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Differential cross sections for DCX reactions on ^{76}Ge, ^{128,130}Te in a particle number projected BCS + QRPA approach

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Abstract. Nuclear structure effects on double charge exchange reactions are studied in the framework of a proton–neutron quasiparticle random phase approximation and with a particle number projected BCS formalism. The approach is used to calculate differential cross sections corresponding to (π^+, π^-) double charge exchange reactions on ^{76}Ge and ^{128,130}Te at low energies. It is found that the projection on good particle number, together with the inclusion of renormalized particle–particle channels in the quasiparticle random phase approximation, leads to a reduction of the values for the (π^+, π^-) differential cross sections.

1. Introduction

The study of double charge exchange reactions (DCX) on nuclei induced by low energy pions is today a fast growing subject, particularly in view of the information which is currently being produced at new facilities [1]. The chances of probing the nuclear structure with low energy pions and the fact that at least two nucleons are involved in DCX reactions thus leading to a clear identification of nuclear correlations effects of the two-body sort are some of the reasons behind current theoretical and experimental efforts [2]. In a previous publication we have analysed DCX reaction data by introducing nuclear structure methods to compute DCX observables [3]. Therein we have shown that the inclusion of proton–neutron correlations and the use of renormalized two-body interactions, in the context of the quasiparticle random phase approximation (QRPA), leads to an acceptable agreement between theoretical results and data. The way in which these nuclear structure effects are accounted for has been described in detail in [3] and we shall refer the reader to this work for further information concerning the scope of our present work as well as for a review of the previously existing literature. Since many of the DCX data have been produced for the scattering of low energy pions on heavy mass nuclei [2] one has to deal with approximations in describing the nuclear structure involved in the DCX reaction, particularly, because shell model nuclear structure calculations of heavy nuclei are still not feasible. Because of this we have based our previous work on the QRPA

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approximation as a way to compute the necessary nuclear structure information. This approximation has been extensively used to describe a variety of nuclear phenomena and it is now described in textbooks [4]. However, an improvement of the theory is still needed and in this paper we are addressing ourselves to the question of particle number violating effects which are inherently posed by the QRPA approach. In this respect we have investigated the effects on DCX differential cross sections associated with the projection on good particle number of the BCS quasiparticle mean field. For the QRPA sector of the calculations, based on these particle number projected BCS factors, we have adopted the method of [5] . Our effective two-body interaction, used to compute BCS and QRPA nuclear structure elements of the DCX calculations, is based on the Bonn one-boson-exchange potential [6]. For the pion sector we have adopted s- and p-wave interaction terms [7] and the pion fields are expanded in plane waves thus neglecting absorption effects [7]. Nucleonic excitations, other than the spin–isospin flip of the nucleons induced by the interaction with the pion field, are neglected since we are dealing with the scattering of low energy pions. Within these approximations we have computed differential cross sections for DCX reactions on ^{76}Ge and ^{128,130}Te at low (50 MeV) pion energy [8]. The details of the formalism are briefly presented in section 2. Results are discussed in section 3. Conclusions are drawn in section 4.

2. Formalism

We have divided this brief description of the formalism in two subsections (2.1) and (2.2) where we are presenting the nuclear structure procedure, (2.1), and the way in which we have computed DCX form factors and differential cross sections, (2.2). The notation which we have adopted in this section is the standard one and further details of the formalism can be found in [3].

2.1. Particle number projected BCS and QRPA equations [9]

From the pairing channels of an effective two-body interaction; hereon given by the solution of the Bethe–Goldstone equation for the Brückner G matrix constructed from the Bonn one-boson-exchange potential [6]; one can write BCS state dependent equations with good particle number [3]. As we have already shown [9] the projection on good particle number of the BCS equations can be performed by introducing the projector [5]

$$\hat{P}_N = \frac{1}{2\pi}\int \mathrm{d}\theta \mathrm{e}^{\mathrm{i}N\theta}\hat{S}_N(\theta) \tag{1}$$

in a factorized form, namely:

$$\hat{S}_N(\theta) = \langle \hat{S}_N(\theta)\rangle \exp\Big(\sum_j \alpha_j(\theta)A_j^\dagger\Big)\exp\Big(\sum_j \beta_j(\theta)N_j\Big)\exp\Big(\sum_j \alpha_j(\theta)A_j\Big) \tag{2}$$

where $A_j^\dagger$ (A_j) are operators which create (annihilate) a pair of quasiparticles coupled to zero angular momentum and N_j is the quasiparticle number operator [5]. The coefficients $\alpha_j(\theta)$ and $\beta_j(\theta)$ are defined by [9]

$$\alpha_j(\theta) = \frac{\sqrt{\Omega_j}u_j v_j(1 - e^{2i\theta})}{u_j^2 + v_j^2 e^{-2i\theta}} \tag{3a}$$

$$e^{\beta_j(\theta)} = \frac{e^{-i\theta}}{u_j^2 + v_j^2 e^{-2i\theta}} \tag{3b}$$

and

$$\langle \hat{S}_N(\theta) \rangle = \Pi_l (d_l(\theta))^{\Omega_l} \tag{3c}$$

with

$$d_l(\theta) = u_l^2 + v_l^2 e^{-2i\theta} . \tag{3d}$$

We can thus define the BCS energy functional

$$E = \frac{\langle \hat{P}_N \hat{H} \rangle}{\langle \hat{P}_N \rangle} \tag{4}$$

and make the variation

$$\nabla(u_j, v_j)E = 0 \tag{5a}$$

where

$$\nabla(u_j, v_j) = \frac{\partial}{\partial v_j} - \frac{v_j}{u_j}\frac{\partial}{\partial u_j} \tag{5b}$$

in order to determine particle number projected BCS parameters [9]. These equations have been obtained after a straightforward application of the method discussed in [5]. The corresponding u and v factors and the quasiparticle energies which are obtained in this way are then going to be taken as the BCS mean field representation of the initial (A, N, Z), intermediate $(A, N-1, Z+1)$ and final $(A, N-2, Z+2)$ nuclear states involved in the DCX reaction [3]. In order to go beyond this BCS approach one has to introduce residual interactions among the nucleons, mainly for the description of the intermediate nuclear states which are involved in the DCX. These are excited states of the double-odd nucleus with $(N-1, Z+1)$ neutrons and protons, respectively. This is done by solving, for the same two-body interaction, QRPA equations for unlike (proton–neutron) quasiparticle pairs [10]. The procedure has been discussed in detail in [3] and we shall avoid repeating it here. Let us indicate that in the QRPA procedure which we have adopted particle–particle proton–neutron channels of the residual two-body interaction are included, as well as the dominant particle–hole ones. It has been shown that they are responsible for a strong attraction in the $J^\pi = 1^+$ channel. This strong attraction has also been shown to be the main mechanism leading to the suppression of other nuclear observables, notably for the case of the nuclear double beta decay transition with the emission of two neutrinos [10]. In the case of DCX reactions the set of excited states belonging to the intermediate double-odd nucleus is not restricted to a certain multipolarity and one has to solve as many QRPA equations as multipole states are required to compute the form factors, as we shall see next.

2.2. DCX form factors [3]

The spin–isospin dependence of the pion–nucleus interactions can be represented by an s-wave, $h_s(q)$, and a p-wave, $h_p(q)$, transition operator, which can be written as [7]:

$$h_s(q) = 4\pi \frac{\lambda_1}{m_\pi} \sqrt{2} \Phi_N^\dagger \omega_q \tau_+ \Phi_N \tag{6a}$$

and

$$h_p(q) = -\sqrt{2}\mathrm{i} \frac{f_\pi}{m_\pi} \Phi_N^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{q}\, \mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{x}} \tau_+ \Phi_N \tag{6b}$$

