Comparison between effective Hamiltonians in symmetry restoring methods: Intrinsic collective excitations in deformed nuclei

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The comparison between effective Hamiltonians, constructed within the framework of symmetry restoring techniques, is discussed for the case of intrinsic $K^\pi=1^+$ vibrations in deformed systems. The methods, which are based on self-consistent and global rotational invariance conditions, allow for the separation of rotational and vibrational degrees of freedom.

I. INTRODUCTION

The description of intrinsic $K^\pi=1^+$ excitations in deformed systems has been discussed recently.[1-5] It has been motivated both by new data about strong low-lying $M1$ transitions[6] and by the very interesting and challenging theoretical problem associated with the decoupling between rotational and vibrational degrees of freedom in deformed basis.[7,8] These intrinsic $K^\pi=1^+$ states have been described rather satisfactorily in terms of spin-independent quadrupole-quadrupole (QQ) forces.[4] Although the choice of quadrupole forces can be justified by the fact that the $\mu=\pm1$ components of the quadrupole operator, $Q_{2\mu}$, induce strong two quasiparticle correlations,[1-5] its use base on first principles, i.e., rotational invariance, has not been demonstrated yet. Currently, the description of $K^\pi=1^+$ states in deformed systems can be attempted in different manners, namely, (i) with model two-body Hamiltonians supplemented by constraints;[1-5] (ii) with effective interactions extracted from one-body Hamiltonians under global symmetry restoring conditions.[9]

It is our aim to demonstrate herein that, with the random phase approximation (RPA), both methods are equivalent. However, some significant differences appear in a more detailed description of the decoupling mechanism associated with the interplay between rotational and vibrational degrees of freedom.

In this paper we are going to describe, analytically, the equivalences and theoretical limitations posed by both methods. Concerning these aspects we shall show that the occurrence of a spurious mode at zero energy, generated by the action of the angular momentum operator on the correlated ground state, when it is included as a constraint of the equation of motion, does not suffice for a complete decoupling between rotational and vibrational modes. This theoretical shortcoming of method (i) could be avoided by adopting the procedure of method (ii). We shall also show that global symmetry restoring conditions are not automatically guaranteed by a renormalization of the quadrupole coupling constants or by the adoption of relationships between the matrix elements of quadrupole and angular momentum operators in the quasiparticle basis.

The formalism is discussed in Sec. II, where the comparison between the results of methods (i) and (ii) is presented. The theoretical results, which are relevant for calculations of $M1$ transitions in deformed systems, are discussed in Sec. III. We are also presenting in Sec. III an example of the decoupling mechanism for the case of a simplified Hamiltonian. Finally, some conclusions are drawn in Sec. IV.

Since we have organized this material on a theoretical basis, we shall present numerical results somewhere else.[10]

II. FORMALISM

The treatment of multipole-multipole (MM) forces of a quadrupole type, H_{QQ}, in the random phase approximation (RPA), for the case of $K^\pi=1^+$ states in axially symmetric deformed basis, has been discussed in detail in Refs. 1 and 2. The choice of these model interactions has been justified, previously, in analogy with the use of quadrupole forces in deformed Nilsson's basis.[5] Although the results of calculations performed with these interactions describe fairly well bulk properties of the experimentally observed $K^\pi=1^+$ states,[1,2] the question concerning the validity of the theoretical procedure can be raised, namely, with reference to the use of non-self-consistent model interaction in deformed basis. On the other hand, self-consistent residual interactions, H_{SR}, which are suitable for the microscopical description of the same states, have been introduced by Baznat *et al.*[9] Therefore we are confronted with two different possibilities. In this context the study of the removal of the spurious state becomes important. The removal of the spurious state associated with collective rotational degrees of freedom, for the case of H_{QQ}, could be partially achieved by an adequate choice of the coupling constants.[3,4] However, additional constraints are required in order to eliminate spurious components from intrinsic RPA wave functions.[2,16] Since the structure of self-consistent residual interactions H_{SR} is obtained under global symmetry restoring conditions, the spurious state would be automatically decoupled from intrinsic excitations, as we shall show later on. In order to determine the characteristic features of each approximation we are going to show, in the fol-

lowing subsections, some of the physical consequences of the use of schematic and self-consistent model interactions.

