

## Finite temperature random phase approximation with the inclusion of scattering terms

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Effects associated with the inclusion of scattering terms, in the random phase approximation treatment of a two-body interaction at finite temperature, are investigated. Numerical results, for the case of quadrupole excitations in  $^{116}\text{Sn}$ , are discussed. It is found that these terms could influence the low-energy region of the intensity distribution, for multipole excitations at finite temperature.

### I. INTRODUCTION

The extension of the standard random phase approximation (RPA) method to the case of nuclear correlations at finite temperature has received some attention.<sup>1,2</sup> Finite temperature RPA (FTRPA) equations have been obtained in both matrix<sup>3</sup> and functional<sup>4</sup> forms and applications of the formalism can be found in the literature.<sup>5-7</sup> Although minor changes in the nuclear response at finite temperature,<sup>8</sup> with respect to the zero-temperature case, have been reported,<sup>1,2</sup> some questions can still be raised concerning the basic elements of the theory. Particularly, the validity of the mapping of the RPA Hamiltonian at finite temperature, namely, with reference to the interplay between fermionlike and bosonlike terms of it, is currently being reviewed. Recent papers<sup>9,10</sup> have dealt with this problem in the framework of the thermofield dynamics (TFD).<sup>11</sup> One of the main conclusions of Ref. 9 is related to the inclusion, in the FTRPA treatment of a residual two-body interaction, of the so-called "scattering terms" or  $H_{31}$  terms of the force<sup>12</sup> in a foot of equality with pair-creation-pair-annihilation and double pair-creation (-annihilation) terms of the form  $H_{22}$  and  $H_{40}$ ,<sup>12</sup> respectively. Following this method<sup>9</sup> the three terms, namely,  $H_{22}$ ,  $H_{40}$ , and  $H_{31}$ , should be included in dealing with FTRPA equations. Moreover, and in a consistent manner, scattering terms should be also included in the definition of FTRPA phonons and transition operators.<sup>9</sup> Since the numerical examples given in Ref. 9 correspond to a schematic two-level model situation, the influence of these new terms upon observables extracted from the FTRPA approach is not established yet, although the consistency requirement of the TFD theory seems to indicate the need of such an inclusion.<sup>9</sup>

In the present work we would like to discuss the results of the above-mentioned formalism for a realistic case. We have applied TFD concepts to the FTRPA treatment of quadrupole excitations in a superfluid nucleus,  $^{116}\text{Sn}$ , and we have calculated intensity distributions for quadrupole transitions in this nucleus. We have used, for the microscopic description of quasiparticle and boson degrees of freedom, a Hamiltonian which includes a single-particle term and an isospin-independent  $\delta$  force<sup>13</sup> and we have solved state-dependent BCS (Ref. 12) and FTRPA

equations in the temperature domain  $0 \leq T \leq 2$  MeV. The formalism is briefly reviewed in Sec. II and the results are discussed in Sec. III. Conclusions, related to the influence of the new terms added to the FTRPA equations upon the quadrupole response function, are drawn in Sec. IV.

### II. FORMALISM

We can write, for the Hamiltonian, the expression

$$H = \sum_{\alpha} \epsilon_{\alpha} c_{\alpha}^{\dagger} c_{\alpha} + \sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta} c_{\alpha}^{\dagger} c_{\beta}^{\dagger} c_{\delta} c_{\gamma}, \quad (1)$$

in standard notation, where  $V_{\alpha\beta\gamma\delta}$  are antisymmetrized matrix elements of the residual two-body interaction. We have adopted, for this residual two-body interaction, an isospin-independent  $\delta$  force.<sup>13</sup> The RPA treatment of (1) has been developed long ago<sup>12</sup> and here on we shall introduce only the definitions which are needed for our discussion. In the following we shall handle (1) in terms of the building blocks of the FTRPA, namely, fermion and boson degrees of freedom. First, we have to solve finite temperature BCS equations, in its state-dependent version,<sup>12</sup> in order to account for quasiparticle degrees of freedom which originates in the monopole channel of the residual interaction. Let us write (1) in a temperature-dependent quasiparticle basis.<sup>9</sup> We therefore have