where τ_+ is the nuclear isospin raising operator, Φ_N are nuclear wavefunctions, q is the momentum of the pion and f_π and m_π are the pion–nucleus coupling constant and the pion mass, respectively. This form for the pion–nucleus interaction is of course limited to the low energy domain we are interested in and it does not include nucleonic excitations [7]. Next we can write, after expanding the pion field operator in plane waves and casting the transition operators (6*a*) and (6*b*) in terms of nucleon and pion creation (annihilation) operators, more explicit forms which include form factors and QRPA charge transition densities [3]. The corresponding equations read

$$h_s(q) = -4\pi \frac{\lambda_1}{m_\pi} \sqrt{2} \omega_q \sum_{p,n} \sqrt{2_{j_p} + 1}\, \delta_{p,n} \rho_{p,n}^{00} \tag{7a}$$

and

$$h_p(q) = -\sqrt{2}\mathrm{i} \frac{f_\pi}{m_\pi} \sum_{p,n} F_{p,n}^{JM}(q) \rho_{p,n}^{JM} \tag{7b}$$

where $\rho_{p,n}^{JM}$ is the charge transition density:

$$\rho_{p,n}^{JM} = [c_p^\dagger c_n]^{JM} \tag{8}$$

and $F_{p,n}^{JM}(q)$ is the form factor:

$$F_{p,n}^{JM}(q) = \sqrt{4\pi}\sqrt{6}\, Y_{JM}^*(\Omega_q) G_{p,n}^J(q) \tag{9}$$

In equation (9) Y_{JM} is the spherical harmonic depending on the solid angle Ω_q and $G_{p,n}^J(q)$ is the reduced matrix element of the $\sigma\nabla_\pi$ operator acting on single particle and pion wavefunctions [3]. The explicit form of (9) in terms of QRPA wavefunctions requires, in our formalism, the adoption of the following approximations: (i) the charge transition density [3] is transformed to the BCS quasiparticle basis and then to the correlated QRPA basis, by writing the corresponding proton–neutron configurations in terms of QRPA phonon states [3, 10] and (ii) the DCX reaction is described as a sequential mechanism going from the ground state of the initial (N, Z) nucleus to the ground state of the final $(N-2, Z+2)$ nucleus through all possible excited states of the intermediate (N-1, Z+1) nucleus [3]. Accordingly one can write the DCX transition amplitude as a second order process in the pion–nucleus vertex:

$$F(k,k')$$
$$= \sum_{m,m'} \frac{\langle f,0^+;\pi^-(k')|\hat{O}|m,JM\rangle\langle mJM|m'JM\rangle\langle m'JM|\hat{O}|i,0^+;\pi^+(k)\rangle}{E_i+\epsilon_k-\frac{1}{2}(E_m^J+E_{m'}^J)} \tag{10}$$

for each set (m, JM) of excited states of the intermediate nucleus. The sum over the indexes (m, m'), in (10), indicates that the excited states of the intermediate $(N-1, Z+1)$ nucleus have been described as QRPA unlike quasiparticle pair excitations built on both initial $(i, 0^+)$ and final $(f, 0^+)$ ground states. The overlap factor $\langle m, JM|m', JM\rangle$ and the average QRPA energy $\frac{1}{2}(E_m^J + E_m'^J)$ are introduced in (10) in order to account for the QRPA spreading of these two different set of states, namely: m and m'. In this respect the adopted method is similar to the one which we have developed for the study of nuclear double beta decay transitions [10, 11]. The full scattering amplitude $F(k, k')$ is written as:

$$F(k,k') = \sum_J (F_J^{(s)}(k,k') + F_J^{(p)}(k,k')) \tag{11}$$

and with it the DCX differential cross section is given by

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = |F(k,k')|^2. \tag{12}$$

Finally, kinematics corrections should be included in order to account for the transformation of (12) from the pion–nucleus centre of mass frame to the laboratory frame. This is done as has been indicated in [3].

