PHYSICAL REVIEW C VOLUME 9, NUMBER 6 JUNE 1974

Generalized semimicroscopic model in odd-mass indium isotopes

S. M. Abecasis*
Laboratorio de Radiaciones, IIAE, Departamento de Física, Facultad de Ciencias Exactas y Naturales, Ciudad Universitaria, Pabellón 1, Nuñez, Buenos Aires, Argentina

and

O. Civitarese and F. Krmpotić
Departamento de Física, Facultad de Ciencias Exactas, Universidad Nacional de La Plata, La Plata, Argentina
(Received 11 July 1973; revised manuscript received 27 February 1974)

A generalization of the classical semimicroscopic model is developed to explain simultaneously quasivibrational and quasirotational nuclear states. The wave functions obtained are used to calculate level sequences, electromagnetic properties, and spectroscopic factors. Experimental data are reasonably well fitted.

[NUCLEAR STRUCTURE ^{115,117}In; calculated levels, J, π, $B(E2)$, $B(M1)$, q, μ, and S. Generalized semimicroscopic model.]

1. INTRODUCTION

According to the classical particle-phonon coupling model, the one-phonon multiplet $|g_{9/2}^{-1}12; IM\rangle$ with total angular momentum I ranging from $\frac{5}{2}^+$ to $\frac{13}{2}^+$ is expected in the vicinity of 1.3 MeV for the odd-mass indium isotopes. For weak coupling and quadrupole harmonic vibrations, the reduced $E2$ transition probability for exciting the multiplet state with angular momentum I should be proportional to $(2I+1)$. Moreover, the magnitude of the $B(E2)$ values for decay of each of these states should be same as that of the $B(E2; {}^12 \rightarrow {}^10+)$ in the even tin core. However, the Coulomb excitation of ^{113}In and ^{115}In (Ref. 1) shows that although a considerable fraction of the one-phonon strength lies in the region between 1.0 and 1.5 MeV, the intensity rule is violated.

Additional experimental evidences argue against the weak particle-phonon coupling; the most important are: (i) rather strong $B(M1)$ transition to the ground state (Ref. 2 and references quoted therein); (ii) fragmentation of the $g_{9/2}$ single-particle strength[3]; (iii) renormalization of the effective charge for the ground-state quadrupole moment, $e_{\text{eff}} = 5.5e_p$ (Ref. 4); (v) existence of additional states—apart from the multiplet— in the energy region of the one-phonon multiplet.

Dietrich *et al.*[2] have recently performed a calculation of the ^{115}In spectrum involving a rather strong coupling of the $g_{9/2}$ proton hole to the quadrupole vibrational states of ^{116}Sn. Their calculations on the level positions and $B(E2)$ values as well as the ground-state quadrupole and the spectroscopic factors observed in the ^{116}Sn$(d, {}^3$He$)$-^{115}In reaction are in good agreement with experiment. The $B(M1)$ are reasonably well reproduced. However, neither this analysis nor the others performed with the same spirit,[5,6] can explain the low-lying $\frac{1}{2}+$ and $\frac{3}{2}+$ excited states experimentally observed.[1,7,8]

On the basis of a highly enhanced $B(E2; \frac{1}{2} \rightarrow \frac{3}{2})$ transition strength [≈100 single-particle units (s.p.u.)] experimentally observed by Bäcklin, Fogelberg, and Malmskog,[1] they have proposed that these levels are members of a rotational band characterized by the Nilsson state $K\pi[Nn_z\Lambda] = \frac{1}{2}+[431]$. Several investigators (Ref. 9 and reference quoted therein), following this suggestion, have tried to identify some other higher-lying states as being members of this rotational sequence.

On the other hand, the ^{115}Cd$(^3$He, $d)^{117}$In reaction studies performed by Ishimatsu *et al.*[10] and by Harar and Horoshko[11] revealed that the excited positive-parity states in ^{117}In cannot be explained by a simple juxtaposition of vibrational and rotational states. In addition, Meyer, Struble, and Smith[8] have obtained an enhancement of less than 40 s.p.u. for the $B(E2, \frac{1}{2}+ \rightarrow \frac{3}{2}+)$ in ^{115}In. This experiment when considered with the recent measurement of the quadrupole moment of the $\frac{3}{2}$ state[12] is inconsistent with a pure rotational nature of these levels.

From the considerations above it can be deduced that the experimental data do not lend support either to the classical particle-phonon coupling interpretation or to the rotational model when they are independently considered. Then, a mixing of rotational with vibrational levels would yield a better description of these isotopes. This approach was followed in the works of Sen[9] and

Mang *et al.*[13]

As far as the intriguing states are concerned, a similar situation was found in the energy spectra of odd-mass cobalt isotopes. Actually, by coupling of a $f_{7/2}$ proton hole to the even-mass nickel nucleus considered as the vibrational core, it has not been possible to reproduce the existence of a $\frac{1}{2}-$ and two $\frac{3}{2}-$ excited states observed in ^{57}Co in the vicinity of 1.5 MeV. This difficulty has recently been overcome in the calculation made by Gomez[14] where two-hole–one-particle proton configurations are coupled to quadrupole vibrational cores. Good general agreement of calculated and experimental properties has been achieved.

