] we shall avoid discussing them here and we shall concentrate our attention on the nuclear structure problem associated with the theoretical estimate of the $2\nu\beta\beta$ decay transition in $^{100}Mo$. The reader is kindly referred to the already published work for further details about the current status of the experiments [17] as well as on the models [15–17] which have been applied. A common feature of the experimental data about $2\nu\beta\beta$ decay transitions is the large order of magnitude of the corresponding half-lives, which are in the range $10^{21}$–$10^{24}$ years [10–14, 17]. This means that the associated nuclear matrix elements are strongly suppressed. This suppression effect has been studied in considerable detail [2–8] and various microscopic mechanisms have been proposed in order to explain it. This effect has been discussed first by the group at Caltech, in the context of schematic two-body interactions [5]. We have

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shown that this suppression is related to the renormalization of particle–particle channels of the residual proton–neutron interaction and that a realistic effective two-body interaction, like the interaction constructed from the Bonn one-boson exchange potential [18], could indeed provide the necessary strength for this attractive particle–particle channel to become operative [6]. Furthermore, this suppression has been found to be present in models which are qualitatively different from the quasiparticle random phase approximation (QRPA) [7, 8]. Recently, we have presented a particle number projected formalism of the QRPA(PQRPA) [19, 20] wherein the suppression of $2\nu\beta\beta$ nuclear matrix elements is also found. Therefore little can be said about the underlying physics except for the fact that QRPA calculations with the inclusion of renormalized particle–particle channels of the residual proton–neutron interaction could explain the above mentioned suppression. In order to determine physical values for the associated coupling constants, studies of single beta decay transitions have been conducted [21] which have shown that the required renormalization was indeed compatible with the findings of the two neutrino double beta decay calculations [6, 19, 20]. In this work we would like to discuss the case of the $2\nu\beta\beta$ decay mode in $^{100}$Mo, since data are available which show that for this case the half-life is much shorter than for other $2\nu\beta\beta$ decay transitions [22–25]. The essentials of the formalism which we have used are briefly described in section 2 and the results of our calculations are presented and discussed in section 3. Conclusions are drawn in section 4.

## 2. Formalism

The half-life for a $2\nu\beta\beta$ decay transition can be written in a factorized form which includes leptonic and nuclear contributions [26]

$$[\tau_{1/2}(2\nu\beta\beta)]^{-1} = F\,|M_{GT}|^2 \tag{1}$$

where $F$ is a leptonic phase space integral and $M_{GT}$ is the nuclear matrix element [6]

$$M_{GT} = \sum_{jk} \frac{\langle 0_f^+ || \tau^+\sigma || 1_k^+ \rangle \langle 1_k^+ | 1_j^+ \rangle \langle 1_j^+ || \tau^+\sigma || 0_i^+ \rangle}{M_e c^2 + \frac{1}{2}Q_{\beta\beta} + E_j - E_i}. \tag{2}$$

In equation (2) we have used standard notation [6]. It includes QRPA amplitudes for the intermediate $1^+$ states which are described as excitations of the initial and final nuclei. The vacuum for these states is a correlated vacuum and the QRPA assumes that the $1^+$ states constructed from the initial nucleus and the ones constructed from the final one are indeed the same. This is of course not true but the consistency of the approximation is enforced by the overlap factors $\langle 1_k^+ | 1_j^+ \rangle$ included in equation (2). Since the proton–neutron excitations of the intermediate odd–odd nucleus are described in terms of unlike (proton–neutron) quasiparticle pair excitations, spurious effects associated with the violation of the particle number symmetry have to be eliminated. In order to cure for these spurious effects we have developed a formalism [19, 20] which is based on: (i) the restoration of the particle number symmetries at the level of the BCS approximation for the quasiparticle mean fields, by using particle number projection techniques [27, 28], and (ii) the construction of particle number conserving wavefunctions for the intermediate $1^+$ states in the odd–odd nucleus, by solving particle number projected QRPA equations (PQRPA) [20, 29]. It should be noted that in case (i) the violation of the particle number symmetry is a consequence of the adoption of the BCS quasiparticle mean fields to describe single particle degrees of freedom in presence of pairing correlations while in case (ii) the violation of the particle number is to be associated with the fact that the QRPA excitations describe not only an intermediate nucleus but rather a chain of nuclei with $N \pm 1$, $N \pm 3$, etc and $Z \pm 1$, $Z \pm 3$, etc. In order to carry on with the formulation of (i) and (ii), in the specific case of a nucleus with open shells both in protons and neutrons, let us write the final expression for the equations which we have solved. The formalism has been discussed in detail in our recent publications [19, 20] and we would like to refer the reader to these references for further details. The solution of number projected BCS state-dependent equations can be obtained once the projected norm, $I_0$, and projected energy, $I_E$, are calculated. The equations which have to be solved to determine particle number projected BCS occupation factors and quasiparticle energies are of the form:

$$I_0 \nabla(u_j, v_j) I_E - I_E \nabla(u_j, v_j) I_0 = 0, \tag{3}$$

where $\nabla(u_j, v_j)$ is given by

$$\nabla(u_j, v_j) = \frac{\partial}{\partial v_j} - \left(\frac{v_j}{u_j}\right) \frac{\partial}{\partial u_j}, \tag{4}$$

and the projected norm and energy integrals are given by

$$I_0 = \langle \mathrm{BCS} | \hat{P}_N | \mathrm{BCS} \rangle = \frac{1}{2\pi} \int_0^{2\pi} \mathrm{d}\phi \, \mathrm{e}^{\mathrm{i}N\phi} \prod_j (u_j^2 + v_j^2 \, \mathrm{e}^{-2\mathrm{i}\phi})^{\Omega_j} \tag{5a}$$

and

$$\begin{aligned} I_E = \frac{1}{2\pi} \int_0^{2\pi} \mathrm{d}\phi \, \mathrm{e}^{\mathrm{i}N\phi} \Big( & \sum_k 2\Omega_k \varepsilon_k v_k^2 \, \mathrm{e}^{-2\mathrm{i}\phi} f(k, \phi) \\ & - \sum_{kk'} \sqrt{\Omega_k \Omega_{k'}} \, G(kk, k'k', 0) v_k v_{k'} u_k u_{k'} \, \mathrm{e}^{-2\mathrm{i}\phi} f(k, k', \phi) \\ & - \sum_{kk'} \sum_J (2J+1) G(kk', kk', J) v_k^2 v_{k'}^2 u_k u_{k'} \, \mathrm{e}^{-4\mathrm{i}\phi} f(k, k', \phi) \Big). \end{aligned} \tag{5b}$$

In these equations the factors $f(k, \phi)$ are blocked BCS overlaps, $\phi$ is the gauge angle, the quantities $G(kk, k'k', 0)$ and $G(kk', kk', J)$ are particle–particle matrix elements of the two-body interaction, $N$ is the number of particles, $v_k$ and $u_k$ are BCS occupation factors, $\varepsilon_k$ are single particle energies and the index $k$ represents the full set of quantum numbers which are needed to define a single particle state. The projected functionals $I_0$ and $I_E$ can be obtained in analytical form and then we can perform the variation

$$\nabla(u_j, v_j) E_0^N = 0. \tag{6}$$

This variation determines the values of the occupation numbers $u_j$ and $v_j$ under the condition of particle number conservation. From it equation (3) can readily be obtained, since $E_0^N = I_E / I_0$. Next we have to solve the corresponding equations, as they are given by the QRPA model, to describe excitations in the basis of proton–neutron quasiparticle pairs coupled to $J^\pi = 1^+$. The conventional (unprojected) QRPA equations have been presented before [6, 21] and for the projected ones the formalism which we have developed prescribes the following structure for