3. Results and discussion

Let us start with the discussion of the present results by defining the basic elements entering in the nuclear structure part of our calculations. We have selected a single particle model space which includes, both for protons and neutrons, two complete harmonic oscillator major shells with $N_{\rm osc}$ = 4 and 5 and the $0\mathrm{i}_{13/2}$ state. The single particle energies are eigenvalues of a Coulomb corrected spherical Woods–Saxon potential. For the two-body interactions we have taken nuclear matter G-matrix elements, corrected to account for finite size and pairing effects. These matrix elements have been obtained by solving the Bethe–Goldstone equation for the Bonn one-boson-exchange potential [6]. Pairing BCS equations, both in the conventional and in the particle number projected formalism described in the previous section, have been solved, for proton and neutron states. The corresponding pairing renormalization parameters, $g_{\rm pair}(\mathrm{n})$ and $g_{\rm pair}(\mathrm{p})$, for neutrons (n) and protons (p), which multiply the corresponding monopole pairing matrix elements of the G matrix of the Bonn potential, have been adjusted in order to reproduce experimental odd–even mass differences for the nuclei involved in the decay processes which we have mentioned before. The values of these pairing renormalization factors have been fixed at: $g_{\rm pair}(\mathrm{n}) = 1.14$ and $g_{\rm pair}(\mathrm{p}) = 1.0$, with small departures from these values for the different nuclei which we have considered. Thus the matrix elements only slightly deviate from the values given by the Bonn potential. The main difference between the conventional state-dependent BCS parameters and the ones obtained by using the above described projection on good particle number is given by an increase of the particle number projected gap values, as compared with the unprojected ones. It also means a larger diffuseness of the occupation factors, for the particle number projected results, as compared with the unprojected ones. The QRPA calculations have been performed as described in [10], with a fixed value for the renormalization factor corresponding to particle–hole channels of the residual interaction, $g_{\rm ph} = 1.0$ (the bare G matrix of the Bonn potential), and with the inclusion of particle–particle channels with a renormalization given by the factor $g_{\rm pp}$. This factor varies, in our calculations, in the interval $0.8 \leqslant g_{\rm pp} \leqslant 1.1$. These values for $g_{\rm pp}$ and $g_{\rm ph}$ are then taken as defined for the QRPA matrix equations corresponding to all multipolarities included in the calculation of excited states (m, JM) of the participant intermediate nuclei. With these elements we have calculated the angular dependence of the DCX differential cross section (12) and the results, for the set of renormalization factors which are indicated in the captions, are shown in figures 1, 2 and 3.

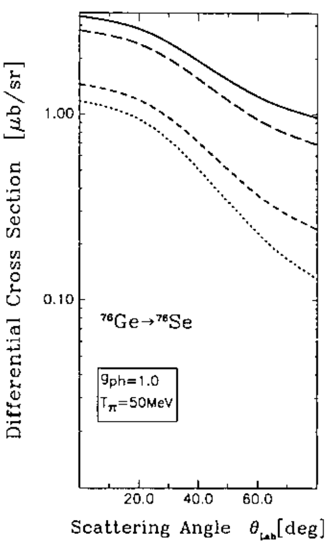

Figure 1. DCX differential cross section for the case of the $^{76}\mathrm{Ge} \to {}^{76}\mathrm{Se}$ ground state to ground state transition. The value of the renormalization factor for particle–hole QRPA channels, $g_{\rm ph}$ and the energy of the incoming pion, T_π, are shown in the inset. ——, conventional QRPA results (without particle number projection) corresponding to the renormalization factor $g_{\rm pp} = 1.0$; – – –, QRPA results including particle number projection and with the same value for $g_{\rm pp}$; - - - -, conventional QRPA results (without particle number projection) for $g_{\rm pp} = 0.8$; · · · · ·, QRPA results using particle number projected BCS parameters and for $g_{\rm pp} = 0.8$. Data are indicated by an error bar and they are taken, when available, from [8].

