Symmetry-restoring treatment of the pairing Hamiltonian in the quasiparticle representation

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(Received 31 August 1987)

In analogy with a symmetry-restoring treatment of rotational invariance for deformed Hamiltonians, we discuss the case of a separable monopole pairing Hamiltonian in the quasiparticle basis. The results are compared to estimates of a conventional treatment based on the quasiparticle random phase approximation.

I. INTRODUCTION

The method of Marshalek and Weneser[1] was used by Pyatov *et al.*[2,3] to reconstruct, in the boson approximation, the rotational invariance of deformed average fields. It has also been shown[2,3] that general equations can be obtained which relate the single-particle matrix elements of the angular momentum operator with those of the multipole operator which generates a rotational symmetry restoring effective interaction. These equations are valid for any average field and are invariant against turning on residual interactions.[2,3] In this context the rotational invariance of the Hamiltonian is violated by the adoption of a deformed, oriented in space, single-particle average field. The rotational invariance must be restored, in principle, by residual interactions which will allow for rotations of the average field thus providing a rotational energy. Similar considerations apply for the case of the pairing Hamiltonian, where the adoption of a quasiparticle basis, with the subsequent "orientation" in the sense of particle number, violates particle number conservation. In both cases a collective rotation in normal or in gauge space, respectively, would result from the occurrence of a zero energy mode. This zero energy mode is to be associated with the generator of the broken symmetry, i.e., the angular momentum or the number operator, for rotational or pairing degrees of freedom, respectively.

The coupling of this zero energy mode with intrinsic modes of the system has been investigated in detail[4,5] by using boson mapping techniques and particle number conserving versions of the BCS formalism.

It is the aim of the present paper to show that the method of Refs. 2 and 3 can also be applied for the case of the pairing Hamiltonian. The equivalence between the above-mentioned cases of broken symmetries is shown in Sec. II where an effective, number symmetry restoring, interaction is constructed from the commutator of the quasiparticle Hamiltonian, the "deformed" average field, and the two quasiparticle components of the number operator. The random phase approximation (RPA) treatment of the resulting Hamiltonian and the contribution of the zero energy mode, represented by a quadratic term in the collective number variable, are discussed in Sec. II together with the corresponding mapping of collective and intrinsic excitations in terms of canonically conjugate operators, like the ones introduced by Marshalek and Weneser.[1] Numerical applications of the formalism are discussed in Sec. III. Some conclusions are drawn in Sec. IV.

II. FORMALISM

A. Conventional BCS plus RPA treatment of the pairing Hamiltonian

In order to set up the formalism, we shall briefly discuss in this section the main features of the BCS treatment of the pairing Hamiltonian; namely, the appearance of a zero energy mode in addition to the nonadiabatic RPA modes. The problem is by now a well-known one and it has been treated in detail in Ref. 6. To start with, let us write the following pairing Hamiltonian:

$$H=\sum_{j,m}\varepsilon_j a^{\dagger}_{jm}a_{jm}-G\sum_{jj',mm'>0}a^{\dagger}_{jm}a^{\dagger}_{j\tilde{m}}a_{j'\tilde{m}'}a_{j'm'}\ , \tag{1}$$

where ε_j are single-particle energies, G is the pairing force constant, and $a^{\dagger}_{jm}(a_{jm})$ are creation (annihilation) operators of fermions in the single-particle orbits $j\equiv(Nlj)$; $a_{j\tilde{m}}$ are the usual time reversed operators $a_{j\tilde{m}}=(-)^{j-m}a_{j-m}$. The BCS transformation

$$\begin{bmatrix}a^{\dagger}_{jm}\\ a_{j\tilde{m}}\end{bmatrix}=\begin{bmatrix}U_j & V_j\\ -V_j & U_j\end{bmatrix}\begin{bmatrix}\alpha^{\dagger}_{jm}\\ \alpha_{j\tilde{m}}\end{bmatrix}\ , \tag{2}$$

to the quasiparticle basis $\alpha^{\dagger}_{jm}(\alpha_{jm})$ leads to the transformed Hamiltonian