$A$ and $B$ matrix elements [20]

$$\begin{aligned} A^{N-1,Z+1}_{\alpha,\beta} &= \langle \mathrm{BCS}(N,Z)|\, b_\alpha(JM)\hat{P}_{N-1}\hat{P}_{Z+1}[\hat{H}, b^\dagger_\beta(JM)]\,|\mathrm{BCS}(N,Z)\rangle \\ B^{N-1,Z+1}_{\alpha,\beta} &= \langle \mathrm{BCS}(N,Z)|\, b_\alpha(JM)\hat{P}_{N-1}\hat{P}_{Z+1}[\hat{H}, b_\beta(\overline{JM})]\,|\mathrm{BCS}(N,Z)\rangle \end{aligned} \tag{7}$$

and they can be written in terms of particle number projected overlaps

$$\begin{aligned} A^{N-1,Z+1}_{\alpha,\beta} = \sum_{pn}\sum_{p'n'} \eta^\alpha_{pn}\eta^\beta_{p'n'}\{&\delta(pp')\delta(nn')[E_p+E_n]I_{Z+1}(p)I_{N-1}(n) \\ &- g_{pp}G(pn,p'n',J)u_pu_{p'}u_nu_{n'}I_{Z+1}(\mathrm{e}^{\beta_p}N_{p'})I_{N-1}(\mathrm{e}^{\beta_n}M_{n'}) \\ &- g_{pp}G(pn,p'n',J)v_pv_{p'}v_nv_{n'}I_{Z+1}(\mathrm{e}^{\beta_p}M_{p'})I_{N-1}(\mathrm{e}^{\beta_n}M_{n'}) \\ &- g_{ph}F(pn,p'n',J)u_pu_{p'}v_nv_{n'}I_{Z+1}(\mathrm{e}^{\beta_p}N_{p'})I_{N-1}(\mathrm{e}^{\beta_n}M_{n'}) \\ &- g_{ph}F(pn,p'n',J)v_pv_{p'}u_nu_{n'}I_{Z+1}(\mathrm{e}^{\beta_p}M_{p'})I_{N-1}(\mathrm{e}^{\beta_n}N_{n'})\} \end{aligned} \tag{8a}$$

$$\begin{aligned} B^{N-1,Z+1}_{\alpha,\beta} = \sum_{pn}\sum_{p'n'} \eta^\alpha_{pn}\eta^\beta_{p'n'}\{&g_{pp}G(pn,p'n',J)(u_pv_{p'}u_nv_{n'}+v_pu_{p'}v_nu_{n'}) \\ &- g_{ph}F(pn,p'n',J)(u_pv_{p'}v_nu_{n'}+v_pu_{p'}u_nv_{n'})\} \\ &\times \tfrac{1}{2}[I_{Z+1}(p)I_{N-1}(n)+I_{Z+1}(p')I_{N-1}(n')] \end{aligned} \tag{8b}$$

where

$$\begin{aligned} I_{N_q}(\mathrm{e}^{\beta_l}M_k) &= \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi\, \mathrm{e}^{\mathrm{i}N_q\phi}\,\mathrm{e}^{\mathrm{i}\beta_l(\phi)}M_k(\phi)\langle\hat{S}_q(\phi)\rangle \\ I_{N_q}(\mathrm{e}^{\beta_l}N_k) &= \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi\, \mathrm{e}^{\mathrm{i}N_q\phi}\,\mathrm{e}^{\mathrm{i}\beta_l(\phi)}N_k(\phi)\langle\hat{S}_q(\phi)\rangle \\ I_{N-1}(n) &= \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi\, \mathrm{e}^{\mathrm{i}(N-1)\phi}\,\mathrm{e}^{\mathrm{i}\phi}M_n(\phi)\langle\hat{S}_N(\phi)\rangle \\ I_{Z+1}(p) &= \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi\, \mathrm{e}^{\mathrm{i}(Z+1)\phi}\,\mathrm{e}^{-\mathrm{i}\phi}N_p(\phi)\langle\hat{S}_Z(\phi)\rangle \end{aligned} \tag{9}$$

with

$$\begin{aligned} &\langle\hat{S}_q(\phi)\rangle = \prod_l [d_l(\phi)]^{\Omega_l} \\ &M_l(\phi) = \frac{\mathrm{e}^{2\mathrm{i}\phi}}{d_l(\phi)} \qquad N_l(\phi) = \frac{1}{d_l(\phi)} \\ &\mathrm{e}^{\beta_j(\phi)} = \frac{\mathrm{e}^{-\mathrm{i}\phi}}{u_j^2+v_j^2\,\mathrm{e}^{-2\mathrm{i}\phi}} \qquad d_l(\phi) = (u_l^2+v_l^2\,\mathrm{e}^{-2\mathrm{i}\phi})^{\Omega_l} \end{aligned} \tag{10}$$

with $\Omega_j = (2j+1)$, $q = N$ or $Z$, $N_q = N-1$ or $Z+1$.