$$H = H_0 + H_{11} + H_{20} + H_{\text{res}}, \quad (2)$$

where each term in the right-hand side of (2) represents, in the notation given in Refs. 9 and 12, the following: constant ( $H_0$ ), one-body ( $H_{11}$ ), single pair-creation and -annihilation ( $H_{20}$ ), and two-body ( $H_{\text{res}}$ ) contributions, respectively. We have followed the method outlined in Ref. 12 to solve the monopole part of the Hamiltonian (2) and we can write, for the quasiparticle term, the usual expression

$$H_{\text{qp}} = \sum_{\alpha} E_{\alpha} a_{\alpha}^{\dagger} a_{\alpha}, \quad (3)$$

where  $E_{\alpha}$  are quasiparticle energies and  $a_{\alpha}^{\dagger}$  ( $a_{\alpha}$ ) represent creation (annihilation) operators in the quasiparticle basis. Next, we have to introduce boson degrees of freedom and to do it let us now start with the discussion of

our main problem which is the treatment of the residual interaction,  $H_{\text{res}}$ , at finite-temperature and in the presence of pairing correlations. The structure of  $H_{\text{res}}$  can be defined by

$$H_{\text{res}} = \sum_{JM} H_{40}^{JM} + H_{22}^{JM} + H_{31}^{JM} + H_{qp-qp}^{JM}, \quad (4)$$

where

$$\begin{aligned} H_{40}^{JM} &= \sum_{\substack{a \geq b \\ c \geq d}} \{ h_{40}(abcdJ): A^\dagger(abJM) A^\dagger(cd\overline{JM}): \text{H.c.} \}, \\ H_{22}^{JM} &= \sum_{\substack{a \geq b \\ c > d}} h_{22}(abcdJ): A^\dagger(abJM) A(cdJM):, \\ H_{31}^{JM} &= \sum_{\substack{a \geq b \\ c \geq d}} \{ [h_{31}(abcdJ): A^\dagger(abJM) N^\dagger(cd\overline{JM}): + \tilde{h}_{31}(abcdJ): A^\dagger(abJM) N(cdM):] + \text{H.c.} \}, \\ H_{qp-qp}^{JM} &= \sum_{\substack{a \geq b \\ c \geq d}} \{ [h_{qp-qp}(abcdJ): N^\dagger(abJM) N^\dagger(cd\overline{JM}): + \text{H.c.}] + \tilde{h}_{qp-qp}(abcdJ): N^\dagger(abJM) N(cdJM): \}, \end{aligned} \quad (5)$$