A. Multipole-multipole forces

Let us write the multipole-multipole Hamiltonian, H_{MM}, in the quasiparticle representation. Following the notation given in the Appendix, we have,

$$H_{\mathrm{MM}}=H_{\mathrm{qp}}+H_{\mathrm{QQ}} , \tag{1}$$

where H_{qp} is the unperturbed single quasiparticle term defined by

$$H_{\mathrm{qp}}=\sum_{i,z} E_i B(ii;z) , \tag{2}$$

and H_{QQ} is the quadrupole-quadrupole interaction ($\mu=\pm 1$) given by

$$H_{\mathrm{QQ}}=-\sum_{m,z,z'} m\chi_{zz'}Q(m,z)Q(m,z') . \tag{3}$$

The index m takes the values $m=\pm 1$ and it denotes linear combinations of two quasiparticle operators $A^{\dagger}(ik,mz)$, which are defined in the Appendix.

The RPA treatment of H_{MM}, Eq. (1), can be performed[1] introducing phonon operators defined by

$$\Gamma_{\nu}^{\dagger}(m)=\tfrac{1}{2}\sum_{ik,z}[\psi_{\nu}(ki,mz)A^{\dagger}(ik,mz) -\phi_{\nu}(ki,mz)A(ik,mz)] \tag{4}$$

and solving the RPA equation of motion

$$[H_{\mathrm{MM}},\Gamma_{\nu}^{\dagger}(m)]=w_{\nu}\Gamma_{\nu}^{\dagger}(m) , \tag{5}$$

we can obtain the energy w_{ν} of the νth correlated state from the RPA secular equation[1]

$$(1-\chi(+)S_{\nu}(n))(1-\chi(+)S_{\nu}(p))-\chi^2(-)S_{\nu}(n)S_{\nu}(p)=0 , \tag{6}$$

where $\chi^{(\pm)}$ are linear combinations of the isoscalar and isovector coupling constants $\chi(\pm)=\chi(0)\pm\chi(1)$, see the Appendix, and

$$S_{\nu}(z)=4\sum_{i,k}^{(z)}\frac{E_{ik}q^2(ki,mz)}{E_{ik}^2-w_{\nu}^2} , \tag{7}$$

where E_{ik} are quasiparticle pair energies, and $q(ki,mz)$ are the matrix elements of the quadrupole operator Q in the quasiparticle basis.

In terms of these quantities we have, for the phonon amplitudes,

$$\psi_{\nu}(ki,mz)=\frac{N_{\nu}(z)q(ki,mz)}{E_{ik}-w_{\nu}} , \qquad \phi_{\nu}(ki,mz)=\frac{mN_{\nu}(z)q(ki,mz)}{E_{ik}+w_{\nu}} , \tag{8}$$

where

$$N_{\nu}(n)=\left[\frac{w_{\nu}}{2}\left[S'_{\nu}(n)+\frac{1}{a_{\nu}^2}S'_{\nu}(p)\right]\right]^{-1/2} ,$$

$$S'_{\nu}(z)=4\sum_{ik}^{(z)}\frac{E_{ik}q^2(ki,mz)}{(E_{ik}^2-w_{\nu}^2)^2} , \tag{9}$$

$$a_{\nu}=\frac{\chi(-)S_{\nu}(p)}{1-\chi(+)S_{\nu}(n)} ,$$

and $N_{\nu}(p)=a_{\nu}N_{\nu}(n)$.

The quantities $S_{\nu}(z)$ and $S'_{\nu}(z)$ do not depend on m, since $q^2(ki,+z)=q^2(ki,-z)$.[1] Thus, the $m=\pm 1$ phonons are degenerated for a given energy w_{ν}. However, the construction of the intrinsic states requires both types of phonons. The linearized RPA Hamiltonian, H_{RPA}, can be written as

$$H_{\mathrm{RPA}}=\mathrm{const}+\sum_{m,\nu} w_{\nu}\Gamma_{\nu}^{\dagger}(m)\Gamma_{\nu}(m) . \tag{10}$$

As has been pointed out in Ref. 3, among the solutions of (5) there is a zero-energy solution: The spurious 1^+ state, which is generated by the action of the angular momentum operator, J, on the unperturbed ground state. The condition

$$[H_{\mathrm{MM}},J(m)]=0 , \tag{11}$$

with $J(m)=J_+-mJ_-$, would thus imply that the spurious mode will occur at zero energy and that a global rotational invariance will be satisfied by the schematic model Hamiltonian H_{MM}. However, since H_{qp} has been introduced like a non-self-consistent mean field which is not rotational invariant and because the model residual quadrupole-quadrupole interaction is also non-self-consistent and noninvariant under rotations, condition (11) will be approximately fulfilled. Consequently a relationship between matrix elements of the quadrupole and angular momentum operators should be established.[1] This relationship is exactly fulfilled for the case of Nilsson's basis,[5] but it has not been verified for axially symmetric deformed single-particle potentials[11] and we do not have an aprioristic argument to support it.