In view of the situation above we felt stimulated to make a detailed study of the odd-mass indium isotopes with a new version of the semimicroscopic model (SMM). We limited ourselves to the consideration of ^{115}In and ^{117}In nuclei since they are the best studied experimentally among the odd-mass indium isotopes. In turn only their positive-parity states are analyzed since those of negative parity have been satisfactorily explained elsewhere.[5,6,9,15]

Our first intent was the enlargement of the configuration space by the inclusion of 2h-1p seniority-zero proton configurations $g_{9/2}{}^{-2}(0)g_{7/2}$, $g_{9/2}{}^{-2}(0)d_{5/2}$, $g_{9/2}{}^{-2}(0)d_{3/2}$, and $g_{9/2}{}^{-2}(0)s_{1/2}$.

This is similar to the procedure followed by Gomez[14] in his calculations of the odd-mass cobalt isotopes. For a relatively strong coupling strength we were able to obtain the energies of the $^{1}\frac{1}{2}+$ and $^{1}\frac{3}{2}+$ states in the vicinity of the first multiplet while an over-all agreement of the total level sequence could not be attained for reasonable values of the single-particle energies. In addition, the quadrupole moment of the $^{1}\frac{3}{2}+$ state was too small.

This last fact indicates that more collectiveness is needed which would be accomplished by the inclusion of 2h-1p seniority-three states. Our basic assumption is that the two holes in the $Z=50$ closed shell together with the tin core bring about an average vibrational field. This hypothesis is supported by the quasivibrational pattern exhibited by the even-mass cadmium isotopes. In the zeroth-order approximation, the lowest states are seniority-zero states. Due to the particle-phonon interaction these states receive significant contributions of the seniority-two states which make them more collective. Consequently, an approximate way to take into account the effects of the 2h-1p seniority-three modes of excitation is to suppose that some low-lying states of odd-mass indium isotopes are originated from the coupling of a proton—which has available the orbits above the $g_{9/2}$ shell—to the cadmium vibrational field.

Therefore we developed a version of the SMM which is suitably generalized so as to include the degrees of freedom associated with the 2h-1p modes of excitation and which will be hereafter referred to as the generalized semimicroscopic model (GSMM). A brief report on the present work was presented at the International Conference on Nuclear Physics held in Münich.[16]

2. FORMULATION AND ASSUMPTIONS

In our investigation the usual space of the hole-phonon coupling model is enlarged by the inclusion of the degrees of freedom associated with the incoherent two-hole–one-particle (2h-1p) modes of excitation. To avoid counting the same states twice, the underlying random-phase-approximation structure of the vibrating tin core is assumed to be exclusively due to a pure neutron character.[17]

A. Model Hamiltonian and matrix elements

We present here an approximate way to include the 2h-1p seniority-three modes of excitation taking up with the available computational facilities. Our total Hamiltonian is

$$H = H(\text{core}, h) + H(p) + H(p, h), \tag{1}$$

where

$$H(\text{core}, h) = H_c + H_{sp}(h) - \sum_{\mu} Q^{k}_{2\mu}(h)\alpha^{\dagger}_{2\mu} + H_p - \chi \sum_{\mu} Q^{r}_{2\mu}(h)Q^{r\dagger}_{2\mu}(h), \tag{2}$$

$$H(p) = H_{sp}(p) - \sum_{\mu} [Q^{k}_{2\mu}(p)\alpha^{\dagger}_{2\mu} - \chi Q^{r}_{2\mu}(p)Q^{r\dagger}_{2\mu}(h)], \tag{3}$$

and

$$H(p, h) = -\sum_{\mu} Q^{k}_{2\mu}(p, h)\alpha^{\dagger}_{2\mu} - \chi \sum_{\mu} Q^{r}_{2\mu}(p, h)Q^{r\dagger}_{2\mu}(h). \tag{4}$$

Here we labeled $Q^{r;k}_{2\mu}(p)$, $Q^{r;k}_{2\mu}(h)$, and $Q^{r;k}_{2\mu}(p, h)$, the particle scattering, the hole scattering, and the pair creation plus pair annihilation terms, respectively, for the one-body operator $Q^{r;k}_{2\mu}$ (Ref. 18).

The Hamiltonian (2) represents the energy of the system formed by the tin core coupled to one or two valence hole protons. In the first case it describes the properties of the indium isotopes[6] and in the second case those of the doubly even cadmium nuclei[4,19] in the standard semimicro-

scopic description. Hence, its terms have the following meaning: (i) H_c is the Hamiltonian associated with the harmonic quadrupole vibrations of the tin core. (ii) $H_{sp}(h)$ is the single-particle Hamiltonian associated with the motion of the extra core holes in an average effective spherical potential. (iii) $H_{int}(h) = -\sum_\mu Q^k_{2\mu}(h)\alpha^\dagger_{2\mu}$ is the interaction Hamiltonian associated with the coupling of holes to the tin vibrational field with amplitude $\alpha_{2\mu}$. The zero-point amplitude α_0 is related to the quadrupole phonon excitation energy $\hbar\omega$ and to the quadrupole deformation parameter β by means of

$$\alpha_0 = [(\hbar\omega/2C)]^{1/2} = \beta/\sqrt{5}\,.$$

(iv) The Hamiltonian associated with the residual two-body interaction not included into the effective average potential is a sum of the pairing force H_P and the quadrupole-quadrupole (Q-Q) force[20]

$$H_Q = -\chi \sum_\mu Q^r_{2\mu} Q^{r\dagger}_{2\mu}$$

with strength χ.