In equation (7) we have denoted the particle number projected BCS ground state as $|\mathrm{BCS}(N,Z)\rangle$ and the structure of this particle number projected ground state is defined by two independent projections, namely: $|\mathrm{BCS}(N,Z)\rangle = \hat{P}_N\hat{P}_Z\,|\mathrm{BCS}\rangle$. The quantities $N$ and $Z$ correspond to the neutron and proton numbers, respectively, of the initial double-even nucleus. The second particle number projection, which has been indicated by the projection operators $\hat{P}_{N-1}\hat{P}_{Z+1}$ is performed in order to eliminate spurious components in the wavefunctions of the intermediate double-odd nucleus.

The quantities $\eta^{\alpha}_{pn}$ which appear in equations (8*a*) and (8*b*) are amplitudes of the linear combination of two quasiparticles

$$b^{\dagger}_{\alpha}(JM) = \sum_{pn} \eta^{\alpha}_{pn}[a^{\dagger}_{p}a^{\dagger}_{n}]^{JM} \tag{11}$$

which diagonalize the projected norm matrix

$$\begin{aligned} N_{\alpha,\beta} &= \langle \mathrm{BCS}(N,Z)|\, b_{\alpha}(JM)\hat{P}_{N-1}\hat{P}_{Z+1}b^{+}_{\beta}(JM)\,|\mathrm{BCS}(N,Z)\rangle \\ &= \sum_{pn}\sum_{p'n'} \eta^{\alpha}_{pn}N_{pn,p'n'}\eta^{\beta}_{p'n'} \end{aligned} \tag{12}$$

where

$$N_{pn,p'n'} = \delta(pp')\delta(nn')I_{Z+1}(p)I_{N-1}(n).$$

The associated creation operator, for a particle number projected correlated state $J^{\pi} = 1^{+}$ of the double-odd nucleus, has been defined by [20]

$$\Gamma^{\dagger}_{\omega}(JM) = \sum_{\alpha} [X^{\omega}_{\alpha}b^{\dagger}_{\alpha}(JM) - Y^{\omega}_{\alpha}b_{\alpha}(\overline{JM})]. \tag{13}$$

In deriving these equations we have written the two-body matrix elements of the Hamiltonian in terms of particle–hole and particle–particle channels, by following the notation which has been presented in [6, 20]. The quantities $G(pn, p'n', J)$ and $F(pn, p'n', J)$ are particle–particle and particle–hole matrix elements of the effective two-body interaction, respectively. With these matrix elements $A$ and $B$, both for the particle number projected case and for the conventional unprojected one, we have solved the corresponding eigenvalue problem [20, 29] and we have obtained the wavefunctions which are needed to evaluate $M_{GT}$, cf equation (2).

## 3. Results and discussion

The half-life, $\tau_{1/2}(2\nu\beta\beta)$, for the two-neutrino double beta decay mode in $^{100}$Mo has been determined recently by the INR–Baksan group [22], by the OSAKA–ELEGANTS experiments [23] and by the group at the University of California at Irvine [24]. The reported values are shown in table 1. The order of magnitude of these experimentally determined half-lives is out of the range of values which have been determined for other $2\nu\beta\beta$ transitions [25] by two to three orders of magnitude.

**Table 1.** Experimental values for the half life, $\tau_{1/2}(2\nu\beta\beta)$, for the two neutrino double beta decay mode of $^{100}$Mo.

| Experiment | $\tau_{1/2}(2\nu\beta\beta)$ (years) | CL (%) | Reference |
|---|---|---|---|
| INR–Baksan | $3.3 \times 10^{18}$ | 95 | [22] |
| OSAKA–ELEGANTS IV | $0.93\binom{+0.60}{-0.26} \times 10^{19}$ | 68 | [23] |
| OSAKA–ELEGANTS V | $1.16\binom{+0.34}{-0.22} \times 10^{19}$ | 68 | [23] |
| UC–Irvine | $1.16\binom{+0.34}{-0.08} \times 10^{19}$ | 68 | [24] |

Data from INR–Baksan [22] are preliminary data with 95% CL. Details concerning the experimental set-up which has been used in each case have been discussed in [22, 24].