However, condition (11) has been replaced[1-5] by the secular equation (6) for the $w_{\nu}=0$ case, which can be written as

$$1-\chi(+)(S_0(n)+S_0(p)) +(\chi^2(+)-\chi^2(-))S_0(n)S_0(p)=0 , \tag{12}$$

where $S_0(z)$ is given by (7) with $w_{\nu}=0$.

This choice of the coupling constants will give us a spurious 1^+ state at zero energy. The question is how good, or reliable, this procedure would be if one is dealing with intrinsic wave functions. The answer to this question can be formulated in terms of the conmutator between the angular momentum and the phonon operators

$$[J(m),\Gamma_{\nu}^{\dagger}(m)]=\sum_{ikz}\frac{2w_{\nu}N_{\nu}(z)q(ki,mz)j(ki,mz)}{E_{ik}^2-w_{\nu}^2} , \tag{13}$$

where $j(ki,mz)$ are matrix elements of the angular momentum operator, $J(m)$, in the quasiparticle basis. The overlap (13) measures the amount of spuriosity carried by intrinsic states. It should also be noted that (13) does not vanishes for values of $w_\nu \neq 0$, as it should if the spurious 1^+ state would be effectively decoupled from the spectrum of intrinsic 1^+ excitations. If we now use Eqs. (11) and (12) to determine the relationship between matrix elements of J and Q, we can write for it

$$q(ki,mz)=m\frac{E_{ik}j(ki,mz)}{F(m,z)} \tag{14}$$

where

$$F(m,z=\tfrac{n}{p})=\chi(\pm)\bar{E}(m,n)+\chi(\mp)\bar{E}(m,p)\ , \tag{15}$$

with $\bar{E}(m,z)=4m\sum_{ik}^{(z)} q(ki,mz)j(ki,mz)$.

We can therefore argue that condition (11) does not guarantee a complete decoupling between spurious and intrinsic 1^+ states although it fixes the spurious 1^+ state at zero energy. This shortcoming could be due to the lack of self-consistency exhibited by the multipole-multipole model interaction, H_{QQ}. A possible solution to this problem has been attempted to in Refs. 16 and 17 where an additional term of the form

$$H_{\text{rotation vibration}}\cong\sum_{\nu m}\chi(m)J(m)(\Gamma_\nu^\dagger(m)-m\,\Gamma_\nu(m))$$

has been introduced in order to minimize the overlap between rotational and vibrational degrees of freedom.

B. Self-consistent residual interaction

The use of effective, self-consistent, and symmetry restoring interactions for the description of intrinsic excitations in deformed systems has been proposed, for the case of nuclear rotations, by Baznat *et al.*,[9] and recently extended to rotations in gauge space.[12] The starting point is the definition of an effective Hamiltonian which is written as

$$H_{\text{SC}}=H_{\text{qp}}+H_{\text{SR}}\ , \tag{16}$$

where H_{qp} is the quasiparticle Hamiltonian (2) and H_{SR} is a symmetry restoring term. The structure of this term is obtained from the conmutator

$$[H_{\text{qp}},J(m)]\ . \tag{17}$$

This conmutator has the structure of a one-body operator in the quasiparticle basis, namely, it includes scattering terms which are proportional to B, and pair creation (annihilation) terms which are proportional to $A^\dagger(A)$. In terms of (17), we can define H_{SR} by

$$H_{\text{SR}}=-\frac{2}{\gamma}\sum_m[H_{\text{qp}},J(m)][H_{\text{qp}},J(m)]\ , \tag{18}$$

where the coupling constant γ is determined from the condition[9]

$$[H_{\text{SC}},J(m)]=0\ , \tag{19}$$

which is satisfied by construction, provided

$$\gamma=4\langle\{J^\dagger(m),[H_{\text{qp}},J(m)]\}\rangle=16\sum_{ikz}E_{ik}j^2(ki,mz)\ . \tag{20}$$

The coupling constant γ, is independent of m, since $j^2(ki,+z)=j^2(ki,-z)$. The spectrum of intrinsic excitations associated to H_{SC} can be obtained after performing a RPA linearization in a new phonon basis, $\tilde{\Gamma}_\nu^\dagger(m)$, namely:

$$[H_{\text{SC}},\tilde{\Gamma}_\nu^\dagger(m)]=\tilde{w}_\nu\tilde{\Gamma}_\nu^\dagger(m)\ , \tag{21}$$

with

$$\tilde{\Gamma}_\nu^\dagger(m)=\tfrac{1}{2}\sum_{ikz}[\tilde{\psi}_\nu(ki,mz)A^\dagger(ik,mz)-\tilde{\phi}_\nu(ki,mz)A(ik,mz)]\ .$$