Taking into account that the cadmium nuclei exhibit a quasivibrational pattern, we approximate the Hamiltonian (2) for this special case by a vibrational Hamiltonian taken to be of the form

$$\tilde{H}(\text{core}) = \hbar\omega \sum_\mu b^\dagger_{2\mu} b_{2\mu} + E_0(\text{Cd})\,, \tag{5}$$

where $\hbar\omega$ and $b^\dagger_{2\mu}$ ($b_{2\mu}$) are the energy and the creation (annihilation) operators of the cadmium vibrational field and E_0(Cd) stands for the energy of the cadmium vacuum.

The Hamiltonian (3) represents the total energy of the valence proton. The first term is a single-particle Hamiltonian associated with the motion of the extra core proton in an average spherical potential. The second and the third terms give the interaction energy of the valence proton with the tin core and with the valence proton holes, respectively. Since the sum of the latter gives the interaction energy of the cadmium vibrational field with the particle, the Hamiltonian (3) can be written as

$$\tilde{H}(p) = \tilde{H}_{sp}(p) - \sum_\mu Q^k_{2\mu}(p)\tilde{\alpha}^\dagger_{2\mu}\,, \tag{6}$$

where $\alpha_{2\mu}$ is the amplitude of the cadmium vibrational field.

In a word, as far as the 2h-1p modes of excitation are concerned we assume that the two proton holes coupled to the tin core create an average vibrational field which acts on the valence proton lying at the next shell above the $Z = 50$ major shell. Consequently, rather than to evaluate the Hamiltonian (1) in the subspace of 2h-1p excitation coupled to tin quadrupole phonons we calculate the approximate Hamiltonian

$$\tilde{H} = \tilde{H}(\text{core}) + \tilde{H}(p) \tag{7}$$

in the configuration space spanned by the cadmium vibrational states and the particle states.

The Hamiltonian (4) gives the interaction energy between the hole state and the 2h-1p states. For its evaluation we assume that the low-lying cadmium vibrational states are yielded by seniority-zero states [see Eq. (15)].

According to the above assumptions our basic vectors are

$$|h^{-1}, NR; IM\rangle = [\,|h^{-1}m_h\rangle \otimes |NRM_R\rangle]_{IM} \tag{8}$$

and

$$|p, \tilde{N}\tilde{R}; IM\rangle = [\,|pm_p\rangle \otimes |\tilde{N}\tilde{R}\tilde{M}_{\tilde{R}}\rangle]_{IM}\,. \tag{9}$$

Here the symbols label: (i) N and $\tilde{N}$, number of phonons of the tin and cadmium vibrators, respectively; (ii) R and $\tilde{R}$, quantum numbers of the resultant angular momenta $\vec{R}$ and $\vec{\tilde{R}}$ from the coupling of N and $\tilde{N}$ phonons, respectively; (iii) $|h^{-1}m_h\rangle$ and $|pm_p\rangle$, the wave functions of the fermions with h and p being the set of quantum numbers necessary to completely describe the proton-hole and proton-particle states with respect to the $Z = 50$ major shell; (iv) I and M, the total quantum numbers of the indium nucleus; and (v) $|NRM_R\rangle$ and $|\tilde{N}\tilde{R}\tilde{M}_{\tilde{R}}\rangle$, the wave functions of the tin and cadmium cores, respectively, assumed to be harmonic quadrupole vibrators.

The matrix elements of the Hamiltonian (2) between the hole states (8) were calculated in the usual way[4]

$$\begin{aligned}\langle h'^{-1}, N'R'; IM|H|h^{-1}, NR; IM\rangle &= [-\epsilon_h + N\hbar\omega]\delta_{NN'}\delta_{RR'}\delta_{hh'} \\ &\quad + (-)^{R'+I+h}\begin{Bmatrix} h' & R' & I \\ R & h & 2 \end{Bmatrix} \\ &\quad \times \langle h'\|k(r)Y_2\|h\rangle\langle N'R'\|\alpha_2\|NR\rangle\,.\end{aligned} \tag{10}$$

Here the symbol $\{\ \}$ is a six-j coefficient; $\langle h'\|k(r)Y_2\|h\rangle$ is the reduced matrix element of the hole coordinates and $\langle N'R'\|\alpha_2\|NR\rangle$ is the reduced matrix element of the collective operator defined by[4]

$$\begin{aligned}\langle N'R'\|\alpha_2\|NR\rangle = \alpha_0[&\langle NR\|b_2\|NR\rangle\delta_{NN'+1} \\ &+ \langle N'R'\|b_2\|NR\rangle\delta_{N'N+1}]\,.\end{aligned} \tag{11}$$