There is a significant aspect, from the nuclear structure point of view, which makes the study of this transition particularly interesting. It is related to the value of $M_{GT}$, which could be extracted from the data, once the phase space integral $F$ which appears in equation (1) is calculated. We have obtained for this factor the value: $F = 2.34 \times 10^{-18}\,\mathrm{MeV}^2\,\mathrm{y}^{-1}$. Consequently for a half-life of the order of $10^{19}$ years one can extract the value of $M_{GT}$, which is therefore fixed by equation (1), and it is found to be of the order of $|M_{GT}| = 0.207\,\mathrm{MeV}^{-1}$.

This is a relatively large value as compared with the values which are needed to explain other $2\nu\beta\beta$ decay transitions [6]. This is an indication about the possible dominant character of some specific proton–neutron configurations of the intermediate excited states. Also it indicates that the single particle states involved should have nearly the same pairing occupation factors, otherwise suppression effects due to the Pauli principle would be larger than in the present case. By this we mean that both particle–hole and particle–particle (proton–neutron) quasiparticle pair configurations are expected to be almost equally weighted by their corresponding BCS occupation factors and that the Pauli suppression of the quasiproton to quasineutron transition would not be operative if BCS occupation factors of the sort $u_n \times v_p$ are comparable with factors of the sort $v_n \times u_p$. In the context of the pn-QRPA formalism almost comparable probabilities associated to particle–hole and particle–particle configurations and the fact that forward (quasineutron to quasiproton) and backward (quasiproton to quasineutron) matrix elements of the $\sigma\tau$ operator are comparable means that backward-going amplitudes, resulting from ground-state correlations, could be rather large. On the other hand, if low-energy proton–neutron excitations are going to be described mainly by few configurations, one should expect to obtain relatively large values for $M_{GT}$ unless a breakdown of the pn-QRPA is induced by attractive proton–neutron interactions.

In the following discussion we would like to show, from the analysis of our results, that the above advanced features do emerge in relation with the $2\nu\beta\beta$ decay of $^{100}$Mo. To start with let us briefly describe the main steps which we have followed in dealing with the calculation of $M_{GT}$ matrix elements in the framework of the QRPA and PQRPA models [19, 20]. For the single particle mean fields we have selected the eigenstates of a Coulomb-corrected Woods–Saxon potential. We have included 15 single particle states, both for protons and neutrons, taken $^{40}$Ca as a core and included three major oscillator shells up to the shell closure at $N = Z = 126$. This single particle basis is required in order to build up enough proton–neutron pair configurations so as to exhaust the Gamow–Teller sum rule (GTSR) up to the 1% limit. For the two-body interaction we have used the nuclear $G$-matrix calculated from the Bonn one-boson exchange potential [18]. Renormalization effects due to finite particle number have been treated in the manner which is described in [6]. A similar procedure has been adopted to determine the strength of proton and neutron pairing channels. Next, we have solved state-dependent BCS equations by using particle–particle matrix elements obtained from this $G$-matrix. We multiplied these $G$-matrix elements, for proton and neutron pairing channels, by factors $g_{\mathrm{pair}}(p)$ and $g_{\mathrm{pair}}(n)$, respectively, in such a way that experimental odd–even mass differences are reproduced. The values for these factors $g_{\mathrm{pair}}(p)$ and $g_{\mathrm{pair}}(n)$ are of the order of 0.98 and 1.10, respectively. We have solved two different sets of equations in order to determine BCS occupation factors and quasiparticle energies, both for protons and neutrons, namely: (i) conventional state-dependent BCS equations and (ii) particle number projected BCS equations. In the case of the particle number projected BCS solutions, equation (3), we have followed the method which has been reported in our previous work [19, 20]. With these occupation factors, quasiparticle energies and matrix elements of the effective two-body interaction we have solved unprojected (projected) QRPA (PQRPA) equations and we have obtained two sets of $1^+$ states, $|1^+\rangle_i$ and $|1^+\rangle_f$, associated with unlike quasiparticle pair excitations of $^{100}$Mo and $^{100}$Ru, respectively. The constant, $g_{\rm ph}$, for particle–hole channels of the residual interaction has been fixed at the value $g_{\rm ph} = 1.2$ and with this value we have reproduced the excitation energy of the giant Gamow–Teller resonance in the odd–odd nucleus $^{100}$Tc, as it is given by the experimentally known systematics [30]. Our configuration space, for unlike quasiparticle pairs, contains 61 configurations of proton–neutron pairs coupled to $1^+$. The dependence of $M_{GT}$ upon the constant multiplying the attractive proton–neutron particle–particle $G$-matrix elements of the residual interaction, $g_{\rm pp}$, has been studied. Thus we have varied it within the range $0 \leqslant g_{\rm pp} \leqslant 1$.