$$H=H_0+H_{11}+H_{(22+40)}+H_{31}+H_{\rm res}\ . \tag{3}$$

The terms on the right-hand side (rhs) of Eq. (3) are defined by

$$\begin{aligned}
H_0&=-(\Delta^2/G)+\sum_j 2\Omega_j V_j^2(\tilde{\varepsilon}_j-GV_j^2/2)\ ,\\
H_{11}&=\sum_j E_j\hat{N}_j\ ,\\
H_{(22+40)}&=\sum_{jj'}r_{jj'}\hat{P}^{\dagger}_j\hat{P}_{j'}+s_{jj'}(\hat{P}^{\dagger}_j\hat{P}^{\dagger}_{j'}+\hat{P}_j\hat{P}_{j'})\ ,\\
H_{31}&=\sum_{jj'}m_{jj'}(\hat{P}^{\dagger}_j\hat{N}_{j'}+N_{j'}\hat{P}_j)\ ,\\
H_{\rm res}&=-(G/4)\sum_{jj'}q_jq_{j'}{:}\hat{N}_j\hat{N}_{j'}{:}\ ,
\end{aligned} \tag{4}$$

where Δ is the pairing gap, E_j are quasiparticle energies, and the factors Ω_j, $r_{jj'}$, $s_{jj'}$, $m_{jj'}$, and q_j are defined by

$$\begin{aligned}
&\Omega_j = j+\tfrac{1}{2} ,\\
&r_{jj'} = -G(U_j^2U_{j'}^2+V_j^2V_{j'}^2) ,\\
&s_{jj'} = (G/2)(U_j^2V_{j'}^2+V_j^2U_{j'}^2) ,\\
&m_{jj'} = (G/2)(U_j^2-V_j^2)q_{j'} ,\\
&q_j = 2U_jV_j ,
\end{aligned} \tag{5}$$

respectively. The single-particle energies $\tilde{\varepsilon}_j$ are defined as $\tilde{\varepsilon}_j=\varepsilon_j-\lambda$, λ being the Lagrange multiplier which guarantees the "on the average" conservation of the number of particles, and the operators $\hat{N}_j$, $\hat{P}^\dagger_j$, and $\hat{P}_j$ are given by

$$\begin{aligned}
&\hat{N}_j = \sum_m \alpha^\dagger_{jm}\alpha_{jm} ,\\
&\hat{P}^\dagger_j = \sum_{m>0} \alpha^\dagger_{jm}\alpha^\dagger_{j\bar{m}} ,\\
&\hat{P}_j = (\hat{P}^\dagger_j)^\dagger ,
\end{aligned} \tag{6}$$

respectively. These operators obey conmutation relations

$$\begin{aligned}
&[\hat{P}_j,\hat{P}^\dagger_{j'}] = \delta_{jj'}(\Omega_j-\hat{N}_j) ,\\
&[\hat{N}_j,\hat{P}^\dagger_{j'}] = \delta_{jj'}2\hat{P}^\dagger_j ,
\end{aligned} \tag{7}$$

which follow from the SU(2) structure of the operators.[7]

The collective solutions of the Hamiltonian

$$H_{\mathrm{RPA}}(\mathrm{BCS}) = H_{11}+H_{(22+40)} \tag{8}$$

can be obtained by solving the RPA equation of motion

$$[H_{\mathrm{RPA}}(\mathrm{BCS}),\Gamma^\dagger_\nu] = w_\nu\Gamma^\dagger_\nu , \tag{9}$$

where

$$\Gamma^\dagger_\nu = \sum_j(\lambda_{j\nu}\hat{P}^\dagger_j-\mu_{j\nu}\hat{P}_j) \tag{10}$$

is the operator which creates a vibrational mode of energy w_ν.

Equation (9) leads to the secular equation[6]

$$\mathrm{Det}\begin{vmatrix} 1-GS_{11} & -Gw_\nu S_{12} \\ -Gw_\nu S_{12} & 1-GS_{22} \end{vmatrix} = 0 . \tag{11}$$

The quantities S which appear in Eq. (11) are defined by

$$\begin{aligned}
&S_{11} = \sum_j \frac{\Omega_j k_j^2 2E_j}{4E_j^2-w_\nu^2} ,\\
&S_{12} = \sum_j \frac{\Omega_j k_j}{4E_j^2-w_\nu^2} ,\\
&S_{22} = \sum_j \frac{\Omega_j 2E_j}{4E_j^2-w_\nu^2} ,
\end{aligned} \tag{12}$$

where $k_j = U_j^2-V_j^2$.