The matrix elements of the Hamiltonian H be-

tween the particle-like states (9) are of the form

$$\langle p', \tilde{N}'\tilde{R}'; IM|\tilde{H}|p, \tilde{N}\tilde{R}; IM\rangle = [\tilde{N}\hbar\omega + E_0 + \epsilon_p]\,\delta_{\tilde{N}\tilde{N}'}\delta_{\tilde{R}\tilde{R}'} + (-)^{\tilde{R}'+I+p+1}\begin{Bmatrix} p' & \tilde{R}' & I \\ \tilde{R} & p & 2 \end{Bmatrix} \times \langle p'\|k(r)Y_2\|p\rangle\langle \tilde{N}'\tilde{R}'\|\alpha_2\|\tilde{N}\tilde{R}\rangle , \quad (12)$$

with

$$\langle \tilde{N}'\tilde{R}'\|\alpha_2\|\tilde{N}\tilde{R}\rangle = \tilde{\alpha}_0[\langle \tilde{N}'\tilde{R}'\|\tilde{b}_2\|\tilde{N}\tilde{R}\rangle\delta_{\tilde{N}\tilde{N}'+1} + \langle \tilde{N}'\tilde{R}'\|\tilde{b}_2\|\tilde{N}\tilde{R}\rangle\delta_{\tilde{N}'\tilde{N}'+1}] . \quad (13)$$

Finally, the matrix elements of the Hamiltonian (7) can be written as

$$\langle p', \tilde{N}'\tilde{R}'; IM|H(p,h)|h^{-1}NR; IM\rangle = \left(\frac{2}{2h+1}\right)^{1/2}(-)^{R'+I+h}\begin{Bmatrix} p' & R' & I \\ R & h & 2 \end{Bmatrix} \times \langle p'\|k(r)Y_2\|h\rangle\langle N'R'\|\alpha_2\|NR\rangle , \quad (14)$$

where the last factor is defined by Eq. (11). They are evaluated using the zeroth-order approximation for the low-lying cadmium vibrational states

$$|\tilde{N}\tilde{R}\tilde{M}_{\tilde{R}}\rangle = \big|\,|(h)^{-2}0,\ NR\,|R=\tilde{R}, M_R=\tilde{M}_{\tilde{R}}\rangle . \quad (15)$$

The eigenstates of the total Hamiltonian (1) with total angular momentum I are linear combinations of the basic vectors (8) and (9):

$$|{}^{\tau}IM\rangle = \sum_{h,NR}\eta(h^{-1}, NR; {}^{\tau}I)|h^{-1}, NR; IM\rangle + \sum_{p,\tilde{N}\tilde{R}}\eta(p, \tilde{N}\tilde{R}; {}^{\tau}I)|p, \tilde{N}\tilde{R}; IM\rangle , \quad (16)$$

where the expansion coefficients η are obtained by diagonalizing the energy matrices and the superindex τ distinguishes between states of same angular momentum.

B. Spectroscopic factors

The eigenstates (16) have been used to calculate the spectroscopic factors $S({}^{\tau}I)$ for (i) proton pick-up reactions and (ii) proton stripping reactions. For the first case they are given by

$$S({}^{\tau}I) = |\eta(h, 0\,0; {}^{\tau}I)|^2 \quad (17)$$

and for the second case

$$S({}^{\tau}I) = |\eta(p, 0\,0; {}^{\tau}I)|^2 \quad (18)$$

when the transferred proton falls into a level above the $Z = 50$ major shell, while

$$S({}^{\tau}I) = \frac{2}{2h+1}|\eta(h, 0\,0; {}^{\tau}I)|^2 \quad (19)$$

for the reverse situation.

C. Electromagnetic transition operators

The quadrupole electric transition operators are given by

$$Q^e_{2\mu}(\text{core}, h) = q_e(\text{core})\alpha^{\dagger}_{2\mu} + e_{\text{eff}}Q^r_{2\mu}(h), \quad (20)$$

for the hole-like states (21);

$$\tilde{Q}^e_{2\mu}(\text{core}, p) = \tilde{q}_e(\text{core})\tilde{\alpha}^{\dagger}_{2\mu} + e_{\text{eff}}Q^r_{2\mu}(p) , \quad (21)$$