The forward and backward-going amplitudes, $\tilde{\psi}_\nu(ki,mz)$ and $\tilde{\phi}_\nu(ki,mz)$, can be written as

$$\begin{aligned}\tilde{\psi}_\nu(ki,mz)&=\frac{8mE_{ik}N_\nu^{\text{SC}}j(ki,mz)}{\gamma(E_{ik}-\tilde{w}_\nu)}\ ,\\ \tilde{\phi}_\nu(ki,mz)&=\frac{8E_{ik}N_\nu^{\text{SC}}j(ki,mz)}{\gamma(E_{ik}+\tilde{w}_\nu)}\ ,\end{aligned} \tag{22}$$

where

$$N_\nu^{\text{SC}}=\left[\frac{32\tilde{w}_\nu P'_\nu}{\gamma^2}\right]^{-1/2}\ ,$$

and

$$P'_\nu=4\sum_{ikz}\frac{E_{ik}^3j^2(ki,mz)}{(E_{ik}^2-\tilde{w}_\nu^2)^2}\ .$$

The corresponding RPA dispersion relation can be written as

$$\gamma=4P_\nu\ , \tag{23}$$

where

$$P_\nu=4\sum_{ikz}\frac{E_{ik}^3j^2(ki,mz)}{E_{ik}^2-\tilde{w}_\nu^2}\ .$$

C. Comparison between the RPA solutions of H_{MM} and H_{SC}

In order to establish a correspondence between both Hamiltonians, H_{MM} and H_{SC}, we can perform a comparison between their RPA wave functions, which leads to the following equations:

$$q(ki,mz)=\left[\frac{2}{\gamma\chi_{zz'}}\right]^{1/2}E_{ik}j(ki,mz)\ , \tag{24}$$

and, consequently, from Eqs. (14) and (24), we have

$$F^2(m,z)=\frac{\gamma\chi_{zz}}{2}\ . \tag{25}$$

Therefore Eq. (25) implies that the RPA structure of both Hamiltonians, H_{MM} and H_{SC}, will coincide, provided

$F^2(mz)$=const. The solutions for $F^2(mz)$, obtained from the RPA treatment of $H_{\rm MM}$ with the coupling constants $\chi(\pm)$ adjusted in order to get the spurious 1^+ state at zero energy, have been found to differ from a constant value.[10] It means that, for the above mentioned equivalence to be valid, we have to force Eq. (24) to be fulfilled. Clearly it will not be valid for a realistic basis.

The reason for this discrepancy between both Hamiltonians lies in the fact that a zero-energy mode generated by $H_{\rm MM}$ is not orthogonal to intrinsic states $\Gamma^\dagger_\nu(m)$. This is not the case for $H_{\rm SC}$, where the orthogonality condition (13) for the new phonon operators $\tilde{\Gamma}^\dagger_\nu(m)$ is automatically fulfilled; as it can be verified straightforwardly.[18] Moreover, from Eq. (20), we can see that all the information related to the restoring symmetry mechanism is contained, for $H_{\rm SC}$, at the level of the single quasiparticle basis, by the matrix elements of J, which is the operator associated with the broken symmetry.

In order to illustrate the above mentioned differences between both methods let us now calculate the rotational contribution by extracting the $w_\nu=0$ term from the corresponding RPA Hamiltonians.

For the case of the RPA solution of $H_{\rm MM}$ we have obtained, from Eqs. (8)–(10) and (14)

$$\sum_m (w_\nu \Gamma^\dagger_\nu(m)\Gamma_\nu(m))_{w_\nu=0}=\frac{2}{(S'_0(n)+S'_0(p))}J^2 \ . \tag{26}$$

It should be noted that this equation has been obtained under strongly limitative conditions, namely, (1) the isovector channel of the quadrupole force has to be switched off, (2) the factors $F(m,z)$, Eq. (15), should be fixed at the arbitrary value $F(m,z)=1$, with the adequate dimensions, as imposed by the dispersion relation Eq. (12), and (3) the relationship, Eq. (14), should be exactly fulfilled.

At this point it should be noted that conditions (1)–(3) cannot be avoided in dealing with the separation of a rotational term from the RPA spectrum but they are not satisfied for an arbitrary deformed mean field, except for Nilsson's one.[5]

The corresponding moment of inertia can be written as

$$\mathcal{J}=\sum_{ikz}\frac{j^2(ki,mz)}{E_{ik}} \ . \tag{27}$$

For the second case, the RPA treatment of $H_{\rm SC}$ gives the result

$$\sum_m (w_\nu \Gamma^\dagger_\nu(m)\Gamma_\nu(m))_{w_\nu=0}=\frac{8}{P'_0}J^2 \ , \tag{28}$$

without the requirement of additional conditions, and the associated moment of inertia coincides with the above given value, Eq. (27).