Equation (11) admits a solution $w_\nu=0$, since for it $S_{22}\mid_{w_\nu=0}=(1/G)$ and the BCS gap equation is recovered. In this case the amplitudes $\lambda_{j\nu}$ and $\mu_{j\nu}$ diverge as[6]

$$\lim_{w_\nu\to 0}\begin{bmatrix}\lambda_{j\nu}\\ \mu_{j\nu}\end{bmatrix} \cong (w_\nu)^{-1/2}\Bigg|_{w_\nu=0} , \tag{13}$$

with

$$\begin{aligned}
&\lambda_{j\nu} = \frac{\Lambda_\nu}{(2E_j-w_\nu)}(a_\nu k_j-b_\nu) ,\\
&\mu_{j\nu} = \frac{\Lambda_\nu}{(2E_j+w_\nu)}(a_\nu k_j+b_\nu) ,\\
&a_\nu = Gw_\nu S_{12} ,\\
&b_\nu = -1+GS_{11} ,
\end{aligned}$$

and

$$\Lambda_\nu = \left\{\sum_j \Omega_j\left[\left(\frac{a_\nu k_j-b_\nu}{2E_j-w_\nu}\right)^2-\left(\frac{a_\nu k_j+b_\nu}{2E_j+w_\nu}\right)^2\right]\right\}^{-1/2} .$$

Therefore, among the physical nonzero energy solutions of Eq. (11), intrinsic vibrational modes, we obtain a zero energy mode of infinite amplitude, i.e., a collective rotational mode. In the present case the appearance of this mode results from the choice of the quasiparticle average field H_{11} which breaks the particle number symmetry. The structure of this zero energy mode is to be related to the action of the two quasiparticle, $2qp$, terms of the number operator upon the BCS ground state. In fact, we have for

$$\hat{N}_{2qp} = \sum_j q_j(\hat{P}^\dagger_j+\hat{P}_j) ,$$

that

$$[H_{11},\hat{N}_{2qp}] \neq 0 , \tag{14}$$

which implies that the above-mentioned conditions of dynamical symmetry breaking are fulfilled once a formal analogy is established between the deformed average field and the angular momentum, for the rotational case, and H_{11} and $\hat{N}_{2qp}$, for the pairing case, respectively.

B. Symmetry restoring effective interaction

The spontaneous symmetry breaking, in this case represented by the violation of the particle number conservation in the quasiparticle basis, manifests itself in the appearance of a zero energy mode. In order to restore this broken symmetry, additional terms should be added to the Hamiltonian H_{11}. The structure of these terms has been studied, among other methods, within the framework of the quantization of systems with constraints,[8] which is particularly suitable for perturbative treatments of the coupling term H_{31} of Eq. (3) within a nuclear field theory.[9] Here we aim at a more restricted scope than that of Ref. 8. Particularly, we would like to show the consequences of the use of Pyatov's method[2,3] in dealing with the construction of effective symmetry restoring interactions for the pairing Hamiltonian. We start with the evaluation of the conmutator

$$[H_{11},\hat{N}_{2qp}] = 2\Delta\sum_j(\hat{P}^\dagger_j-\hat{P}_j) . \tag{15}$$

Next, we can introduce the effective Hamiltonian[2,3]

$$H_{\text{eff}}=H_{11}+H_{\text{sym}} ,$$

with

$$H_{\text{sym}}=\gamma[H_{11},\hat{N}_{2qp}]^{\dagger}[H_{11},\hat{N}_{2qp}] , \tag{16}$$

where the coupling constant γ is fixed by the condition

$$\langle\tilde{0}\,|\,[[H_{11},\hat{N}_{2qp}],\hat{N}_{2qp}]\,|\,\tilde{0}\rangle=(1/2\gamma) , \tag{17}$$

where $|\,\tilde{0}\rangle$ is the quasiparticle vacuum.

In this way H_{eff} is, by construction, a symmetry conserving Hamiltonian in the quasiparticle basis. Its structure is similar to that of H_{RPA}(BCS), Eq. (8), and it reads

$$H_{\text{eff}}=H_{11}+g\sum_{jj'}(\hat{P}^{\dagger}_{j}\hat{P}_{j'}+\hat{P}_{j}\hat{P}^{\dagger}_{j'})-g\sum_{jj'}(\hat{P}^{\dagger}_{j}\hat{P}^{\dagger}_{j'}+\hat{P}_{j}\hat{P}_{j'}) , \tag{18}$$

where $g=4\Delta^2\gamma$ for $\gamma=-G/16\Delta^2$, as it is determined by Eq. (17). The RPA treatment of H_{eff} can be performed in the basis of phonon operators

$$\tilde{\Gamma}^{\dagger}_{\nu}=\sum_{j}(X_{j\nu}\hat{P}^{\dagger}_{j}-Y_{j\nu}\hat{P}_{j}) . \tag{19}$$