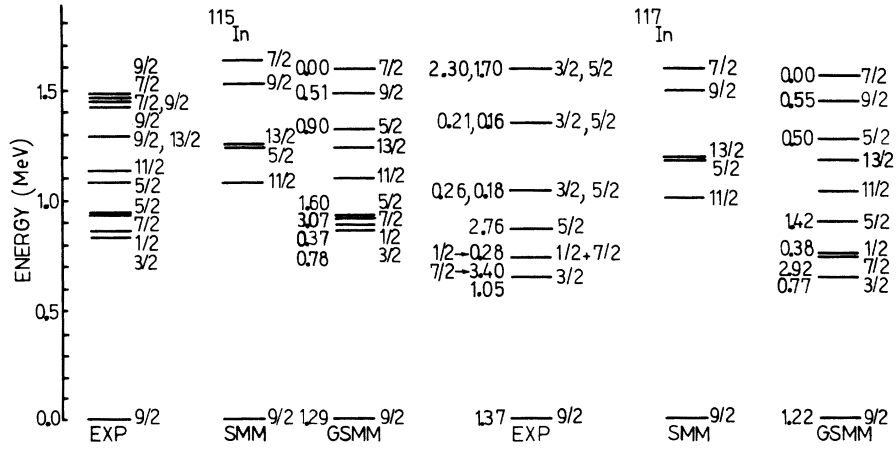

FIG. 1. Experimental and calculated even-parity levels of ^{115}In and ^{117}In. The first and fourth columns show the experimental energy levels for ^{115}In {Refs. 1, 2, 28–30, and V. Sergeev, J. Becker, L. Erikson, L. Gidefelt, and L. Holmberg [Nucl. Phys. A202, 385 (1973)]} and ^{117}In (Ref. 11), respectively. The remaining columns indicate our theoretical results obtained with the SMM and the GSMM. The numbers at the left of the levels indicate the spectroscopic factors $(2j+1)S_j({}^{\tau}I)$ for Cd(^{3}He, d)In reactions.

for the particle-like states (9); and

$$Q^e_{2\mu}(p,h) = e_{\text{eff}} Q^r_{2\mu}(p,h), \tag{22}$$

for the crossing terms. Here e_{eff} is the effective charge of the proton and

$$q_e(\text{core}) = (3/4\pi) Z e R_0^2, \tag{23}$$

$$\tilde{q}_e(\text{core}) = (3/4\pi) \tilde{Z} e \tilde{R}_0^2 \tag{24}$$

are the electric quadrupole moment generated by the collective motion of the nucleon of the tin and cadmium cores, respectively.

TABLE I. Comparison of experimental and calculated quadrupole electric and dipole magnetic moments and $B(E2)$ and $B(M1)$ values for the low-lying positive-parity states of ^{115}In.

	Q [e b]			μ [μ_N]		
		Theory			Theory	
$^{\kappa}I$	Experiment	SMM	GSMM	Experiment	GMM	GSMM
$^{1}\frac{9}{2}$	0.830 [a]	0.928	0.911	5.536 [b]	5.757	5.756
$^{1}\frac{3}{2}$	|0.60 ± 0.08| [c]		−0.539	0.53 ± 0.09 [d]		0.725
$^{1}\frac{1}{2}$						1.832
$^{1}\frac{7}{2}$			−1.157			2.137
$^{1}\frac{5}{2}$			−0.391			3.784
$^{1}\frac{11}{2}$		0.618	0.613		5.819	5.828
$^{2}\frac{5}{2}$		0.607	0.070		4.467	3.919
$^{2}\frac{9}{2}$		0.598	0.471		5.459	5.354

		$B(E2)$ [e^2b^2]$\times 10^{-2}$			$B(M1)$ [μ_N]^2	
$^{\kappa}I_i$	$^{\kappa}I_f$	Experiment	SMM	GSMM	Experiment	GMM	GSMM
		≥40 [e]					
$^{1}\frac{1}{2}$	$^{1}\frac{3}{2}$			24.63	$0.008^{+0.0016}_{-0.0012}$ [e]		0.007
		<16 [f]					
$^{1}\frac{3}{2}$	$^{1}\frac{7}{2}$	36 [g]		25.91			
$^{2}\frac{5}{2}$	$^{2}\frac{7}{2}$		5.14	1.60	0.115 [h]	0.564	0.408
$^{1}\frac{5}{2}$	$^{1}\frac{9}{2}$	0.45 ± 0.05 [h]		2.38			
$^{2}\frac{5}{2}$	$^{1}\frac{9}{2}$	3.77 ± 0.33 [h]	5.26	2.91			
$^{2}\frac{7}{2}$	$^{1}\frac{9}{2}$	1.55 ± 0.40 [h]	1.20	1.28		0.230	0.227
$^{1}\frac{9}{2}$	$^{2}\frac{9}{2}$	1.24 ± 0.20 [h]	2.06	2.33		0.022	0.017
$^{1}\frac{9}{2}$	$^{3}\frac{9}{2}$	0.60 ± 0.15 [h]		0.00			0.006
$^{1}\frac{9}{2}$	$^{1}\frac{11}{2}$	10.0 ± 1.1 [h]	10.01	9.88	0.298 ± 0.064 [h]	0.205	0.191
$^{2}\frac{9}{2}$	$^{1}\frac{11}{2}$		2.80	2.69	0.489 ± 0.086 [h]	0.435	0.450
$^{1}\frac{9}{2}$	$^{1}\frac{13}{2}$	5.0 ± 0.5 [h]	6.97	7.02			
					0.689 ± 0.080 [h]		
$^{1}\frac{11}{2}$	$^{1}\frac{13}{2}$		2.77	2.70		0.355	0.352
					0.5 ± 0.1 [g]		

[a] G. F. Fuller and V. W. Cohen, Nucl. Data B9, Appendix 1 (1965).

[b] V. S. Shirley, in *Table of Nuclear Moments, Hyperfine Structure, and Nuclear Radiations*, edited by E. Mathias and D. A. Shirley (North-Holland, Amsterdam, 1968).

[c] Reference 12.