with

$$\begin{aligned} h_{40}(abcdJ) &= 2u_\alpha u_\beta v_\gamma v_\delta G(abcdJ) - (u_\alpha v_\beta v_\gamma u_\delta + v_\alpha u_\beta u_\gamma v_\delta) F(abcdJ) + (v_\alpha u_\beta v_\gamma u_\delta + u_\alpha v_\beta u_\gamma v_\delta) \theta(abJ) F(bacdJ), \\ h_{22}(abcdJ) &= 2[-(u_\alpha u_\beta u_\gamma u_\delta + v_\alpha v_\beta v_\gamma v_\delta) G(abcdJ) - (u_\alpha v_\beta u_\gamma v_\delta + v_\alpha u_\beta v_\gamma u_\delta) F(abcdJ) \\ &\quad + (v_\alpha u_\beta u_\gamma v_\delta + u_\alpha v_\beta v_\gamma u_\delta) \theta(abJ) F(bacdJ)], \\ h_{31}(abcdJ) &= 2[-(u_\alpha u_\beta v_\gamma u_\delta - v_\alpha v_\beta u_\gamma v_\delta) G(abcdJ) + (v_\alpha u_\beta u_\gamma u_\delta - u_\alpha v_\beta v_\gamma v_\delta) F(abcdJ) \\ &\quad - (u_\alpha v_\beta u_\gamma u_\delta - v_\alpha u_\beta v_\gamma v_\delta) \theta(abJ) F(bacdJ)], \\ \tilde{h}_{31}(abcdJ) &= 2[-(u_\alpha u_\beta u_\gamma v_\delta - v_\alpha v_\beta v_\gamma u_\delta) G(abcdJ) + (u_\alpha v_\beta u_\gamma u_\delta - v_\alpha u_\beta v_\gamma v_\delta) F(abcdJ) \\ &\quad - (v_\alpha u_\beta u_\gamma u_\delta - u_\alpha v_\beta v_\gamma v_\delta) \theta(abJ) F(bacdJ)], \\ h_{qp-qp}(abcdJ) &= -(u_\alpha v_\beta v_\gamma u_\delta + v_\alpha u_\beta u_\gamma v_\delta) G(abcdJ) + (v_\alpha v_\beta u_\gamma u_\delta + u_\alpha u_\beta v_\gamma v_\delta) F(abcdJ) \\ &\quad + (u_\alpha u_\beta u_\gamma u_\delta + v_\alpha v_\beta v_\gamma v_\delta) \theta(abJ) F(bacdJ), \\ \tilde{h}_{qp-qp}(abcdJ) &= 2[-(u_\alpha v_\beta u_\gamma v_\delta + v_\alpha u_\beta v_\gamma u_\delta) G(abcdJ) - (u_\alpha u_\beta u_\gamma u_\delta + v_\alpha v_\beta v_\gamma v_\delta) F(abcdJ) \\ &\quad - (v_\alpha v_\beta u_\gamma u_\delta + u_\alpha u_\beta v_\gamma v_\delta) \theta(abJ) F(bacdJ)]. \end{aligned} \quad (6)$$

The quantities  $F(abcd, J)$ ,  $G(abcd, J)$ , and  $\theta(abJ)$  are defined as in Ref. 12 and  $u_\alpha$  and  $v_\alpha$  are temperature-dependent BCS occupation numbers. The operators  $A^\dagger$ ,  $A$ ,  $N^\dagger$ , and  $N$  are defined by

$$\begin{aligned} A^\dagger(a, b, JM) &= \sum_{m_\alpha m_\beta} \langle j_\alpha j_\alpha m_\alpha j_\beta m_\beta | JM \rangle a_\alpha^\dagger a_\beta^\dagger, \\ A(a, b, JM) &= [A^\dagger(a, b, JM)]^\dagger, \\ N^\dagger(a, b, JM) &= \sum_{m_\alpha m_\beta} \langle j_\alpha m_\alpha j_\beta m_\beta | JM \rangle a_\alpha^\dagger a_\beta, \\ N(a, b, JM) &= (N^\dagger(a, b, JM))^\dagger. \end{aligned} \quad (7)$$

Each term of (4) collects terms which are originated by the thermal Bogoliubov transformation of the original single-particle basis defined by the operators  $(c^\dagger, c)$ .

The RPA method, in its extended TFD version,<sup>9</sup> prescribes that the Hamiltonian

$$H_{\text{RPA}} = H_{qp} + H_{\text{res}} \quad (8)$$

should be linearized in the phonon basis

$$\begin{aligned} \Gamma^\dagger(\nu, \lambda\mu) &= \sum_{\substack{a \geq b \\ c > d}} [X(ab, \nu) A^\dagger(ab, \lambda\mu) \\ &\quad - Y(ab, \nu) A(ab, \overline{\lambda\mu}) \\ &\quad + Z_1(ab, \nu) N^\dagger(ab, \lambda\mu) \\ &\quad - Z_2(ab, \nu) N(ab, \overline{\lambda\mu})], \end{aligned} \quad (9)$$

where an extra term, the so-called scattering term, namely,  $Z_1(ab, \nu) N^\dagger(ab, \lambda\mu) - Z_2(ab, \nu) N(ab, \overline{\lambda\mu})$ , is added to the conventional definition of the phonon operator.