The corresponding RPA secular equation is

$$w_{\nu}^{2}f(w_{\nu})=0 , \tag{20}$$

where

$$f(w_{\nu})=(G/2)\sum_{j}\frac{\Omega_j}{E_j(4E_j^2-w_{\nu}^2)} . \tag{21}$$

The amplitudes $X_{j\nu}$ and $Y_{j\nu}$ are given by

$$X_{j\nu}=\frac{\tilde{\Lambda}_{\nu}}{(2E_j-w_{\nu})} ,$$
$$Y_{j\nu}=-\frac{\tilde{\Lambda}_{\nu}}{(2E_j+w_{\nu})} , \tag{22}$$

with

$$\tilde{\Lambda}_{\nu}=\left[w_{\nu}\sum_{j}\frac{8E_j\Omega_j}{(4E_j^2-w_{\nu}^2)^2}\right]^{-1/2} .$$

We can write H_{eff}, Eq. (16), in the phonon basis $\langle\tilde{\Gamma}^{\dagger}_{\nu},\tilde{\Gamma}_{\nu}\rangle$ and extract from it the contribution to the energy due to the collective rotation, hereby generated by the one-phonon components of $\hat{N}_{2qp}$. The result is

$$H_{\text{eff}}=\text{const}+\sum_{\nu}w_{\nu}\tilde{\Gamma}^{\dagger}_{\nu}\tilde{\Gamma}_{\nu} . \tag{23}$$

In order to show that the contribution to Eq. (23) from the $w_{\nu}=0$ mode is given by the rotational term generated by $\hat{N}_{2qp}$, we shall introduce Marshalek and Weneser's transformation[1]

$$\hat{\mathcal{P}}_{\nu}=(w_{\nu}/2)^{1/2}(\tilde{\Gamma}^{\dagger}_{\nu}+\tilde{\Gamma}_{\nu}) ,$$
$$\hat{\mathcal{L}}_{\nu}=-i(2w_{\nu})^{-1/2}(\tilde{\Gamma}^{\dagger}_{\nu}-\tilde{\Gamma}_{\nu}) , \tag{24}$$

and, consequently, we have

$$\tilde{\Gamma}^{\dagger}_{\nu}=(2w_{\nu})^{-1/2}\hat{\mathcal{P}}_{\nu}+i(w_{\nu}/2)^{1/2}\hat{\mathcal{L}}_{\nu}$$
$$\tilde{\Gamma}_{\nu}=(\tilde{\Gamma}^{\dagger}_{\nu})^{\dagger} . \tag{25}$$

The inversion of Eq. (19) and its expansion in terms of Eq. (25) gives, for the pair creation (annihilation) operators $\hat{P}^{\dagger}_{j}$ ($\hat{P}_{j}$) of Eq. (6), the result

$$\hat{P}^{\dagger}_{j}=\Omega_j\sum_{\nu}(2w_{\nu})^{-1/2}(X_{j\nu}+Y_{j\nu})\hat{\mathcal{P}}_{\nu}$$
$$+i(w_{\nu}/2)^{1/2}(X_{j\nu}-Y_{j\nu})\hat{\mathcal{L}}_{\nu} ,$$
$$\hat{P}_{j}=(\hat{P}^{\dagger}_{j})^{\dagger} , \tag{26}$$

and with them, the $2qp$ components of the number operator can be written as

$$\hat{N}_{2qp}=\sum_{\nu}\left[\sum_{j}\Omega_j q_j(X_{j\nu}+Y_{j\nu})\right](2/w_{\nu})^{1/2}\hat{\mathcal{P}}_{\nu} . \tag{27}$$

