Properties of the Odd-Mass Iodine Isotopes in a Particle-Phonon Coupling Scheme

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A qualitative description of the low-lying states of odd-mass iodine nuclei is proposed in terms of a three-proton cluster coupled to a harmonic-quadrupole vibrational field. Using reasonable values for the parameters involved, the calculated spectrum of ^{131}I is in good agreement with experimental data. The comparison with experiments is also satisfactory for the one-body stripping strengths, $B(E2)$ transition probabilities, and magnetic-dipole and electric-quadrupole moments. We do not obtain good results concerning the energies of the lightest I isotopes.

I. INTRODUCTION

The particle-vibrator interaction can be considered the most important coupling phenomenon concerning the origin of vibrational anharmonicities and it provides a very natural interpretation of many of the features of spherical nuclei. In this model a few extra-core particles (shell-model cluster) polarize the whole nucleus. This effect is reflected in the renormalization of single-particle operators, splitting of multiplets, and removal of single-particle strengths from the shell-model states through multiplets.[1-3]

Recently, it has been shown that the particle-phonon scheme is capable of explaining many of the properties of odd-mass antimony[4-6] and even-mass tellurium nuclei.[7] In these studies the valence protons have been coupled to a harmonic-quadrupole field and the residual interaction between them has been approximated by a pairing force. In the present communication the same approach, i.e., coupling of a three-proton cluster to a quadrupole vibrator, is applied to the odd-mass iodine isotopes.

This investigation was stimulated by the fact that there are, at present, sufficient experimental data available on the low-energy states of the iodine isotopes in the range of the mass number $A = 125-133$, to allow for a qualitative study of their properties.[8-23] The experimental results indicate that the proposed model can be appropriate for their description. For example, from the study of the reaction ^{126,128,130}Te(^{3}He, d)^{127,129,131}I, Auble, Ball, and Fulmer have concluded that the three-proton clusters, as well as the tin core, strongly participate in dividing up the spectroscopic strengths.[13] This fact was also confirmed from the analysis of the ft values for the allowed decays to the lowest states ($ft \simeq 6-7$), yielding an inverse relationship between the spectroscopic factor and the ft values. In addition, from Coulomb excitation experiments it is known that several $B(E2)$ transitions are strongly enhanced with respect to their single-particle [10-16]; in particular the $B(E2; \frac{7}{2}_1 \rightarrow \frac{5}{2}_1)$ transition is approximately 70–100 times larger than the single-particle estimate.

Previous theoretical studies of odd-mass iodine nuclei were performed within the framework of the quasiparticle-plus-phonon model[24,25] and by coupling a proton to an even-core tellurium nucleus.[26] The model proposed here includes more correlations from the neighboring even nuclei. In fact, it considers the anharmonicities induced in the even Te core through the interaction of the two protons with the tin core, and in addition it takes into account the effects of the broken and promoted pairs. The same coupling scheme was applied some years ago to odd Au nuclei by Alaga and Ialongo[27] and quite recently to odd Ga isotopes by Paar[28] and Almar *et al.*[29] Paar has also explained the $j-1$ anomaly for ^{107,109}Ag[30] and has accounted for the quasirotational features of ^{57}Fe,[31] using the same picture. The coupling of a three-particle cluster to a harmonic-quadrupole field also provides a good description of the odd Mo and Cs nuclei.[32,33]

II. MODEL

A detailed description of the model can be found in Ref. 2; we simply sketch the procedure and give the pertinent definitions here. The total Hamiltonian of the system is written as

$$H = H_{sp} + H_{vib} + H_{res} + H_{int} .$$

Here, H_{sp} describes the energy of the valence shell particles; H_{vib}, the free quadrupole-vibrational field; H_{res}, the residual (pairing) two-body interaction; and

$$H_{\rm int} = -\left(\frac{\hbar\omega}{2C}\right)^{1/2} \sum_{\mu=-2}^{2} \sum_{i} k(r_i) Y_2^{\mu *}(\theta_i, \phi_i) \times [b_2^{\mu\dagger} + (-)^\mu b_2^{-\mu}],$$

the particle-field interaction.