In this fashion, FTRPA matrix equations can be written as

$$\begin{bmatrix} \tilde{A} & \tilde{B} \\ \tilde{B}^* & \tilde{A}^* \end{bmatrix} \begin{bmatrix} \tilde{X} \\ \tilde{Y} \end{bmatrix} = \omega_\nu \begin{bmatrix} \tilde{S} & 0 \\ 0 & -\tilde{S} \end{bmatrix} \begin{bmatrix} \tilde{X} \\ \tilde{Y} \end{bmatrix}, \quad (10)$$

where we have used the following definitions:

$$\begin{aligned}
\tilde{A} &= \begin{pmatrix} A_{ab,cd} & C_{ab,cd} \\ E_{ab,cd} & G_{ab,cd} \end{pmatrix}, \\
\tilde{B} &= \begin{pmatrix} B_{ab,cd} & D_{ab,cd} \\ F_{ab,cd} & H_{ab,cd} \end{pmatrix}, \\
\tilde{S} &= \begin{pmatrix} S_{ab,cd} & 0 \\ 0 & T_{ab,cd} \end{pmatrix}, \\
\tilde{X} &= \begin{pmatrix} X(ab, \nu) \\ Z_1(ab, \nu) \end{pmatrix}, \\
\tilde{Y} &= \begin{pmatrix} Y(ab, \nu) \\ Z_2(ab, \nu) \end{pmatrix},
\end{aligned} \tag{11}$$

with

$$\begin{aligned}
A_{ab,cd} &= \langle [A(ab, \lambda\mu), [H, A^\dagger(cd, \lambda\mu)]] \rangle, \\
B_{ab,cd} &= -\langle [A(ab, \lambda\mu), [H, A(cd, \bar{\lambda}\mu)]] \rangle, \\
C_{ab,cd} &= \langle [A(ab, \lambda\mu), [H, N^\dagger(cd, \lambda\mu)]] \rangle, \\
D_{ab,cd} &= -\langle [A(ab, \lambda\mu), [H, N(cd, \bar{\lambda}\mu)]] \rangle, \\
E_{ab,cd} &= \langle [N(ab, \lambda\mu), [H, A^\dagger(cd, \lambda\mu)]] \rangle = C_{cd,ab}^*, \\
F_{ab,cd} &= -\langle [N(ab, \lambda\mu), [H, A(cd, \bar{\lambda}\mu)]] \rangle = D_{cd,ab}, \\
G_{ab,cd} &= \langle [N(ab, \lambda\mu), [H, N^\dagger(cd, \lambda\mu)]] \rangle, \\
H_{ab,cd} &= -\langle [N(ab, \lambda\mu), [H, N(cd, \bar{\lambda}\mu)]] \rangle, \\
S_{ab,cd} &= \langle [A(ab, \lambda\mu), A^\dagger(cd, \lambda\mu)] \rangle, \\
T_{ab,cd} &= \langle [N(ab, \lambda\mu), N^\dagger(cd, \lambda\mu)] \rangle.
\end{aligned} \tag{12}$$

We can write these matrix elements, in terms of quasiparticle energies, quasiparticle thermal occupation factors, and matrix elements of the residual interaction and their

expressions are

$$\begin{aligned}
A_{ab,cd} &= (E_a + E_b)(1 - f_c - f_d)\delta(a, c)\delta(b, d) \\
&\quad + h_{22}(abcdJ), \\
B_{ab,cd} &= [h_{40}(abcdJ) + h_{40}(cdabJ)], \\
C_{ab,cd} &= \tilde{h}_{31}(cdabJ), \\
D_{ab,cd} &= -\theta(cdj)\tilde{h}_{31}(dcabJ), \\
G_{ab,cd} &= (E_a - E_b)(f_b - f_a)\delta(a, c)\delta(b, d) \\
&\quad + \tilde{h}_{qp-qp}(abcdJ), \\
H_{ab,cd} &= 2h_{qp-qp}(abcdJ), \\
S_{ab,cd} &= (1 - f_c - f_d)\delta(a, c)\delta(b, d), \\
T_{ab,cd} &= (f_b - f_a)\delta(a, c)\delta(b, d),
\end{aligned} \tag{13}$$