[d] T. Badica, C. Ciortea, S. Dima, A. Gelberg, S. Petrovici, and I. Popescu, in *Proceedings of the International Conference on Nuclear Physics, Münich, 1973*, edited by J. de Boer and H. J. Mang (North-Holland/American Elsevier, 1973), p. 146.

[e] Reference 1.

[f] Reference 8.

[g] V. Sergeev, J. Becker, L. Erikson, L. Gidefelt, and L. Holmberg, Nucl. Phys. A202, 385 (1973).

[h] Reference 2.

Then the reduced matrix elements take the form

$$\langle \tau' I' \| Q_2^{el} \| \tau I \rangle = [(2I+1)(2I'+1)]^{1/2}$$
$$\times \Bigg\{ \sum_{\substack{h\,NR \\ h'N'R'}} \eta(h'N'R';\tau' I')\eta(hNR;\tau I) \Big[(-)^{h'+I+R+1} e_{\rm eff} \begin{Bmatrix} h & h' & 2 \\ I' & I & R \end{Bmatrix} \langle h' \| r^2 Y_2 \| h \rangle \delta_{NN'} \delta_{RR'}$$
$$+ (-)^{h+I'+R} q_e \begin{Bmatrix} I & I' & 2 \\ R' & R & h \end{Bmatrix} \langle N'R' \| \alpha_2 \| NR \rangle \delta_{hh'} \Big]$$
$$+ \sum_{\substack{p\tilde{N}\tilde{R} \\ p'\tilde{N}'\tilde{R}'}} \eta(p'\tilde{N}'\tilde{R}';\tau' I')\eta(p\tilde{N}\tilde{R};\tau I) \Big[(-)^{p'+I+\tilde{R}} e_{\rm eff} \begin{Bmatrix} p & p' & 2 \\ I' & I & \tilde{R} \end{Bmatrix} \langle p' \| r^2 Y_2 \| p \rangle \delta_{\tilde{N}\tilde{N}'} \delta_{\tilde{R}\tilde{R}'}$$
$$+ (-)^{p+I'+\tilde{R}} \tilde{q}_e \begin{Bmatrix} I & I' & 2 \\ \tilde{R}' & \tilde{R} & p \end{Bmatrix} \langle \tilde{N}'\tilde{R}' \| \alpha \| \tilde{N}\tilde{R} \rangle \delta_{pp'} \Big]$$
$$+ \sum_{\substack{h\,NR \\ p'N'R'}} \eta(p'N'R';\tau' I')\eta(hNR;\tau I)(-)^{p'+I+R+1}(2/2h+1)^{1/2} e_{\rm eff} \begin{Bmatrix} h & p' & 2 \\ I' & I & R \end{Bmatrix} \langle p' \| r^2 Y_2 \| h \rangle \delta_{NN'} \delta_{RR'}$$
$$+ \sum_{\substack{p\,NR \\ h'N'R'}} \eta(h', N'R';\tau' I')\eta(p, NR;\tau I)(-)^{h'+I+R+1} \left(\frac{2}{2h'+1} \right)^{1/2} e_{\rm eff} \begin{Bmatrix} p & h' & 2 \\ I' & I & R \end{Bmatrix} \langle h' \| r^2 Y_2 \| p \rangle \delta_{NN'} \delta_{RR'} \Bigg\} . \tag{25}$$

Similarly, the matrix elements of the dipole magnetic transition operator M_1^m take the form

$$\langle \tau' I' \| M_1^m \| \tau I \rangle = \mu_N (3/4\pi)^{1/2}$$
$$\times \Bigg\{ \sum_{\substack{h\,NR \\ h'N'R'}} \eta(h', N'R';\tau' I')\eta(h, NR;\tau I) \Big[g_R \langle I' \| \hat{I} \| I \rangle + (-)^{I+h'+R'+1} [(2I+1)(2I'+1)]^{1/2} \begin{Bmatrix} h' & 1 & h \\ I & R' & I' \end{Bmatrix}$$
$$\times \langle h' \| \hat{j} \| h \rangle (g_l - g_R) \delta_{NN'} \delta_{RR'} + (-)^{I+h'+R'+1}$$
$$\times [(2I+1)(2I'+1)]^{1/2} \begin{Bmatrix} h' & 1 & h \\ I & R' & I' \end{Bmatrix} \langle h' \| \hat{s} \| h \rangle (g_s - g_l) \delta_{NN'} \delta_{RR'} \Big]$$
$$+ \sum_{\substack{p\tilde{N}\tilde{R} \\ p'\tilde{N}'\tilde{R}'}} \eta(p', \tilde{N}'\tilde{R}';\tau' I')\eta(p, \tilde{N}\tilde{R};\tau I) \Big[g_{\tilde{R}} \langle I' \| \hat{I} \| I \rangle + (-)^{I+p'+\tilde{R}'+1} [(2I+1)(2I'+1)]^{1/2} \begin{Bmatrix} p' & 1 & p \\ I & \tilde{R}' & I' \end{Bmatrix}$$
$$\times \langle p' \| \hat{j} \| p \rangle (g_l - g_{\tilde{R}}) \delta_{\tilde{N}\tilde{N}'} \delta_{\tilde{R}\tilde{R}'} + (-)^{I+p'+\tilde{R}'+1}$$
$$\times [(2I+1)(2I'+1)]^{1/2} \begin{Bmatrix} p' & 1 & p \\ I & \tilde{R}' & I' \end{Bmatrix} \langle p' \| \hat{s} \| p \rangle (g_s^{\rm eff} - g_l) \delta_{NN'} \delta_{RR'} \Big]$$
$$+ \sum_{\substack{h\,NR \\ p'N'R'}} \eta(p', N'R';\tau' I')\eta(h, NR;\tau I)(-)^{I+p'+R'+1} \left(\frac{2}{2h+1} \right)^{1/2}$$
$$\times [(2I+1)(2I'+1)]^{1/2} \begin{Bmatrix} p' & 1 & h \\ I & R' & I' \end{Bmatrix} \langle p' \| \hat{s} \| h \rangle (g_s^{\rm eff} - g_l) \delta_{NN'} \delta_{RR'}$$
$$+ \sum_{\substack{p\,NR \\ h'N'R'}} \eta(h', N'R';\tau' I')\eta(p, NR;\tau I)(-)^{I+h'+R'+1} \left(\frac{2}{2h'+1} \right)^{1/2}$$
$$\times [(2I+1)(2I'+1)]^{1/2} \begin{Bmatrix} h' & 1 & p \\ I & R' & I' \end{Bmatrix} \langle h' \| \hat{s} \| p \rangle (g_s^{\rm eff} - g_l) \delta_{NN'} \delta_{RR'} \Bigg\} . \tag{26}$$