In summary, we are dealing with two different approaches. In the first, described in Sec. II A, we have started from a nonsymmetry invariant representation of the quasiparticle term, i.e., $H_{\rm qp}$, in an axially symmetric deformed Woods-Saxon field, and from a model multipole-multipole residual interaction, $H_{\rm QQ}$. The treatment of this Hamiltonian in the RPA basis determines a spurious state at zero energy and a set of intrinsic excitations, which are not completely decoupled from the spurious state. In the second, described in Sec. II B we have started from the same $H_{\rm qp}$ and constructed an effective, symmetry restoring, interaction that in the RPA treatment gives an intrinsic spectrum which is decoupled from the spurious state.

While the first method does require the validity of a relationship between matrix elements of the multipole operators and those of the angular momentum operator, which in general will not be fulfilled for any given deformed single particle or quasiparticle mean field; the second one does not require this sort of relationship and it is able to produce effective residual interactions which approximately restore, at the RPA level, the symmetry broken by the deformed quasiparticle mean field. In some respect the second method could be representative of a more general technique to restore symmetries.[13]

In Sec. III we are going to describe some other differences between both methods, particularly, reflected upon transition probabilities.

III. RESULTS AND DISCUSSION

In the previous section we have already pointed out some of the major differences between the solutions for $H_{\rm MM}$ and $H_{\rm SC}$, the multipole-multipole and self-consistent Hamiltonians, respectively. A numerical comparison between the spectrum of intrinsic excitations for both Hamiltonians will be presented somewhere else[10] and herein we shall discuss, analytically, some consequences of both formalisms.

A. $M1$ transition probabilities

As pointed out in Ref. 1, rotationally invariant wave functions, with $K^\pi=1^+$, can be written as

$$|IMK^\pi=1^+,\nu\rangle = \left[\frac{2I+1}{32\pi^2}\right]^{1/2}\sum_{m=\pm}[\mathcal{D}^I_{M1}(w)-m(-)^I\mathcal{D}^I_{M-1}(w)] \times(\tfrac{1}{2})^{1/2}[\Gamma^\dagger_\nu(m=+)+m\Gamma^\dagger_\nu(m=-)]|0\rangle \ , \tag{29}$$

with $w_\nu\neq 0$, based on the realization of the unified model[14] for $\Gamma^\dagger_\nu(m=+)$ and $\Gamma^\dagger_\nu(m=-)$ as members of an intrinsic time reversal pair. The tensor components of the intrinsic $M1$ operators can be written as

$$M'(M1,\mu)=(3/4\pi)^{1/2}\mu_N m_\mu \ , \tag{30}$$

where μ_N is the Bohr magneton and m_μ are the tensor components of the magnetic dipole operator,

$$\mu=\sum_z [g_l(z)\hat{j}(z)+\tfrac{1}{2}(g_s(z)-g_l(z))\hat{\sigma}(z)] \ . \tag{31}$$

After some algebra, we get, for the associated reduced transition probabilities in the laboratory frame, the result

$$B(M1,\nu)\uparrow=(3/4\pi)\mu_N^2 m_\nu^2 \ , \tag{32}$$

with

$$m_\nu=\sum_{kiz} m_+(ki,z)\begin{cases}\psi_\nu(ki,-z)+\phi_\nu(ki,-z)\\ \tilde{\psi}_\nu(ki,-z)+\tilde{\phi}_\nu(ki,-z)\end{cases}, \tag{33}$$

where m_+ are the matrix elements of the tensor components of μ in the quasiparticle basis, and the upper (lower) RPA amplitudes are associated to $H_{\rm MM}(H_{\rm SC})$, more explicitly:

$$m_\nu=\sum_{kiz} m_+(ki,z)q(ki,-z)N_\nu(z)\frac{2w_\nu}{E_{ik}^2-w_\nu^2}, \tag{34}$$

for the case of $H_{\rm MM}$, and

$$m_\nu=-\sum_{kiz} m_+(ki,z)j(ki,-z)\frac{16N_\nu^{\rm SC}}{\gamma}\frac{\tilde{w}_\nu E_{ik}}{E_{ik}^2-\tilde{w}_\nu^2}, \tag{35}$$

for the case of $H_{\rm SC}$, respectively. Equations (34) and (35) will, of course, not give the same result unless the relationship between $q(ki,mz)$ and $j(ki,mz)$, Eq. (14), would be exactly fulfilled. It means that the main differences between both formalisms will be reflected upon the transition probabilities. To a certain degree these differences could be attributed to the noncomplete removal of the spurious state from the intrinsic spectra of $H_{\rm MM}$. In order to set up a limit for them it would be necessary to perform systematic numerical calculations for both Hamiltonians,[10] but it becomes evident that these differences will be particularly noticeable for $K^\pi=1^+$ states dominated by few two-quasiparticle configurations. From the results of Ref. 1 it appears to be the case, in view of the weak collectivity associated to low-lying $K^\pi=1^+$ states in rare-earth nuclei.