The Hamiltonian is diagonalized in the basis

$$|\{[(j_1 j_2)J_{12}, j_3]J, NR\}IM\rangle .$$

Here the symbols $j_i \equiv (n_i, l_i, j_i)$ represent quantum numbers of the proton states, and the angular momenta J_{12} and J correspond to successive couplings $\vec{j}_1 + \vec{j}_2 = \vec{J}_{12}$ and $\vec{J}_{12} + \vec{j}_3 = \vec{J}$. The symbols N and R represent the phonon number and the angular momentum of the N phonon state, respectively; I and M stand for the total angular momentum of the nucleus.

The eigenvector n of the Hamiltonian H are of the form:

$$|I_n M\rangle = \sum_{j_1 j_2 j_3 J_{12} NR} \eta_3(j_1 j_2, J_{12}, j_3, J, NR; I_n) \times |\{[(j_1 j_2)J_{12}, j_3]NR\}IM\rangle .$$

In order to evaluate one-nucleon stripping processes the wave function of the ground state of the target nucleus is obtained by calculating the above Hamiltonian in the representation of two particles and N quadrupole phonons. It has the form

$$|0_1 0\rangle = \sum_{j_1 j_2 J_{12} NR} \eta_2(j_1 j_2, J_{12}, NR; 0_1) \times |[(j_1 j_2)J_{12}, NR]00\rangle$$