Here the symbols have the following meaning: (i) μ_N, the nuclear magneton; (ii) g_l, g_R^{eff}, g_R ($g_{\tilde{R}}$), are the effective gyromagnetic ratios for the orbital, spin, and tin (cadmium) core, respectively. For the expressions of the reduced matrix elements of the operators $\hat{l}$, $\hat{j}$, and $\hat{s}$, see Ref. 4.

3. RESULTS AND DISCUSSION

In our calculation, the single-particle wave functions were generated from a Woods-Saxon well with spin-orbit coupling. The corresponding energies were considered to be $\epsilon(d_{5/2}) - \epsilon(g_{7/2}) = 0.50$ and 0.75 MeV for ^{115}In and ^{117}In, respectively, and $\epsilon(d_{3/2}) - \epsilon(g_{7/2}) = 2.60$ MeV and $\epsilon(s_{1/2}) - \epsilon(g_{7/2}) = 2.95$ MeV. The first value was regarded as an adjustable parameter to reproduce the spin sequence of the $^1\frac{3}{2}^+$, $^1\frac{1}{2}^+$, $^1\frac{7}{2}^+$, and $^1\frac{5}{2}^+$ states. The remaining two are those of Reehal and Sorensen.[21] The energy difference between the lowest unperturbed particle-like and hole-like states $\Delta E = [E_0(\text{Cd}) + \epsilon(g_{7/2})] - [E_0(\text{Sn}) - \epsilon(g_{9/2})]$ was estimated to be $S_p(Z=51, N) - S_p(Z=49, N)$, yielding values of -2.40 and -2.34 MeV for ^{115}In and ^{117}In, respectively. Here $S_p(Z, N)$ denotes the proton separation energy of a nucleus (Z, N). Our values were obtained from the mass tables of Wapstra and Gove.[22] The phonon energies and the amplitudes of the vibrational motion $\alpha_0 = \beta/\sqrt{5}$ were taken from experiment.[23]

To investigate the extent to which the GSMM succeed in improving the agreement with experiment, calculations were also performed with the SMM. The resulting spectra and the spectroscopic factors for the one-body stripping reactions Cd(^{3}He, d)In are compared in Fig. 1 with experimental data. In Table I are listed the quadrupole electric and dipole magnetic reduced transition probabilities and moments for ^{115}In. For the electric case we present the results obtained with the effective proton charge $e_p^{\text{eff}} = 2e$; e is the proton charge. For the magnetic case we give the values obtained with $g_l = 1$, $g_s = 3.911$, and $g_R = Z/A$. The enhanced $E2$ transitions between the $^1\frac{3}{2}$, $^1\frac{1}{2}$, $^1\frac{7}{2}$, $^1\frac{5}{2}$, $^3\frac{9}{2}$, $^4\frac{13}{2}$, and $^2\frac{11}{2}$ positive-parity states, the intensity ratios involving crossover-to-stopover $E2$ transitions, as well as their large and negative quadrupole moments allow us to assume the existence of a quasirotational structure within a quasivibrational picture.

Our approximations appear quite reasonable in view of the assumption concerning the harmonicity of the tin vibrating core as well as the previous calculations of even-even cadmium nuclei.[4,19] The latter have shown that the lowest five states in cadmium are mostly a $(g_{9/2})_0^{-2}$ pair coupled to zero-, one- and two-phonon states and also that the particle and collective parts of the electric quadrupole transition operator contribute coherently to the stopover transitions $^12 \rightarrow {}^10$, $^20 \rightarrow {}^12$, $^22 \rightarrow {}^12$, and $^14 \rightarrow {}^12$ whereas they add incoherently for the crossover transition $^22 \rightarrow {}^10$. Although it is also true that there is coherence for the quadrupole moment of the ^{12} state, its most recent measurement[24] gives a value similar to that observed in even-even tin nuclei.[25,26] It is relevant to notice that the introduction of anharmonic effects in cadmium isotopes[27] drastically affects only the static quadrupole moment.