The dependence of the matrix element $M_{GT}$ upon the factor $g_{\rm pp}$ is shown in figure 1. Both QRPA and PQRPA results show cancellation nearby the value $g_{\rm pp} = 1$. This trend is somehow a common-place feature which has been found practically in all previous calculations of $2\nu\beta\beta$ matrix elements [1–8, 19, 20] and it cannot be attributed to the approximations which are involved in the QRPA description. It has been found in a variety of other model descriptions [16, 17] and it is well understood in terms of the strong attraction which is induced by particle–particle $1^+$ channels of the proton–neutron interaction.

It seems, from the results which are shown in figure 1, that PQRPA and QRPA results do not differ much.

It has to be mentioned that in the interval of $g_{\rm pp}$ values shown in figure 1 the associated GTSR $= 3(N - Z)$ is conserved both for QRPA and PQRPA calculations. It should also be mentioned that the value of the factor $g_{\rm pp}$ for which the QRPA collapses does not coincide with the value at which the matrix element $M_{GT}$ is completely suppressed. To make the point clearer let us show the dependence of the

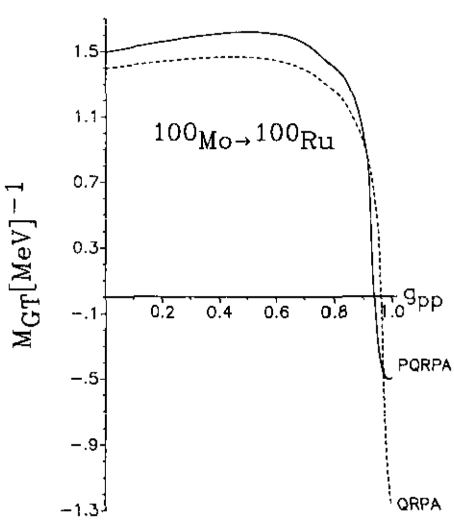


**Figure 1.** Dependence of the matrix element, $M_{GT}$, equation (2), upon the coupling constant for attractive proton–neutron particle–particle interactions, $g_{\rm pp}$. Full and broken curves correspond to particle number projected (PQRPA) and unprojected (QPRA) results, respectively.

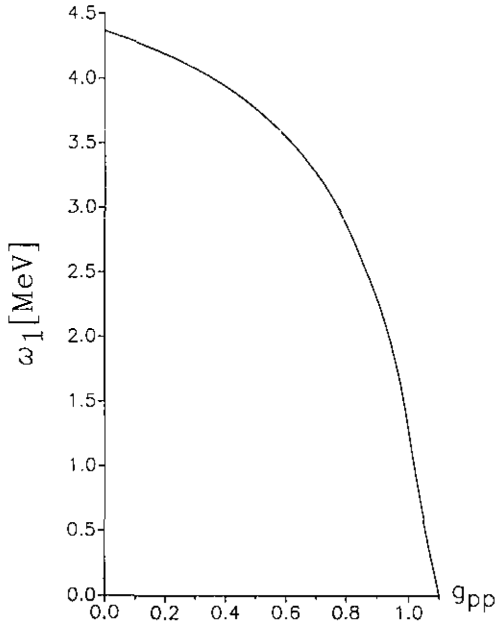


**Figure 2.** Energy $\omega_1$, of the first QRPA $1^+$ states in $^{100}$Tc, as a function of the coupling constant $g_{pp}$.

first QRPA root, for $1^+$ excitations in $^{100}$Tc, upon the particle–particle strength $g_{pp}$. The results are shown in figure 2. It is evident that the attraction which is coming from proton–neutron particle–particle interactions dominates over the repulsion which is coming from proton–neutron particle–hole channels of the two-body force.