B. Results for a simplified model

The above discussed formalism can be illustrated for the case of a simplified model Hamiltonian.[15] We shall assume that the deformed single-particle Hamiltonian $H_{\rm sp}$ represents a system of 2Ω particles distributed in a split $j=\frac{3}{2}$ multiplet, namely

$$H_{\rm sp}=\varepsilon\sum_\nu (a_\nu^\dagger a_\nu+a_{\bar\nu}^\dagger a_{\bar\nu}-b_\nu^\dagger b_\nu-b_{\bar\nu}^\dagger b_{\bar\nu}), \tag{36}$$

where the energy spacing $\varepsilon(m_j=\pm\frac{3}{2})-\varepsilon(m_j=\pm\frac{1}{2})$ is fixed at the value 2ε and the number of available single-particle states is twice the number of particles. The lower levels ($m_j=\pm\frac{1}{2}$) are degenerate and they are fully occupied while the upper levels ($m_j=\pm\frac{3}{2}$) are also degenerate and empty. The operators $a_\nu^\dagger(a_\nu)$ create (annihilate) a particle in a $m_j=\frac{3}{2}$ state and the operators $b_\nu^\dagger(b_\nu)$ create (annihilate) a particle in a $m_j=\frac{1}{2}$ state; a reversed state ($m_j=-\frac{3}{2}$ and $m_j=-\frac{1}{2}$) is denoted with $\bar\nu$. Various properties associated with nuclear rotations in this model space have been described in terms of group theoretical classifications by Krumlinde and Szymanski.[15] As it has been shown in Ref. 15, rotational effects can be introduced, in this model space, by defining a rotational term of the form $aI^2/2$, where a is inverse proportional to the moment of inertia. In order to show that the formalism of Sec. II B correctly decouples rotational and intrinsic excitations we shall apply it to the Hamiltonian (36). The angular momentum operators J_+ and J_- can be written as

$$\begin{aligned}J_+&=(3)^{1/2}\sum_\nu (a_\nu^\dagger b_\nu-b_{\bar\nu}^\dagger a_{\bar\nu})+2\sum_\nu b_\nu^\dagger b_{\bar\nu},\\ J_-&=(J_+)^\dagger.\end{aligned} \tag{37}$$

With these expressions, we can define $J(m)=J_+-mJ_-$ and calculate the conmutator of Eq. (17); the result is

$$[H_{\rm sp},J(m)]=2\varepsilon(6)^{1/2}\left[\sum_\nu (A^\dagger(\nu,m)+mA(\nu,m))\right], \tag{38}$$

where the pair operators

$$\begin{aligned}A^\dagger(\nu,m)&=(2)^{-1/2}(a_\nu^\dagger b_\nu+ma_{\bar\nu}^\dagger b_{\bar\nu}),\\ A(\nu,m)&=(A^\dagger(\nu,m))^\dagger,\end{aligned} \tag{39}$$

correspond, in this model space, to those of the Appendix. From Eq. (37) we have, for the matrix elements $j(\nu,m)$, the value $j(\nu,m)=(6)^{1/2}$. We can now construct the Hamiltonian (16), from Eq. (38). The result is

$$\begin{aligned}H_{\rm SC}=H_{\rm sp}&-C_\gamma\sum_{\nu\omega m}A^\dagger(\nu,m)A(\omega,m)\\ &-(C_\gamma/2)\sum_{\nu\omega m}m(A^\dagger(\nu,m)A^\dagger(\omega,m)\\ &+A(\nu,m)A(\omega,m))+\text{const.},\end{aligned} \tag{40}$$

with $C_\gamma=\varepsilon/\Omega$; it has been obtained from the value of γ fixed by Eq. (20).