As expected, with transformation (24), H_{eff} reduces to

$$H_{\text{eff}}=\tfrac{1}{2}\sum_{\nu}(\hat{\mathcal{P}}^{2}_{\nu}+w_{\nu}^{2}\hat{\mathcal{L}}^{2}_{\nu}) , \tag{28}$$

finally, the contribution from $w_{\nu}=0$ reads

$$H_{\text{eff}}(w_{\nu}=0)=\tfrac{1}{2}\hat{\mathcal{P}}^{2}_{0}=(1/2\chi)\hat{N}^{2}_{2qp}(\nu=0) , \tag{29}$$

where

$$(1/\chi)=\left[\sum_{j}\frac{\Omega_j q_j^2}{E_j}\right]^{-1} \tag{30}$$

is the corresponding moment of inertia associated to the rotational pairing mode. Equation (30) has to be compared with the value $(1/\chi)_{\text{BCS+RPA}}$ obtained from the conventional BCS plus RPA treatment; namely,

$$(1/\chi)_{\text{BCS+RPA}}=\frac{2}{\Delta^2}\frac{\left[\sum_{j}\Omega_j/E_j^3\right]-2G\left[\sum_{j}\Omega_j k_j/2E_j^2\right]^2}{\left[\sum_{j}\Omega_j/E_j^3\right]^2} . \tag{31}$$

Therefore, Eq. (23) is transformed in terms of the operators $\hat{\mathcal{L}}_{\nu}$ and $\hat{\mathcal{P}}_{\nu}$ into the form

$$H_{\text{eff}}=(1/2\chi)\hat{N}^{2}_{2qp}(\nu=0)+\tfrac{1}{2}\sum_{\nu\neq 0}(\hat{\mathcal{P}}^{2}_{\nu}+w_{\nu}^{2}\hat{\mathcal{L}}^{2}_{\nu}) , \tag{32}$$

which guarantees the explicit separation of a rotational term, associated to the number operator, and a vibrational term, given by the intrinsic excitations with $w_{\nu}\neq 0$.

It should be noted that Eq. (32) has been obtained from the RPA treatment of the symmetry conserving Hamiltonian H_{eff} of Eq. (16). Since this H_{eff} has been constructed in order to obey condition (17), it would be relevant for our discussion to write it in terms of $\hat{\mathcal{P}}_{\nu}$ and $\hat{\mathcal{L}}_{\nu}$, which play the role of number and angle variables, respectively.

The conmutator (15) reads

$$[H_{11},\hat{N}_{2qp}]=-i\sum_{\nu}h_{\nu}\hat{\mathcal{L}}_{\nu} , \tag{33}$$

with

$$h_\nu=(2\Delta^2)^{1/2}\frac{\sum_j 8E_j\Omega_j/(4E_j^2-w_\nu^2)}{\left[\sum_j 8E_j\Omega_j/(4E_j^2-w_\nu^2)^2\right]^{1/2}},$$

while H_{11} can be written as

$$H_{11}=\sum_{\nu\omega}\varepsilon_{\nu\omega}\hat{\mathcal{P}}_\nu\hat{\mathcal{P}}_\omega+\sigma_{\nu\omega}\hat{\mathcal{L}}_\nu\hat{\mathcal{L}}_\omega , \tag{34}$$

and for $H_{\rm eff}$ the corresponding expression is

$$H_{\rm eff}=\sum_{\nu\omega}\varepsilon_{\nu\omega}\hat{\mathcal{P}}_\nu\hat{\mathcal{P}}_\omega+(\sigma_{\nu\omega}+\gamma h_\nu h_\omega)\hat{\mathcal{L}}_\nu\hat{\mathcal{L}}_\omega . \tag{35}$$

In Eqs. (34) and (35), $\varepsilon_{\nu\omega}$ and $\sigma_{\nu\omega}$ are given by

$$\begin{aligned}\varepsilon_{\nu\omega}&=(w_\nu w_\omega)^{-1/2}\sum_j\Omega_jE_j(X_{j\nu}+Y_{j\nu})(X_{j\omega}+Y_{j\omega}) ,\\ \sigma_{\nu\omega}&=(w_\nu w_\omega)^{1/2}\sum_j\Omega_jE_j(X_{j\nu}-Y_{j\nu})(X_{j\omega}-Y_{j\omega}) ,\end{aligned} \tag{36}$$

respectively. The coefficients for $\hat{\mathcal{P}}_0^2$ and $\hat{\mathcal{L}}_0^2$ in (35) are straightforwardly obtained; namely,

$$\begin{aligned}\varepsilon_{00}&=\tfrac{1}{2} ,\\ \sigma_{00}+\gamma h_0^2&=\left[\sum_j\Omega_j/E_j^3\right]^{-1}\left[\frac{4}{G}+\gamma\frac{64\Delta^2}{G^2}\right]=0 ,\end{aligned} \tag{37}$$

for $\gamma=-G/16\Delta^2$.