and the spectroscopic factor to the final state $|I_n M\rangle$

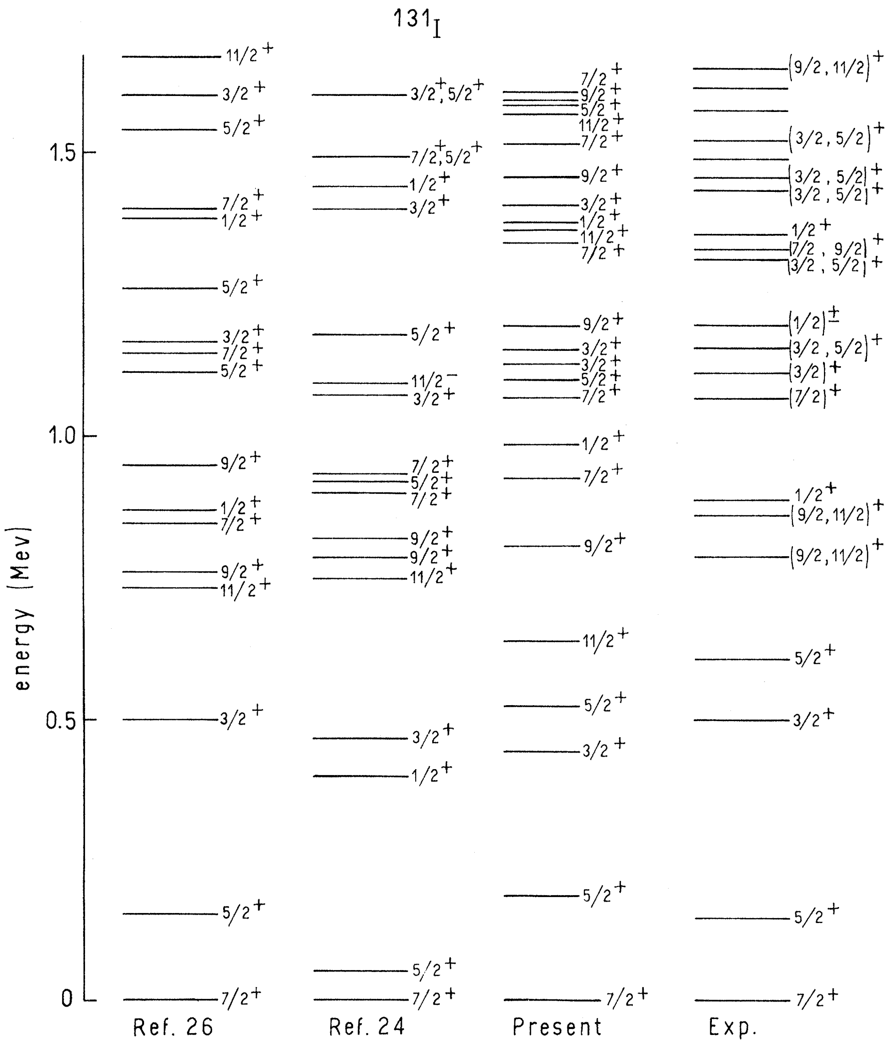

FIG. 1. Experimental and theoretical spectra for ^{131}I. For comparison the calculations with the particle-phonon and quasiparticle-phonon models are also presented. The parameters used in the present calculation are given in the text.

is given by

$$S_j(0_1, I_n) = \left| \sum_{\kappa_2' \kappa_3} \left[\frac{2J+1}{(2j+1)(2J_{12}'+1)} \right]^{1/2} \eta_2(\kappa_2', 0_1)\eta_3(\kappa_3, I_n)\theta_j(J_{12}', J)\delta_{RR'}\delta_{NN'}\delta_{R' J_{12}'} \right|^2,$$

where $\kappa_2 \equiv (j_1 j_2, J_{12}, NR)$ and $\kappa_3 \equiv (j_1 j_2, J_{12}, j_3, NR)$, and $\theta_j(J_{12}', J)$ is the usual shell-model overlap:

$$\theta_j(J_{12}', J) = \frac{\langle \{[(j_1 j_2)J_{12}] j_3\} J \| a_j^\dagger \| (j_1' j_2') J_{12}' \rangle}{(2J+1)^{1/2}}.$$

The spectroscopic factors obey the sum rule

$$\sum_f S_j(0_1, I_f) = 1 - \frac{\langle p \rangle_j}{2j+1},$$

where $\langle p \rangle_j$ is the average number of protons in the orbit nlj of the target nucleus and is given by

$$\langle p \rangle_j = \sum_{\kappa_2} |\eta_2(\kappa_2, 0_1)|^2 (\delta_{jj_1} + \delta_{jj_2}).$$

The electric-quadrupole and magnetic-dipole operators consist of a particle and a vibrator part

$$M(E2, \mu) = e_p^{\text{eff}} \sum_i r_i^2 Y_2^\mu(\theta_i, \phi_i) + \frac{3}{4\pi} e_v^{\text{eff}} [b_2^\mu + (-)^\mu b_2^{-\mu}],$$

$$\vec{M}(M1) = \left(\frac{3}{4\pi}\right)^{1/2} [g_R \vec{R} + g_l \vec{L} + g_s \vec{S}] \mu_N,$$

where e_p^{eff} is the effective proton charge and $e_v^{\text{eff}} = Ze(\hbar\omega/2C)^{1/2}$ is the effective vibrator charge. The symbols $\vec{L}$ and $\vec{S}$ are the total orbital and the total spin angular momentum of the particles, respectively. The quantities g_R, g_l, and g_s^{eff} are the effective gyromagnetic ratios.

III. CALCULATIONS AND COMPARISON WITH EXPERIMENTS

In order to test the consistency of the model we choose a parametrization very close to that used in the calculation of ^{125}Sb and ^{126,128,130}Te nuclei,[5-7] namely:

$\epsilon(g_{7/2}) = 0.$ MeV,

$\epsilon(d_{5/2}) = 0.5$ MeV,

$\epsilon(d_{3/2}) = 1.8$ MeV,

$\epsilon(s_{1/2}) = 2.0$ MeV,

$\epsilon(h_{11/2}) = 1.9$ MeV,

$\hbar\omega = 1.2$ MeV,

$G = 0.15$ MeV,

$a = 0.5$ MeV.

The symbol $\epsilon(lj)$ labels the single-particle energies, $\hbar\omega$ the phonon energy, G the effective pairing strength, and

$$a = \langle k \rangle \left(\frac{\hbar\omega}{8\pi C}\right)^{1/2},$$

the particle-vibrator coupling strength.