The collapse of the QRPA description of pn excitations in the intermediate nucleus $^{100}$Tc is in fact a breakdown induced by attractive proton–neutron two-particle channels. It is of course obvious that the presence of this zero-energy mode, which is reminiscent of the one found in the RPA treatment of pairing excitations, invalidates any attempt to extend the initial pn QRPA solutions beyond the point of collapse at $g_{pp}=1$. Since this behaviour of the QRPA eigenvalues should also be accompanied by a drastic change in the structure of the wavefunctions let us investigate its effect upon them by calculating the contributions to $M_{GT}$ which are obtained when: (i) only the first QRPA $1^+$ state is included in the sum of equation (2), and (ii) by excluding it from the same sum. The results are shown in figure 3, for some values of $g_{pp}$ nearby $g_{pp}=1$. It is in fact observed that while the contribution coming from the first QRPA state is strongly dependent upon $g_{pp}$ the accumulated sum for all the other states remains almost constant. In the neighbourhood of the value $g_{pp}=1$ the contribution coming from the first QRPA root changes its sign. In this example it is also worthwhile to observe that below $g_{pp}=0.90$ the main contribution to $M_{GT}$ is coming from the first excited state and it is an order of magnitude larger than the accumulated sum of contributions coming from the other excited states.

This is due to the fact that in the case of the $2\nu\beta\beta$ decay transition in $^{100}$Mo the extreme single particle model is dominated by a single pair configuration, namely: $[0g_{7/2}(n)0g_{9/2}(p)]_{1^+}$.

This configuration has a relatively large matrix element $\langle 0g_{9/2}(p)\|\,\sigma\,\|0g_{7/2}(n)\rangle$, which is of the order of four, and the inclusion of BCS pairing occupation numbers $u$ and $v$ reduces it to about 1.26 for the direct n to p transition. It is also worth mentioning that the corresponding p to n transition is also large and comparable

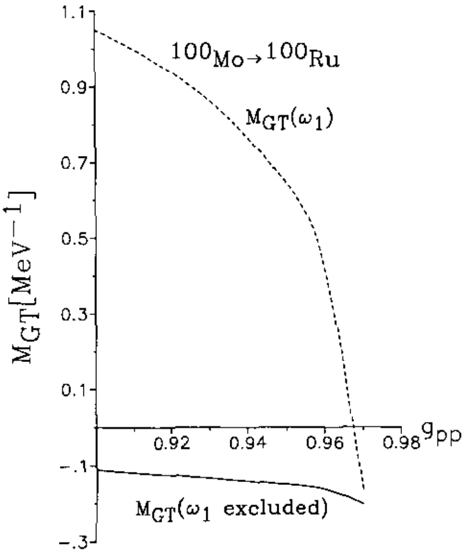


**Figure 3.** Contributions to $M_{GT}$, equation (2), coming from the first excited $1^+$ state in $^{100}$Tc (broken curve) and from all the other excited states excluded the first one (full curve). The results are displayed for different values of the coupling constant $g_{pp}$ near $g_{pp} = 1$, and they have been obtained by using the QRPA formalism.

with the n to p transition. This is due to the fact that both orbitals have nearly the same occupation factors and that since they are members of the same spin–orbit pair the associated matrix elements for $\sigma\tau$ are comparable. It means that the usually found suppression of the $\beta^+$ branch of the decay will not be operative here and this is the reason for the large values of $M_{GT}$ at $g_{pp} = 0$, as compared with other systems [6]. Consequently, since both $\beta^-$ and $\beta^+$ contributions are large, the suppression of the matrix element $M_{GT}$ around $g_{pp} = 1.0$ occurs if both forward- and backward-going amplitudes acquire comparable values and have opposite signs. This observation is in fact confirmed by inspection of the QRPA wavefunctions and we have found that they have large backward-going amplitudes. The sign of this contribution changes near the point $g_{pp} = 1$ and this is why $M_{GT}$ is suppressed. This statement is of course supported by the dependence of $M_{GT}$ upon $g_{pp}$ which is shown in figure 3.