It means that the divergency contained in H_{11} is canceled out by the divergency contained in $H_{\rm sym}$, both represented by terms proportional to $\hat{\mathcal{L}}_0^2$ in Eq. (35).

The coefficients for nonzero energies w_ν, given by Eqs. (22) and (36), lead to the diagonal form of Eq. (32), as expected. In other words, the orientation in number, or gauge, space introduced in H_{11} is removed by $H_{\rm sym}$. Consequently, the symmetry restoring term $H_{\rm sym}$ could be interpreted as a quadratic form in the angle variable $\hat{\mathcal{L}}_\nu$, as it has been already shown by the result of Eq. (33).

In consequence, we have shown that arguments which have been used to reconstruct the rotational invariance of deformed axially symmetric average fields[2,3,10] can also be used for the pairing Hamiltonian. We remind the reader that the treatment of the rotational case has been discussed[10] in terms of the relationship between the matrix elements of the angular momentum and of the quadrupole operators, once a global rotational invariance was imposed. For this reason, in Ref. 10 $H_{\rm sym}\cong[H_{qp},\hat{J}]^\dagger[H_{qp},\hat{J}]$ was employed, rather than a more conventional quadrupole-quadrupole interaction.

In order to complete the analogy with the pairing case, we shall show that the spectrum of nonzero excitations given by $H_{\rm RPA}$(BCS) of Eq. (8) and $H_{\rm eff}$ of Eq. (32) are comparable. This is shown in Sec. III, where results of calculations based on both formalisms are discussed.

TABLE I. Intrinsic RPA energies. The roots of the secular equations corresponding to $H_{\rm RPA}$(BCS) and $H_{\rm eff}$ are shown for the case of $N=14$ particles in the single-particle basis described in the text.

ν	w_ν [$H_{\rm RPA}$(BCS)] (MeV)	w_ν ($H_{\rm eff}$) (MeV)
1	2.841	2.873
2	2.989	3.303
3	3.571	3.811
4	4.032	4.071

TABLE II. Overlaps between RPA wave functions. The second column shows the overlaps between intrinsic states belonging to $H_{\rm RPA}$(BCS) and $H_{\rm eff}$.

ν	[$H_{\rm RPA}$(BCS), $H_{\rm eff}$] (%)
1	91
2	79
3	83
4	95

III. RESULTS AND DISCUSSION

From the results of the above-developed formalism, we can conclude that, like in the case of space rotations, we can either adopt a model residual interaction or construct it directly from a symmetry restoring treatment of the deformed average field. The case of the pairing force Hamiltonian is a particularly interesting one because we know beforehand the structure of the residual interaction; namely, $H_{(22+40)}$(BCS), once H_{11} has been chosen as a deformed single quasiparticle field. Therefore, the symmetry restoring procedure of Sec. II, via $H_{\rm eff}$, should provide a residual two quasiparticle interaction with similar

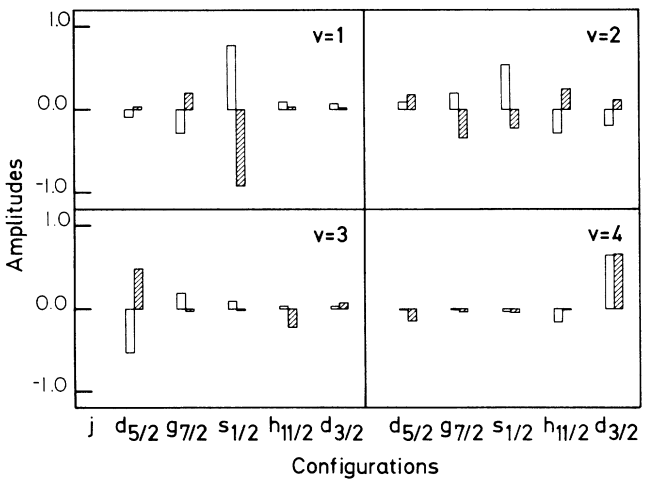

FIG. 1. RPA wave functions. Forward-going RPA amplitudes $\lambda_{j\nu}$ and $X_{j\nu}$ for intrinsic excitations evaluated with $H_{\rm RPA}$(BCS), open histogram, and $H_{\rm eff}$, dashed histogram, are shown. The index ν, like in the tables, indicates the associated state with energy w_ν. The two quasiparticle configurations are indicated at the bottom.