with

$$f_a = [1 + \exp(E_a/T)]^{-1}. \tag{14}$$

It should be noted that the expectation values which appear in (12) have been calculated at finite temperature taking the thermal reference state as the ground state.<sup>9</sup> The quantity  $T$ , which appears in the quasiparticle thermal occupation factor (14), represents the nuclear temperature expressed in units of energy.

Finally, in order to finish with the definition of the quantities that we have calculated, let us introduce the transition operator  $Q_{\lambda\mu}$  which can be written as<sup>2</sup>

$$Q_{\lambda\mu} = Q_{pp, \lambda\mu} + Q_{hh, \lambda\mu} + Q_{ph, \lambda\mu}, \tag{15}$$

where with pp, hh, and ph we have denoted particle-particle, hole-hole, and particle-hole transitions, respectively. The terms which appear in the right-hand side of (15) are defined by

$$Q_{pp, \lambda\mu} = \sum_{j_1, j_2 > j_F} q_\lambda(j_1, j_2) \{ (u_1 u_2 - v_1 v_2) [N^\dagger(21, \lambda\mu) + N(21, \bar{\lambda}\mu)] - (u_1 v_2 + v_1 u_2) [A^\dagger(21, \lambda\mu) + A(21, \bar{\lambda}\mu)] \}, \tag{16}$$

and similar expressions are obtained for  $Q_{hh, \lambda\mu}$  and  $Q_{ph, \lambda\mu}$  except for the limits on configuration indexes ( $j_1, j_2$ ) which should read as  $j_1 \geq j_F > j_2$  ( $Q_{ph, \lambda\mu}$ ) and  $j_1, j_2 \leq j_F$  ( $Q_{hh, \lambda\mu}$ ), respectively, with

$$q_\lambda(j_2 j_1) = (2\lambda + 1)^{-1/2} \langle j_2 || Q_\lambda || j_1 \rangle. \tag{17}$$

The electric multipole moment,  $M(E\lambda)$ , can thus be written as

$$M(E\lambda, \mu) = \sum_{\kappa = pp, ph, hh} e_\kappa^{\text{eff}} Q_{\kappa, \lambda\mu}, \tag{18}$$

where  $e_\kappa^{\text{eff}}$  are effective charges corresponding to each channel,  $\kappa$ , of the operator (15). For the associated transition probability we have

$$B(E\lambda, \nu \rightarrow 0) = \sum_{\mu} |\langle 0 | M(E\lambda, \mu) | \nu \rangle|^2. \tag{19}$$

Furthermore, for the structure of  $B(E\lambda, \nu \rightarrow 0)$  in terms of the phonon amplitudes, we have

$$\begin{aligned}
B(E\lambda, \nu \rightarrow 0) &= (2\lambda + 1) \left| \sum_{\kappa = pp, ph, hh} e_\kappa^{\text{eff}} \left[ \sum_{j_2 \geq j_1} q_\lambda(j_2 j_1) \{ (u_1 u_2 - v_1 v_2) (f_1 - f_2) [Z_1(12, \nu) + Z_2(12, \nu)] \right. \right. \\
&\quad \left. \left. + (1 - f_1 - f_2) (u_1 v_2 + v_1 u_2) [X(12, \nu) + Y(12, \nu)] \right] \right|_\kappa^2.
\end{aligned} \tag{20}$$

Once the transition probability, Eq. (20), is defined, we can calculate the corresponding energy-weighted sum rule, (EWSR) which is given by

$$\text{EWSR} = \sum_{\nu > 0} \omega_{\nu} B(E\lambda, \nu \rightarrow 0). \quad (21)$$