The present results show that the GSMM is able to overcome the most remarkable difficulties met with the SMM, achieving a reasonable agreement with experiment. However, it does not modify seriously the description of the electromagnetic properties of the quasivibrational states except for the $\frac{5}{2}^+$ and $\frac{9}{2}^+$ states.

The comparison with experiment indicates that this admixture would not be the most adequate. This fact can be attributable to single-particle energies employed in this calculation and/or to the approximation, Eq. (15). In a few words, the coupling scheme we discuss seems to have some promising aspects specially concerned with the coexistence of quasirotational and quasivibrational structures. Nevertheless, some work still remains to be done (and it is in progress) to estimate how serious the approximations involved are.

One of us (S. M. A.) wishes to express her indebtedness to Professor H. J. Mang for fruitful discussions and hospitality during her stay in Münich.

---

*Member of the Scientific Research Career of the Consejo Nacional de Investigaciones Científicas y Técnicas, Buenos Aires, Argentina.

[1]A. Bäcklin, B. Fogelberg, and S. G. Malmskog, Nucl. Phys. A96, 539 (1967).

[2]F. S. Dietrich, B. Herskind, R. A. Naumann, R. G. Stokstad, and G. E. Walker, Nucl. Phys. A155, 209 (1970).

[3]C. V. Weiffenbach and R. Tickle, Phys. Rev. C 3, 1668 (1971).

[4]G. Alaga, F. Krmpotić, V. Lopac, V. Paar, and L. Sips, Institute Ruder Bosković, Zagreb, Yugoslavia, Report No. IRB-TP-70, 1970 (unpublished).

[5]B. I. Atalay and L. W. Chiao-Yap, Phys. Rev. C 5, 369 (1972).

[6]A. Covello, V. R. Manfredi, and N. Azziz, Nucl. Phys.

A201, 215 (1973).
[7]E. M. Bernstein, G. G. Seaman, and J. M. Palms, Nucl. Phys. A141, 67 (1970).
[8]R. A. Meyer, G. L. Struble, and N. Smith, Bull. Am. Phys. Soc. 17, 467 (1972).
[9]S. Sen, Nucl. Phys. A191, 29 (1972).
[10]T. Ishimatsu, T. Nakagawa, H. Orihara, H. Ohmura, T. Awaya, and K. Yagi, J. Phys. Soc. Jap. 30, 1519 (1971).
[11]S. Harar and R. N. Horoshko, Nucl. Phys. A183, 161 (1972).
[12]D. Haas and D. A. Shirley, J. Chem. Phys. 58, 3339 (1973).
[13]H. J. Mang, F. Krmpotić, and S. M. Abecasis, Z. Phys. 262, 39 (1973).
[14]J. M. G. Gomez, Phys. Rev. C 6, 149 (1972).
[15]L. Silverberg, Ark. Fys. 20, 341 (1961).
[16]S. M. Abecasis, O. Civitarese, and F. Krmpotić, in *Proceedings of the International Conference on Nuclear Physics, Münich, 1973*, edited by J. de Boer and H. J. Mang (North-Holland/American Elsevier, 1973), p. 86.
[17]W. P. Beres and W. E. Dorenbusch, Phys. Rev. C 3, 952 (1971).
[18]A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, New York, to be published), Vol. II.
[19]G. Alaga, F. Krmpotić, and V. Lopac, Phys. Lett. 24B, 537 (1967).
[20]S. T. Belyaev, K. Dan. Vidensk. Selsk. Mat.-Fys. 31, No. 11 (1958).
[21]B. S. Reehal and R. A. Sorensen, Phys. Rev. C 2, 819 (1970).
[22]A. H. Wapstra and M. B. Gove, Nucl. Data A9, 303 (1971).
[23]P. H. Stelson and L. Grodzins, Nucl. Data A1, 21 (1965).
[24]L. Berant, R. A. Eisenstein, Y. Horowitz, U. Smilansky, P. N. Tandon, J. S. Greeberg, A. M. Kleinfeld, and H. G. Maggi, Nucl. Phys. A196, 312 (1972).
[25]P. H. Stelson, F. K. McGowan, R. L. Robinson, and W. T. Milner, Phys. Rev. C 2, 201 (1970).
[26]A. M. Kleinfeld, R. Covello-Moro, H. Ogata, G. G. Steadiman, and J. de Boer, Nucl. Phys. A154, 499 (1970).
[27]D. R. Bes and G. G. Dussel, Nucl. Phys. A135, 1, 25 (1969).
[28]A. Marcinkowski, A. Bäcklin, and I. Berqvist, Nucl. Phys. 79, 781 (1972).
[29]G. Graeffe, C. W. Tang, C. D. Coryell, and G. E. Gordon, Phys. Rev. 149, 884 (1966).
[30]J. McDonald, D. Porter, and D. T. Stewart, Nucl. Phys. A104, 177 (1967).