All the unperturbed states for which the diagonal terms are less than 3.0 MeV have been considered. In this way the main matrices are of the order of 150×150. The effects of the truncation have been tested by taking different cutoff energies in the vicinityof 3.0 MeV, but no significant difference

TABLE I. Experimental and calculated spectroscopic factors $S_j(0_1 I_n)$ for the lowest-lying states and the corresponding sum rules. The calculations with the particle-phonon and quasiparticle-phonon models were performed with the parameters presented in Refs. 25 and 24, respectively, for the ^{131}I nucleus. The parameters used in the present calculation are given in the text.

	Experiment (Ref. 13)						Theory					
	^{127}I		^{129}I		^{131}I		Present model		Particle-phonon model		Quasiparticle-phonon model	
I_n	S_j	$\sum S_j$	S_j	$\sum S_j$	S_j	$\sum S_j$	S_j	$\sum S_j$	S_j	$\sum S_j$	S_j	$\sum S_j$
$\frac{7}{2}_1$	0.72	0.72	0.66	0.7	0.64	0.64	0.47	0.80	0.63	1.0	0.64	0.81
$\frac{5}{2}_1$	0.59	1.0	0.59	1.1	0.53	1.1	0.40	0.96	0.64	1.0	0.56	0.95
$\frac{3}{2}_1$	0.06	0.9	0.07	0.9	0.07	0.7	0.09	0.96	0.07	1.0	0.15	0.99
$\frac{1}{2}_1$	0.20	0.8	0.21	1.0	0.21	1.0	0.12	0.99			0.17	0.99
$\frac{5}{2}_2$	0.12		0.21		0.47		0.22		0.02		0.00	

TABLE II. Calculated wave function for the low-lying states of ^{131}I. Only those amplitudes that contribute more than 4% are listed. The parameters used in the present calculation are given in the text.

$$|\tfrac{1}{2}{}_1\rangle = 0.66|[(\tfrac{7}{2})^2 0, \tfrac{5}{2}]\tfrac{5}{2}, 12\rangle - 0.41|[(\tfrac{7}{2})^2 2, \tfrac{5}{2}]\tfrac{1}{2}, 00\rangle + 0.38|[(\tfrac{7}{2})^2 0, \tfrac{1}{2}]\tfrac{1}{2}, 00\rangle$$

$$|\tfrac{3}{2}{}_1\rangle = 0.58|[(\tfrac{7}{2})^3]\tfrac{3}{2}, 00\rangle + 0.48|[(\tfrac{7}{2})^3]\tfrac{7}{2}, 12\rangle + 0.27|[(\tfrac{7}{2})^2 0, \tfrac{3}{2}]\tfrac{3}{2}, 00\rangle + 0.24|[(\tfrac{7}{2})^3]\tfrac{5}{2}, 12\rangle$$

$$|\tfrac{5}{2}{}_1\rangle = 0.58|[(\tfrac{7}{2})^2 0, \tfrac{5}{2}]\tfrac{5}{2}, 00\rangle + 0.39|[(\tfrac{7}{2})^3]\tfrac{5}{2}, 00\rangle + 0.32|[(\tfrac{7}{2})^3]\tfrac{7}{2}, 12\rangle - 0.27|[(\tfrac{7}{2})^2 0, \tfrac{5}{2}]\tfrac{5}{2}, 12\rangle - 0.22|[(\tfrac{7}{2})^2 2, \tfrac{5}{2}]\tfrac{7}{2}, 12\rangle$$

$$|\tfrac{5}{2}{}_2\rangle = 0.58|[(\tfrac{7}{2})^3]\tfrac{5}{2}, 00\rangle - 0.44|[(\tfrac{7}{2})^2 0, \tfrac{5}{2}]\tfrac{5}{2}, 00\rangle + 0.37|[(\tfrac{7}{2})^3]\tfrac{7}{2}, 12\rangle + 0.22|[(\tfrac{7}{2})^2 0, \tfrac{5}{2}]\tfrac{5}{2}, 12\rangle$$