Finally, and in order to complete the definitions given in this section, let us introduce the normalization condition associated with the RPA phonon operators, cf. Eq. (9). With the thermal occupation numbers  $f_a$  of Eq. (14) and with the corresponding matrix elements  $S_{ab,cd}$  and  $T_{ab,cd}$  of the norm matrix, Eq. (13), we can write, for the normalization condition associated to the amplitudes  $X$ ,  $Y$ ,  $Z_1$ , and  $Z_2$ , the following equation:

$$\begin{aligned} & \langle [\Gamma(\nu, \lambda\mu), \Gamma^{\dagger}(\omega, \lambda\mu)] \rangle \\ & = \delta_{\nu, \omega} \sum_{a \geq b} \{ (1 - f_a - f_b) [X^2(ab, \omega) - Y^2(ab, \omega)] \\ & \quad + (f_a - f_b) Z_1^2(ab, \omega) - Z_2^2(ab, \omega) \}. \end{aligned}$$

### III. RESULTS AND DISCUSSION

We have applied the above described formalism to the case of quadrupole excitations in  $^{116}\text{Sn}$ . We have adopted, for the single-particle basis, harmonic-oscillator levels with energies given by Nilsson's parametrization<sup>14</sup> with the following set of coupling constants:  $\kappa_N = 0.675 \times 10^{-1}$ ,  $\kappa_Z = 0.671 \times 10^{-1}$ ,  $\kappa_N \mu_N = 0.278 \times 10^{-1}$ , and  $\kappa_Z \mu_Z = 0.363 \times 10^{-1}$ , for neutrons ( $N$ ) and protons ( $Z$ ); we have included shell up to the major oscillator number  $N_{\text{osc}} = 7$  and we have adjusted single-particle energies for few states, nearby the shell closure  $N = Z = 50$ , in order to reproduce observed single-particle energy spacings.<sup>15</sup>

The coupling constant  $V_{ON}$ , corresponding to the neutron pairing channel of the two-body interaction described in Ref. 13, was fixed at the value  $V_{ON} = 114 \text{ MeV fm}^{-4}$ . After solving state-dependent BCS equations, for neutrons, we have obtained an average gap parameter of the order of  $\Delta = 1.16 \text{ MeV}$ . This value corresponds to the zero-temperature case and it reproduces fairly well the data.<sup>15</sup> For finite temperatures we have solved similar equations, in the fashion which is described in Ref. 16. With the above described single-particle basis we have calculated  $J^{\pi} = 2^{+}$  energy spectra and strength distributions, for quadrupole transitions in  $^{116}\text{Sn}$ . The

coupling constants for neutron (proton) two quasiparticle (particle-hole) configurations have been fixed at the value  $V_J (J^{\pi} = 2^{+}) = 94 \text{ MeV fm}^{-4}$ . With this value we have obtained, for the zero-temperature calculations, an energy of the order of  $\omega(2_1^{+}) = 1.329 \text{ MeV}$  which is to be compared with the observed energy  $\omega_{\text{exp}}(2_1^{+}) = 1.294 \text{ MeV}$  for the first excited  $J^{\pi} = 2^{+}$  state in  $^{116}\text{Sn}$ .<sup>15</sup> The diagonalization of the FTRPA equation (10) was performed for various values of the temperature, namely,  $0 \leq T \leq 2 \text{ MeV}$ . The  $T = 0$  value, for the EWSR ( $\lambda^{\pi} = 2^{+}$ ), was found to be of the order of  $1.95 \times 10^5 \text{ MeV fm}^4$  (or  $1161 \text{ MeV s.p.u.}$ ), a value which is fairly similar to the predicted, model-independent, value.<sup>17</sup>  $\text{EWSR}(\lambda^{\pi} = 2^{+}) \cong 71.12 A^{5/3} \text{ MeV fm}^4 = 1.96 \times 10^5 \text{ MeV fm}^4$  (or  $1168 \text{ MeV s.p.u.}$ ). Values of the EWSR ( $\lambda^{\pi} = 2^{+}$ ), which have been obtained with and without the inclusion of scattering terms in the FTRPA equations, are shown in Table I for various values of the nuclear temperature. It is evident from the results shown in Table I that FTRPA values obtained with the inclusion of scattering terms are better than these corresponding to the FTRPA without them. It should be noted that the missing strength at the higher temperature,  $T = 2 \text{ MeV}$ , is of the order of 32% for the usual FTRPA approach. This is a value which is expected in a truncated configuration space, like the one we are using in this calculation. However, the missing strength for the extended FTRPA approach, with the inclusion of scattering terms both in the phonons and in the transition operator, is smaller. It means that, as concluded in Ref. 9, scattering terms are of some significance in dealing with the microscopic structure of collective excitations at finite temperature. It is therefore expected that in an extended basis, with more shells included, this effect would be larger. There is, however, a compromise between the inclusion of scattering terms and the use of an enlarged configuration space. We have checked on this point and we have found that nine oscillator shells would be enough to describe, within a 5% accuracy, quadrupole transitions with a constant EWSR up to temperatures of the order of  $T = 2 \text{ MeV}$  without the inclusion of scattering terms. When these terms were switched on the computer time needed to solve FTRPA equations, at this upper value of  $T$ , was exceedingly long. The results discussed so far seemingly indicate that the TFD version of FTRPA equations yields better results than the conventional FTRPA without scattering terms. In order to determine the influence of the scattering terms upon the