TABLE III. Matrix elements, $|\langle \nu|\hat{T}|0\rangle|^2$, for two-particle transfer. The matrix elements of the transfer operator $\hat{T}=(2)^{-1/2}\sum_j[a_j^\dagger a_j^\dagger]_0^0$, connecting zero and one phonon states, are evaluated with the RPA wave functions corresponding to $H_{\rm RPA}$(BCS) and $H_{\rm eff}$, respectively.

ν	$\lvert\langle\nu\rvert\hat{T}\rvert0\rangle\rvert^2 H_{\rm RPA}$(BCS)	$\lvert\langle\nu\rvert\hat{T}\rvert0\rangle\rvert^2 H_{\rm eff}$
1	0.216	0.059
2	0.185	0.168
3	0.010	0.054
4	0.206	0.335
Total	0.617	0.616

effects upon the RPA intrinsic spectrum, as for $H_{(22+40)}$, but free of collective contributions induced by the number operator. To show it we have performed calculations for a system of 14 particles distributed in the single-particle states $j\equiv(Nlj)=4d_{5/2}$, $4g_{7/2}$, $4s_{1/2}$, $5h_{11/2}$, and $4d_{3/2}$, with energies fixed at the values $\varepsilon_j=0.0$, 0.8, 2.40, 2.50, and 2.80 MeV, respectively. This model space corresponds to shell model orbits above the $N=50$ closed shell and the single-particle energy spacing has been taken from Ref. 11. We have determined BCS quasiparticle energies E_j and occupation numbers U_j and V_j for $G=0.22$ MeV. For this coupling constant we have obtained, for the pairing gap, the value $\Delta=1.42$ MeV. Next we have solved RPA dispersion relations for $H_{\rm RPA}$(BCS), Eq. (11), and for $H_{\rm eff}$, Eq. (20). The results for the intrinsic RPA energies are shown in Table I. The results of both approximations do not differ much although roots corresponding to $H_{\rm eff}$ are slightly shifted upwards. Forward-going amplitudes of intrinsic, nonzero energy modes for both cases are shown in Fig. 1. The dominant components of the first three states, for $H_{\rm eff}$, have opposite phase with respect to those of $H_{\rm RPA}$(BCS), and they are in phase for the last state. Some differences could be observed concerning absolute values of the amplitudes of the states shown in Fig. 1. Their overlaps are shown in Table II. In order to estimate the effect of these differences, in magnitude and in phase, upon a physical observable, we have calculated matrix elements for two-particle transfer processes connecting the RPA vacuum with one-phonon states, in both approximations. The results are shown in Table III. Although some differences are observed concerning the population of individual states, the strengths shown in Table III closely agree for both methods. Finally, as an additional test of the method, we have calculated the quantities $(1/\chi)$ and $(1/\chi)_{\rm BCS+RPA}$, of Eqs. (30) and (31); their values have been found to be of the order of 0.155 and 0.158 MeV, respectively.

IV. CONCLUSIONS

We have shown that the symmetry restoring procedure developed by Pyatov *et al.*,[2,3] for axially symmetric deformed single-particle fields, can also be applied to the case of the pairing Hamiltonian treated in the quasiparticle basis. The analogy between orientations in normal space and in gauge or number space, means between deformed rotational noninvariant single-particle fields and number nonconserving quasiparticle fields, enable us to construct symmetry restoring effective interactions. The overall agreement which has been obtained for the intrinsic two quasiparticle modes, both with the conventional $H_{\rm RPA}$(BCS) and with $H_{\rm eff}$, shows that the zero energy mode can be unambiguously removed.

Finally, we should also note that the symmetry restoring procedure is valid for only one sector of the original Hamiltonian; namely, $H_{\rm RPA}$(BCS). The coupling terms between rotational and vibrational modes, which in the original BCS Hamiltonian are of the form H_{31}, still have to be treated, eventually, in the form which is described in[12] or by more involved techniques.[5] Work is in progress concerning this problem.[13]

ACKNOWLEDGMENTS

This work was supported in part by the Consejo Nacional de Investigaciones Cientificas y Tecnicas (CONICET), Argentina; the authors are fellows of the CONICET, Argentina. We thank Professor A. Zuker for useful discussions.

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