The RPA treatment of $H_{\rm SC}$, Eqs. (21)–(23), allows for the following definitions of forward and backward-going amplitudes,

$$\begin{aligned}\tilde{\psi}_\nu(m)&=\frac{C_\gamma\Lambda_\nu(m)}{(2\varepsilon-\tilde{w}_\nu)},\\ \tilde{\phi}_\nu(m)&=\frac{mC_\gamma\Lambda_\nu(m)}{(2\varepsilon+\tilde{w}_\nu)},\end{aligned} \tag{41}$$

where $\Lambda_\nu(m)=1/C_\gamma(\tilde{w}_\nu P'_\nu)^{1/2}$ and $P'_\nu=2\varepsilon\Omega/(4\varepsilon^2-\tilde{w}_\nu^2)^2$.

The dispersion relation which determines the frequencies $\tilde{w}_\nu$ reads

$$1=C_\gamma P_\nu, \tag{42}$$

where $P_\nu=4\varepsilon\Omega/(4\varepsilon^2-\tilde{w}_\nu^2)$.

Due to the two level structure of the single-particle model basis Eq. (42) has only one solution at $\tilde{w}_\nu=0$. The intrinsic excitations are, in this model, represented by unperturbed two quasiparticle excitations.

We can now extract the $\tilde{w}_\nu=0$ contribution from the RPA Hamiltonian

$$H_{\rm RPA}=\sum_{m,\nu}\tilde{w}_\nu\tilde{\Gamma}_\nu^\dagger(m)\tilde{\Gamma}_\nu(m), \tag{43}$$

and the result is

$$H_{\mathrm{RPA}}=(\varepsilon/2\Omega)\sum_{\nu\omega m}[2A^{\dagger}(\nu,m)A(\omega,m) - m(A^{\dagger}(\nu,m)A^{\dagger}(\omega,m) + A(\nu,m)A(\omega,m))] + \sum_{m\nu\neq 0}\tilde{\omega}_{\nu}\tilde{\Gamma}^{\dagger}_{\nu}(m)\tilde{\Gamma}_{\nu}(m)\ . \quad (44)$$

The first term can also be written as

$$H_{\mathrm{RPA}}(\tilde{\omega}_{\nu}=0)=-(\varepsilon/12\Omega)\sum_{m} mJ^{2}(m)\ , \quad (45)$$

and it is the rotational term of Ref. 15, with the inverse moment of inertia $a=2\varepsilon/3\Omega$. The above described results show that the method of Sec. II B enables us to extract, from the deformed single particle Hamiltonian, a rotational term with a correct value for the moment of inertia.

We can now turn to the case of the quadrupole-quadrupole interaction. Since the single-particle basis defined by Eq. (36) could be interpreted as an extreme Nilsson's case it will be enough to show that the RPA treatment of H_{MM} yields the results which have been presented above. To start with let us write the quadrupole operator in the form

$$Q(m)=\sum_{\nu} q(\nu,m)(A^{\dagger}(\nu,m)+mA(\nu,m))\ , \quad (46)$$

with it the Hamiltonian (1) takes the form

$$H_{\mathrm{MM}}=H_{\mathrm{sp}}-K(0)\sum_{\nu\omega m} m(A^{\dagger}(\nu,m)A^{\dagger}(\omega,m) + A(\nu,m)A(\omega,m) + 2A^{\dagger}(\nu,m)A(\omega,m))\ , \quad (47)$$

with $K(0)=\varepsilon/2\Omega$.

The comparison of Eqs. (40) and (47) shows that both Hamiltonians coincide, since $2K(0)=C_{\gamma}$. Concerning the relationships between the matrix elements $q(\nu,m)$ and $j(\nu,m)$ we have from Eqs. (14) and (15) that $F(m)q(\nu,m)=m2\varepsilon j(\nu,m)$ which implies $q^{2}(\nu,m)=m^{2}$ [or $q(\nu,m)=-m$]. Since Eq. (14) is, for this very crude single-particle space, exactly fulfilled both Hamiltonians (40) and (47) yield, at the RPA level of approximation, the same result. Clearly, for any deformed single-particle mean field or for a case with neutron excess, calculations based on quadrupole-quadrupole interactions will not give the correct separation of rotational terms.[19]

IV. CONCLUSIONS

In this paper we have shown that the use of multipole-multipole interactions, of the spin-independent quadrupole type for the description of $K^{\pi}=1^{+}$ states in deformed basis, requires the fulfillment of proportion conditions between matrix elements of the quadrupole and angular momentum operators. These conditions are not automatically fulfilled for any deformed single-particle field and the choice of isoscalar and isovector coupling constants under global rotational invariance conditions does not guarantee the decoupling between spurious and intrinsic $K^{\pi}=1^{+}$ excitations. On the other hand, we have shown that a symmetry restoring technique allows for the complete decoupling from the intrinsic spectrum of the spurious 1^{+} state generated by the angular momentum operator. The structure of the effective residual interaction, generated in this fashion, is similar but not equivalent to the structure of the model multipole-multipole force.