$$|\tfrac{7}{2}{}_1\rangle = 0.76|[(\tfrac{7}{2})^3]\tfrac{7}{2}, 00\rangle + 0.29|[(\tfrac{5}{2})^2 0, \tfrac{7}{2}]\tfrac{7}{2}, 00\rangle - 0.24|[(\tfrac{7}{2})^3]\tfrac{7}{2}, 12\rangle + 0.24|[(\tfrac{7}{2})^3]\tfrac{11}{2}, 12\rangle$$

$$|\tfrac{7}{2}{}_2\rangle = 0.62|[(\tfrac{7}{2})^2 2, \tfrac{5}{2}]\tfrac{7}{2}, 00\rangle + 0.38|[(\tfrac{7}{2})^2 0, \tfrac{5}{2}]\tfrac{5}{2}, 12\rangle + 0.31|[(\tfrac{7}{2})^2 2, \tfrac{5}{2}]\tfrac{5}{2}, 12\rangle - 0.24|[(\tfrac{7}{2})^2 2, \tfrac{5}{2}]\tfrac{9}{2}, 12\rangle + 0.22|[(\tfrac{7}{2})^2 4, \tfrac{5}{2}]\tfrac{7}{2}, 00\rangle - 0.21|[(\tfrac{7}{2})^2 4, \tfrac{5}{2}]\tfrac{11}{2}, 12\rangle$$

$$|\tfrac{9}{2}{}_1\rangle = 0.66|[(\tfrac{7}{2})^3]\tfrac{9}{2}, 00\rangle - 0.33|[(\tfrac{7}{2})^3]\tfrac{7}{2}, 12\rangle + 0.27|[(\tfrac{7}{2})^2 2, \tfrac{5}{2}]\tfrac{9}{2}, 00\rangle - 0.26|[(\tfrac{7}{2})^3]\tfrac{11}{2}, 12\rangle$$

$$|\tfrac{11}{2}{}_1\rangle = 0.71|[(\tfrac{7}{2})^3]\tfrac{11}{2}, 00\rangle + 0.44|[(\tfrac{7}{2})^3]\tfrac{7}{2}, 12\rangle \quad 0.26|[(\tfrac{7}{2})^3]\tfrac{15}{2}, 12\rangle$$

for the low-lying states has been observed. In the evaluation of the one-body transfer reactions the ground state of the Te nucleus has been calculated within the same model space and employing the same parametrization.

With the set of parameters quoted above, it was possible to reproduce satisfactorily the energy spectra of ^{131}I and the one-body reaction strengths for ^{127,129,131}I. The results are displayed in Fig. 1 and in Table I. The pronounced components in the wave functions obtained for the low-lying states are listed in Table II.

The calculation of the electromagnetic properties was carried out taking for the effective proton and vibrator charges $e_p^{\text{eff}} = 2e$ and $e_v^{\text{eff}} = 2.5e$, respectively, and for the effective gyromagnetic ratios: $g_R = 1$, $g_s^{\text{eff}} = 3.91$, and $g_R = 0$. These parameters are also similar to those used in Refs. 5–7. The reduced $E2$ transition probabilities are given in Table III and the magnetic-dipole moments and the electric-quadrupole moments are listed in Table IV. The experimental data for several iodine nuclei (also shown in the tables) are, in general, fairly well reproduced.

Now, as the $N = 82$ shell becomes more and more empty, the energy spectra of odd-mass I nuclei change gradually, and the quadrupole moment of the $\frac{7}{2}{}_1^+$ state increases. In order to account for these effects we have, in principle, several free parameters at our disposal: single-particle energy spacings, the phonon energy, and the coupling

TABLE III. Experimental and calculated $B(E2)$ transition rates in units of $10^{-50} e^2 \text{cm}^4$. The results for the particle-phonon model and the quasiparticle-phonon model are taken from Refs. 25 and 24, respectively, for the ^{127}I nucleus. The parameters used in the present calculation are given in the text.