TABLE I. Energy-weighted sum rules (EWSR) for quadrupole transitions in  $^{116}\text{Sn}$ . For each temperature  $T$ , unperturbed and FTRPA results are shown which have been obtained without (a) and with (b) the inclusion of scattering terms.

$T$ (MeV)	EWSR (MeV s.p.u.)			
	(a)		(b)	
	Unperturbed	FTRPA	Unperturbed	FTRPA
0.0	1165	1161	1165	1161
0.5	1116	1113	1126	1126
1.0	921	926	993	992
1.5	852	857	943	943
2.0	790	793	889	891

transition density we have calculated the strength distribution function  $S(w)$ , which is defined by

$$\text{EWSR} = \int_0^\infty dw w S(w), \quad (22)$$

for quadrupole transitions in  $^{116}\text{Sn}$ . The energy and temperature dependence of  $S(w)$  is shown in Fig. 1, for two values of the temperature. These results indicate that the inclusion of scattering terms in the FTRPA Hamiltonian does increase contributions to the strength function at low energies and changes in a minor but not negligible amount the strength distribution nearby the giant resonance region. In fact, scattering terms could also increase the FTRPA width of the resonance, as shown in Fig. 1.

The mechanism which emerges from these results can be explained in a very simple way: The increase of  $T$  would in fact produce the collapse of the particle-hole-dominated amplitudes, since for them the thermal occupation factors  $(1-f_a-f_b)$  would decrease as  $T$  increases. On the other hand, new states, of the particle-particle and hole-hole type, would contribute to the transition strength since for them the thermal occupation factors  $(f_a-f_b)$  would be larger for  $T \neq 0$ . The number of particle-particle and hole-hole transitions, nearby the Fermi energy, increase as  $T$  increases thus producing a balance with respect to the thermal blocking induced upon particle-hole transitions. It should be noted, however, that since all types of transitions are present in the definition of the transition operator, as well as in the definition of the phonon operators, each term will, in the quasiparticle representation, give contributions to the ex-