We think that these results could be of some significance concerning the theoretical interpretation of the available data about $M1$ transitions in deformed systems.

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APPENDIX

The expression of the quadrupole operator Q in the quasiparticle basis requires the definition of the following two quasiparticle operators:

$$A^{\dagger}(ik,mz)=(\tfrac{1}{2})^{1/2}(\alpha^{\dagger}_{i}\alpha^{\dagger}_{k}-m\alpha^{\dagger}_{i}\alpha^{\dagger}_{\bar{k}})_{z}\ ,$$
$$\overline{A}^{\dagger}(ik,mz)=(\tfrac{1}{2})^{1/2}(\alpha^{\dagger}_{\bar{k}}\alpha^{\dagger}_{i}+m\alpha^{\dagger}_{k}\alpha^{\dagger}_{\bar{i}})_{z}\ , \quad (A1)$$

in terms of which we can write

$$Q(m,z)=\sum_{i,k}^{(z)} q(ki,mz)(A^{\dagger}(ik,mz)+mA(ik,mz))\ , \quad (A2)$$

where $q(ki,mz)$ are matrix elements of the quadrupole operator

$$Q_{21}(m,z)=(\tfrac{1}{2})(Q^{\dagger}_{21}(z)+mQ_{21}(z))\ , \quad (A3)$$

in the quasiparticle basis. The linear combination (A3), for a given value of z, defines the two intrinsic operators associated with a $K^{\pi}=1^{+}$ state, for $m=+1$ or -1, respectively. For the one quasiparticle operator B we have

$$B(ii;z)=(\alpha^{\dagger}_{i}\alpha_{i}+\alpha^{\dagger}_{\bar{i}}\alpha_{\bar{i}})_{z}\ . \quad (A4)$$

The index $i(k)$ reads for all the quantum numbers which are needed to specify a quasiparticle state in a deformed central field, which we assume to be an axially symmetric Woods-Saxon potential;[11] single quasiparticle energies are denoted by E_i; the operators B and Q are defined in terms of quasiparticle creation and annihilation operators; the index z reads for the type of particles, namely, neutrons ($z=n$) and protons ($z=p$) and $\chi_{zz'}$ are coupling constants associated with the product operator $Q(m,z)Q(m,z')$. The index m takes the values $m=\pm 1$ and it denotes linear combinations of two quasiparticle operators $A^{\dagger}(ik,mz)$.

For the sake of simplicity, we have omitted in Eq. (A2) configurations of the form $(i\bar{k})$, where $\bar{k}$ denotes a reversed state, which are obtained by reversing one of the quasiparticle states included in a given pair (ik) and changing $A^{\dagger}(ik,mz)$ and $A(ik,mz)$ by $\bar{A}^{\dagger}(ik,mz)$ and $\bar{A}(ik,mz)$, respectively. In the present notation these configurations are implicitly included in every sum running over pairs (ik).

Following the notation of Ref. 1 we can define isoscalar ($\tau=0$) and isovector ($\tau=1$) components of the quadrupole operator, namely

$$Q(m,\tau)=Q(m,n)+(-)^{\tau}Q(m,p) , \tag{A5}$$

and in this representation the quadrupole-quadrupole term H_{QQ} [cf., Eq. (3)] can be written as

$$H_{QQ}=-\tfrac{1}{2}\sum_{m,\tau}\chi(\tau)Q^{\dagger}(m,\tau)Q(m,\tau) , \tag{A6}$$

where $\chi(\tau)=\chi_{nn}+(-)^{\tau}\chi_{np}$.

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[18]After some algebra and from the results of Eq. (22), the overlap given by Eq. (13) takes the value $[J(m),\tilde{\Gamma}^{\dagger}_{\nu}(m)]=m\times C_0$, where

$$C_0=\frac{16w_{\nu}N_{\nu}^{SC}}{\gamma}\sum_{ikz}\frac{E_{ik}j^2(ki,mz)}{E_{ik}^2-w_{\nu}^2}$$

is independent of m. Therefore, we get

$$[J_+,\tilde{\Gamma}^{\dagger}_{\nu}(+)+\tilde{\Gamma}^{\dagger}_{\nu}(-)]=[J_-,\tilde{\Gamma}^{\dagger}_{\nu}(+)-\tilde{\Gamma}^{\dagger}_{\nu}(-)]=0 .$$

[19]Concerning the values for $M1$ and transitions probabilities, this example, based on a two level model, will not show the expected differences since for both cases, H_{SR} and H_{QQ}, the intrinsic states are unperturbed particle-hole states. In a realistic case these differences appear, as it has been shown in Ref. 16.