Transition		Experiment			Theory	
	^{125}I	^{127}I	^{129}I	Present model	Particle-phonon model	Quasiparticle-phonon model
$\frac{1}{2}{}_1$ $\frac{5}{2}{}_1$	12.2 ± 0.8 [a]	8.8 [b]		8.3	5.1	34.2
$\frac{3}{2}{}_1$ $\frac{5}{2}{}_1$	7.1 ± 0.3 [a]	5.0 [b]		4.2	1.3	4.0
$\frac{5}{2}{}_2$ $\frac{5}{2}{}_1$		0.72 [b]		0.40	0.84	
$\frac{7}{2}{}_1$ $\frac{5}{2}{}_1$	4.5 ± 0.2 [a]	6.4 ± 1.1 [a]	7.3 ± 0.8 [c]	6.7	1.1	2.5
$\frac{9}{2}{}_1$ $\frac{5}{2}{}_1$		1.4 [b]		2.6	4.6	
$\frac{1}{2}{}_1$ $\frac{3}{2}{}_1$	1.9 ± 0.8 [a]	4.0 [a]		1.4		21.8
$\frac{3}{2}{}_1$ $\frac{7}{2}{}_1$	14.4 ± 0.4 [a]	11.2 ± 0.2 [a]		12.9	11.6	21.5
$\frac{5}{2}{}_2$ $\frac{7}{2}{}_1$				5.3	0.11	
$\frac{7}{2}{}_2$ $\frac{7}{2}{}_1$				0.24	0.48	
$\frac{9}{2}{}_1$ $\frac{7}{2}{}_1$				3.8	0.21	
$\frac{11}{2}{}_1$ $\frac{7}{2}{}_1$				9.7	15.4	

[a] Reference 14. [b] Reference 17. [c] Reference 8.

strengths a and G. However, a glance at the reaction data indicates that we do not have too much freedom. Actually, the spectroscopic strengths of the low-lying states, observed in the Te(^{3}He, d) process, are nearly the same for ^{127,129,131}I nuclei and the corresponding centers of gravity barely change in going from one isotope to another.[13] In addition, in the Te(d, ^{3}He) reaction study on the ^{126,128,130}Te nuclei, only a very slight increase in the filling up of the $(g_{7/2})_0^2$ state is observed as A increases.[34] With this situation in view, several calculations were performed by varying the parameters in the neighborhood of the values given above. In this way only the energy spectrum of ^{129}I was reasonably well reproduced, either by (a) adding more collectively to the lowest states ($a = 0.7$ MeV and $\hbar\omega = 1.0$ MeV) or by (b) lowering the single-particle states $d_{5/2}$ and $d_{3/2}$ [$\epsilon(d_{5/2}) = 0.1$ MeV and $\epsilon(d_{3/2}) = 0.8$ MeV].

For the sake of completeness, the results concerning the energies are confronted with experiment in Fig. 2.

There are some particularly noteworthy features in the results, concerning the properties of the ground state, with these new parametrizations. In both cases the one-body spectroscopic strength is appreciably reduced, whereas the electric-quadrupole moment is increased. The results are: $S_{7/2}(0_1, \frac{7}{2}{}_1^+) = 0.32$, $Q(\frac{7}{2}{}_1^+) = -0.46$ eb and $S_{7/2}(0_1, \frac{7}{2}{}_1^+) = 0.26$, $Q(\frac{7}{2}{}_1^+) = -0.55$ eb for case (a) and (b), respectively.

IV. SUMMARY AND DISCUSSION

We have investigated the consequences of coupling a three-proton cluster to the quadrupole-vibrational field of the tin core. Employing a parametrization close to the one used in the calculation of Sb and Te nuclei, where the same model has been applied, a reasonably good description for the dynamics properties of the low-lying states in odd-mass I nuclei, as well as for the energy spectrum of ^{131}I, was achieved.

By varying the parameters of the model, we do not obtain good results for the energies of the lightest nuclei ($A \leq 127$). In addition, the spectrum of ^{129}I was reproduced, but at the expense of spoiling the agreement between the calculated and experimental values for the spectroscopic strength of the ground state. Consequently, changing the parameters in the same direction will not only lead to unacceptable values, both for the coupling strength a and the single-particle energies, but would also destroy the agreement with experiments, concerning the one-body reaction data. This fact indicates that some additional degree of freedom (particle nature of the phonons, nonharmonic coupling of particles, higher multipolarities of the pairing force, etc.) may play a significant role in determining the energy spectra of odd-mass I nuclei.