In the non-relativistic approach which we have used to calculate DCX differential cross sections it becomes evident that the inclusion of renormalized particle–particle channels of the nuclear two-body interaction produces a strong forward-peaked angular dependence. This effect, which becomes more pronounced when the projection on good particle number is performed, and the dominance of p-wave contributions in the partial wave expansion of the form factors clearly show that both the nuclear structure components as well as the description of the pion–nucleus vertex parts of the calculations are indeed important. The strong reduction of the differential cross section for large angles is induced both by the competition between renormalized particle–hole and particle–particle channels of the two-body interaction as well as by the relative interplay between s and p partial wave effects, as we have noticed from a detailed numerical analysis of the results. However, the very sparse data available do

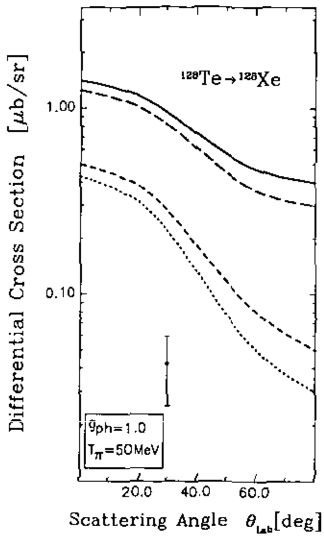

Figure 2. DCX differential cross section for the case of the ground state to ground state transition $^{128}Te \rightarrow ^{128}Xe$. The notation is as described in the caption to figure 1.

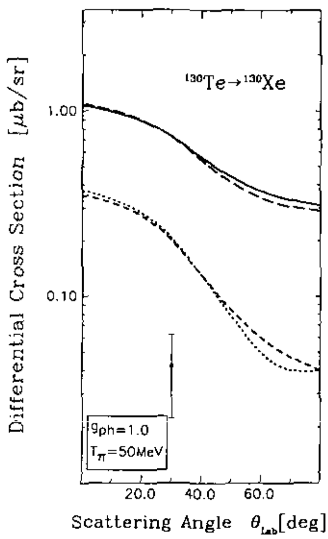

Figure 3. DCX differential cross section for the case of the ground state to ground state transition $^{130}Te \rightarrow ^{130}Xe$. The notation is as described in the caption to figure 1.

not allow us to extract a definite conclusion about the agreement between theory and experiments. Nevertheless the sensitivity of the results on purely nuclear structure based approximations, like the inclusion of full QRPA correlations for the intermediate states and the relatively strong suppression of the transition amplitude induced by the inclusion of particle–particle correlations, seemingly hints that DCX reactions could indeed be taken as a convenient tool for the study of those nuclear structure effects which are also important for double beta decay.

4. Conclusions

In this paper we have reported some results about the angular dependence of DCX differential cross sections for ground state to ground state transitions in the double beta decay nuclei ^{76}Ge and ^{128,130}Te. These DCX reactions, induced by low energy pions, have been described perturbatively as second order processes in the pion–nucleus vertex functions. S- and p-wave interactions have been used to describe the transition densities. The coupling of the incoming and outgoing pion fields with the initial and final nuclear states has been represented by a sequential double charge exchange through excited states of the participant intermediate nuclei. The wavefunctions for these states have been described in the QRPA approach, for unlike quasiparticle pair excitations. BCS occupation factors and quasiparticle energies have been calculated both in a conventional and in a particle number projected way. After computing form factors and transition amplitudes, by means of a partial wave expansion of the pion–nucleus vertex in the non-relativistic approach, we have found that the differential cross sections are strongly dependent upon nuclear correlations, as for example the strength of the correlations in the particle–particle channels in the QRPA approach.

The projection on good particle number decreases the theoretical values of the DCX cross sections, as compared with the unprojected ones, thus reducing the difference between the computed values and the few known data. Unfortunately the available data do not allow for a complete analysis of the agreement between theory and experiments. Nevertheless the sensitivity to nucleon–nucleon correlations found for the DCX differential cross sections is similar to the double beta decay with and without the emission of two neutrinos. Thus the DCX can test the quality of the wavefunctions which are needed as an input for the double beta decay which, in principle, allows a test of Grand Unified models. A weak point of the present approach is that the distortion and the absorption of the pions in nuclei is not yet included. For the small pion energy of $T_\pi = 50$ MeV this might not be a bad approximation. However at higher energies (near the Δ resonance) these effects must be included. Such work is in progress [12].

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