The results obtained with the PQRPA formalism are not very much different from the ones which we have discussed so far. For the QRPA case the matrix element $M_{GT}$ is totally suppressed if one fixes $g_{pp}$ in the interval $0.96 < g_{pp} < 0.97$, where $M_{GT}$ values from $M_{GT} = 0.2647\ \mathrm{MeV}^{-1}$ to $M_{GT} = -0.3728\ \mathrm{MeV}^{-1}$, respectively. If we restrict the value of $g_{pp}$ to $g_{pp} \geqslant 0.9$ the corresponding theoretical lower limit for $\tau_{1/2}(2\nu\beta\beta)$ is of the order of $\tau_{1/2}(2\nu\beta\beta) > 0.50 \times 10^{18}$ y. Since the dependence of $M_{GT}$ upon $g_{pp}$, around $g_{pp} = 1.0$, is so strong the theoretical value for the half-life will change very rapidly and a more accurate prediction becomes unfeasible. The total suppression of $M_{GT}$, for the PQRPA model, occurs near the point $g_{pp} = 0.93$. From this point and up to the point $g_{pp} = 1$ the particle number projected results for $M_{GT}$ are of the order of $-0.5070\ \mathrm{MeV}^{-1}$, which corresponds to a half-life $\tau_{1/2}(2\nu\beta\beta) = 1.66 \times 10^{18}$ y.

From the above presented results it can be said that a slightly larger renormalization of the particle–particle $G$-matrix elements is required by the PQRPA in order to suppress $M_{GT}$, when compared with QRPA results.

The fact that the PQRPA formalism gives a nearly constant value for the matrix element $M_{GT}$ in the neighbourhood of $g_{pp} = 1$ but is still unable to go through the collapse means that another description of the microscopic wavefunctions is needed, particularly near this point. There are several possibilities. In fact one can argue that it should be a mixing between low-lying $1^+$ states of the intermediate nucleus and other excitations of the initial and final nuclei. This mixing could reduce the effect of the ground-state correlations which, as we have seen, are responsible for the increase in the values of the QRPA backward-going amplitudes. Higher order RPA correlations could also be important [31]. Also mean field effects can be described in a different framework like, i.e. in the MONSTER–VAMPIR approach [32] which is free from the limitations which are posed by the quasiparticle mean field approximation.

We think that the theoretical analysis of this $2\nu\beta\beta$ decay transition should be continued since it could offer some hints about the underlying physics of the nuclear double beta decay.

## 4. Conclusion

We have presented the result of our calculations for the $2\nu\beta\beta$ decay transition in $^{100}$Mo. The calculations have been performed by using a conventional QRPA method and a particle number projected formalism [19, 20]. We have used a realistic effective two-body interaction based on the Bonn one-boson exchange potential [18]. It has been shown that the microscopic analysis of this $2\nu\beta\beta$ decay transition shows significant differences when compared to other transitions which we have analysed previously [6].

We can summarize the features which can be extracted from our present work as follows

(i) The decay mode is dominated by a nearly pure proton–neutron configuration.

(ii) Renormalization of the particle–particle interaction between protons and neutrons suffices for the suppression of the relevant nuclear matrix element.

(iii) This renormalization also induces the collapse of the proton–neutron excitations in $^{100}$Tc. Some features which are similar to the so called pairing phase transition are found to be exhibited by the low-lying $1^+$ state in $^{100}$Tc.

(iv) The particle number projected version of the QRPA formalism shows the same suppression of $M_{GT}$ as is found in the unprojected QRPA model [6].

(v) The above-mentioned features are characteristic of this decay where the collectivity of the low-lying $1^+$ excitations in $^{100}$Tc, as given by the QRPA model, is rather poor. They have not been found for other cases [6, 19, 20] which have been analysed by using the same models.

(vi) Due to the unusually strong dependence of the results on $g_{pp}$, around $g_{pp} = 1.0$, the calculations yield only a lower limit for the half life.

Further investigations are in progress concerning the breakdown of the QRPA and its consequences upon $2\nu\beta\beta$ decay observables.

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