From the comparison of the present description with the quasiparticle-phonon[24,25] and particle-

TABLE IV. Experimental and calculated magnetic dipole moments and electric quadrupole moments in units of μ_N and eb respectively. The results for the particle-phonon model and quasiparticle-phonon model are taken from Refs. 25 and 24, respectively, for the ^{127}I nucleus. The parameters used in the present calculation are given in the text.

State	Experiment ^{125}I	^{127}I	^{129}I	^{131}I	^{133}I	Theory: Present model	Particle-phonon model	Quasiparticle-phonon model
				Magnetic dipole moment				
$\frac{7}{2}_1$		2.54 [a]	2.614 [b]	2.74 [b]	2.84 [b]	2.32	1.55	2.75
$\frac{5}{2}_1$	3.0 [b]	2.808 [b]	2.81 [b]	2.77 [b]		2.96	4.38	2.60
$\frac{3}{2}_1$		1.15 [c]				1.21		
$\frac{5}{2}_2$						2.38		
				Electric quadrupole moment (Q)				
$\frac{7}{2}_1$		−0.71 [b]	−0.55 [b]	−0.40 [b]	−0.27 [b]	−0.36	−0.78	−1.08
$\frac{5}{2}_1$	−0.89 [b]	−0.79 [b]	−0.68 [b]			−0.67	−0.59	−0.99
$\frac{3}{2}_1$						0.30		
$\frac{5}{2}_2$						−0.46		

[a] Reference 22. [b] Reference 15. [c] Reference 11.