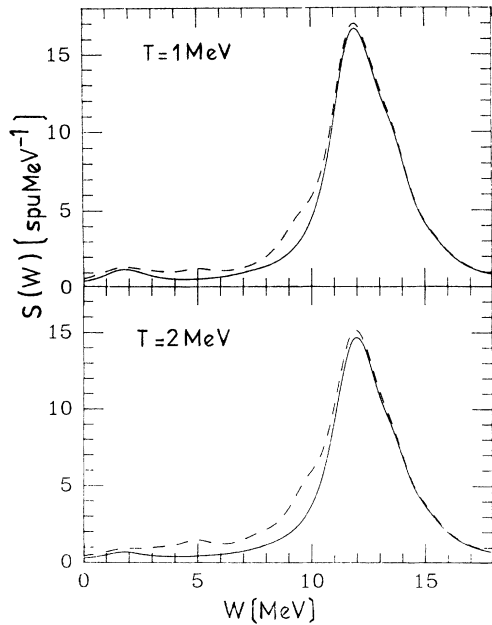


FIG. 1. Intensity distribution,  $S(w)$ , as a function of the excitation energy,  $w$ , for quadrupole transitions in  $^{116}\text{Sn}$ . Solid (dashed) lines correspond to RPA results which have been obtained without (with) the inclusion of scattering terms  $H_{31}$  like it is described in the text. Nuclear temperatures are indicated by  $T$ .

pectation values of the  $H_{22}$ ,  $H_{40}$ , and  $H_{31}$  terms of the Hamiltonian. The final result of this competition, namely, the collapse of particle-hole-dominated amplitudes and the increase of the number of particle-particle and hole-hole amplitudes, is a nearly preserved balance of them. This balance turns out to be the reason for the observed trend of the RPA sum rules which are, in the present case, much closer to their unperturbed values. This is shown in Table I, for the different values of  $T$  considered in the calculations. Therefore, the inclusion of scattering terms in the RPA equations at finite  $T$  seems to compensate for the missing strength which has been reported early.<sup>2</sup>

As we have already said before, a larger configuration space would give better results, but even for this case scattering terms in the RPA equations will significantly contribute to the stability of the EWSR at finite  $T$ .

Finally, let us summarize briefly the main features associated with the behavior of low-lying states. Because of the relatively large number of configurations which are present at finite  $T$ , we would rather like to describe the general picture which can be extracted from the results, instead of a detailed discussion of the components of the wave functions and transition matrix elements. As an example, we have, in the solution of FTRPA equations without scattering terms, at  $T = 1$  MeV, nine states in the energy interval  $0.1 \text{ MeV} \leq E_x \leq 3.21 \text{ MeV}$ . In the same energy interval and at the same temperature, FTRPA equations with the inclusion of scattering terms give 30 states, associated with dominant particle-particle and/or hole-hole character. While the lowest FTRPA eigenvalue, for  $T = 1$  MeV and without scattering terms, is of the order of 0.973 MeV, FTRPA equations at the same temperature and with the scattering terms included give a lowest eigenvalue at 0.164 MeV. For this state, as well as for the other low-lying states, contributions to the matrix elements of the transition operator are larger for particle-particle and hole-hole terms than for particle-hole terms. It means that the character of the low-lying transitions is changed drastically when scattering terms are included in the FTRPA equations. However, because of the low energy associated to these transitions, its effect upon the total EWSR is a minor one.

#### IV. CONCLUSIONS

In this paper we have shown some results concerning the influence of scattering terms upon the FTRPA description of quadrupole excitations in  $^{116}\text{Sn}$ . We have discussed, in detail, the changes which should be introduced in the FTRPA equations in order to account for the inclusion of scattering terms in the definition of phonon operators and in the relevant commutators of the theory. The simultaneous treatment of pair creation-pair annihilation, double pair creation (annihilation), and scattering terms of the residual two-body interaction, in the context of the TFD, has been shown to yield better results than the conventional FTRPA treatment of the same interaction. We have determined quantitatively this effect by calculating EWSR and strength distributions for quadrupole transitions in the tempera-

ture domain  $0 \leq T \leq 2$  MeV. This conclusion, already advanced in connection with the discussion of TFD concepts for a schematic situation,<sup>9</sup> seemingly encourages the systematic application of this extended FTRPA formalism to the description of nuclear degrees of freedom at finite temperature. Although numerical applications of the theory for realistic interactions in large spaces at finite temperatures are time consuming, the TFD method shows some theoretical features which are very interest-

ing, particularly in dealing with the full thermal mapping of a given Hamiltonian.<sup>9</sup>

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