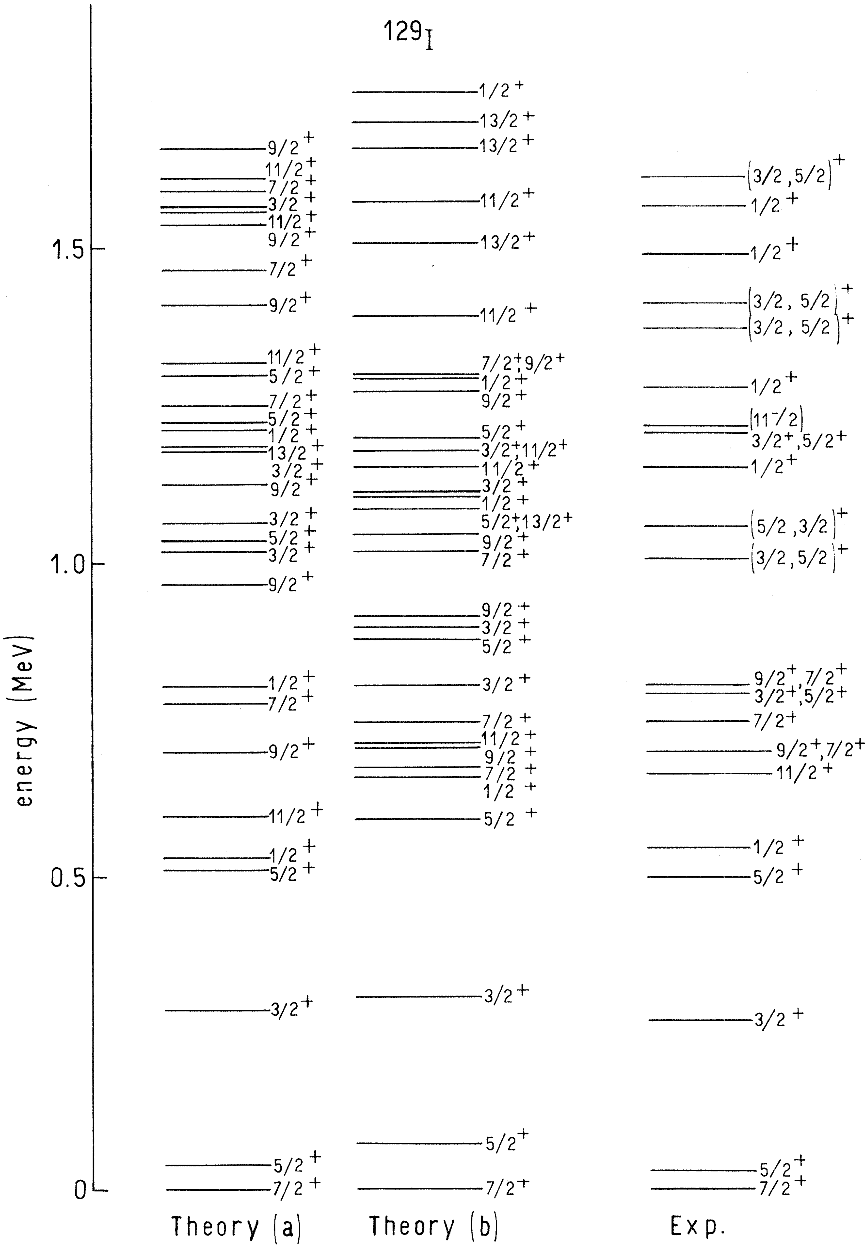

FIG. 2. Experimental and theoretical spectra for ^{129}I. Calculations (a) and (b) correspond to the following choice of parameters: (a) $a = 0.7$ MeV, $\hbar\omega = 1.0$ MeV. (b): $\epsilon(d_{5/2}) = 0.1$ MeV, $\epsilon(d_{3/2}) = 0.8$ MeV. The values for the remaining parameters are given in the text.

phonon[26] schemes several qualitative differences become apparent. We shall point out only some of these which are rather independent of the parametrization employed in each case.

a. Energy and the reaction strength of the $\frac{5}{2}^+_2$ *state.* In the particle-phonon and quasiparticle-phonon pictures the $\frac{5}{2}^+_2$ state arises from the one-phonon multiplets $|g_{7/2}, 12; I\rangle$ or $|d_{5/2}, 12; I\rangle$, depending upon the single-particle energies, and it always lies above the members of the multiplet with $I \geq \frac{7}{2}$. In addition, the single-particle strength removed from the $|d_{5/2}, 00; \frac{5}{2}\rangle$ state is mainly absorbed by the third or fourth $\frac{5}{2}^+$ state. In the model proposed here the $\frac{5}{2}^+_1$ and $\frac{5}{2}^+_2$ states arise in the zeroth-order approximation from the configurations

$$\|[(g_{7/2})^2 0, d_{5/2}]\tfrac{5}{2}, 00; \tfrac{5}{2}\rangle$$

and

$$\|[(g_{7/2})^3]\tfrac{5}{2}, 00; \tfrac{5}{2}\rangle \quad ,$$

respectively. The residual interaction and the vibrational field strongly mix them and give rise in this way to the fragmentation of the spectroscopic strength.

b. $B(E2; \frac{5}{2}^+_1 \to \frac{7}{2}^+_1)$ transition. While in the particle-phonon and quasiparticle-phonon models the magnitude of this transition is governed by the spin-flip matrix element $\langle d_{5/2} \| Y_2 \| g_{7/2}\rangle$, in the present model it occurs predominantly between the members of the shell-model cluster $(g_{7/2})^3 J$. Due to this fact, only the present model provides the necessary enhancement to account for the experimental value.

c. Quadrupole moment of the $\frac{7}{2}^+_1$ state. With the same parametrization the present description gives a result for the $Q(\frac{7}{2}^+_1)$ similar to the one obtained with the quasiparticle-plus-phonon model, but significantly smaller (approximately one third) than that provided by the particle-phonon coupling scheme.

We can conclude consequently, that in order to account for the energy spectrum of ^{131}I and the spectroscopic factors for ^{127,129,131}I it is necessary to consider explicitly the three-particle clustering. On the other hand, although this statement does not emerge from a comparison of the numerical results for electromagnetic properties, the underlying structure of the process might be quite different in the present approach than in the particle-phonon and quasiparticle-phonon coupling schemes.

In conclusion, we feel that the results of this investigation demonstrate the importance of the three-proton-cluster–vibrational-field coupling in the description of the odd-mass iodine nuclei. However, more theoretical and experimental work is needed for a more complete understanding of the properties of these nuclei.

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