A One-Way Quantum Computer

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We present a scheme of quantum computation that consists entirely of one-qubit measurements on a particular class of entangled states, the cluster states. The measurements are used to imprint a quantum logic circuit on the state, thereby destroying its entanglement at the same time. Cluster states are thus one-way quantum computers and the measurements form the program.

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A quantum computer promises efficient processing of certain computational tasks that are intractable with classical computer technology [1]. While basic principles of a quantum computer have been demonstrated in the laboratory [2], scalability of these systems to a large number of qubits [3], essential for practical applications such as the Shor algorithm, represents a formidable challenge. Most of the current experiments are designed to implement sequences of highly controlled interactions between selected particles (qubits), thereby following models of a quantum computer as a (sequential) network of quantum logic gates [4,5].

Here we propose a different model of a scalable quantum computer. In our model, the entire resource for the quantum computation is provided initially in the form of a specific entangled state (a so-called cluster state [6]) of a large number of qubits. Information is then written onto the cluster, processed, and read out from the cluster by one-particle measurements only. The entangled state of the cluster thereby serves as a universal "substrate" for any quantum computation. Cluster states can be created efficiently in any system with a quantum Ising-type interaction (at very low temperatures) between two-state particles in a lattice configuration.

We consider two- and three-dimensional arrays of qubits that interact via an Ising-type next-neighbor interaction [6] described by a Hamiltonian $H_{\text{int}} = g(t) \times \sum_{\langle a,a'\rangle} \frac{1+\sigma_z^{(a)}}{2}\frac{1-\sigma_z^{(a')}}{2} \cong -\frac{1}{4} g(t) \sum_{\langle a,a'\rangle} \sigma_z^{(a)}\sigma_z^{(a')}$ [7] whose strength $g(t)$ can be controlled externally. A possible realization of such a system is discussed below. A qubit at site a can be in two states $|0\rangle_a \equiv |0\rangle_{z,a}$ or $|1\rangle_a \equiv |1\rangle_{z,a}$, the eigenstates of the Pauli phase flip operator $\sigma_z^{(a)}$ $[\sigma_z^{(a)}|i\rangle_a = (-1)^i|i\rangle_a]$. These two states form the computational basis. Each qubit can equally be in an arbitrary superposition state $\alpha|0\rangle + \beta|1\rangle$, $|\alpha|^2 + |\beta|^2 = 1$. For our purpose, we initially prepare all qubits in the superposition $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$, an eigenstate of the Pauli spin flip operator σ_x $[\sigma_x|\pm\rangle = \pm|\pm\rangle]$. H_{int} is then switched on for an appropriately chosen finite time interval T, where $\int_0^T dt\, g(t) = \pi$, by which a unitary transformation S is realized. Since H_{int} acts uniformly on the lattice, entire clusters of neighboring particles become entangled in one single step. The quantum state $|\Phi\rangle_C$, the state of a cluster (C) of neighboring qubits, which is thereby created provides in advance all entanglement that is involved in the subsequent quantum computation. It has been shown [6] that the cluster state $|\Phi\rangle_C$ is characterized by a set of eigenvalue equations

$$\sigma_x^{(a)} \bigotimes_{a' \in ngbh(a)} \sigma_z^{(a')}|\Phi\rangle_C = \pm|\Phi\rangle_C\,, \tag{1}$$

where $ngbh(a)$ specifies the sites of all qubits that interact with the qubit at site $a \in C$. The eigenvalues are determined by the distribution of the qubits on the lattice. The equations (1) are central for the proposed computation scheme. As an example, a measurement on an individual qubit of a cluster has a random outcome. On the other hand, Eqs. (1) imply that any two qubits at sites $a, a' \in C$ can be projected into a Bell state by measuring a subset of the other qubits in the cluster. This property will be used to define quantum channels that allow us to propagate quantum information through a cluster.

We show that a cluster state $|\Phi\rangle_C$ can be used as a substrate on which any quantum circuit can be imprinted by one-qubit measurements. In Fig. 1 this scheme is illustrated. For simplicity, we assume that in a certain region of the lattice each site is occupied by a qubit. This requirement is not essential as will be explained below [see (d)]. In the first step of the computation, a subset of qubits is measured in the basis of σ_z which effectively removes them. In Fig. 1 these qubits are denoted by "$\odot$."

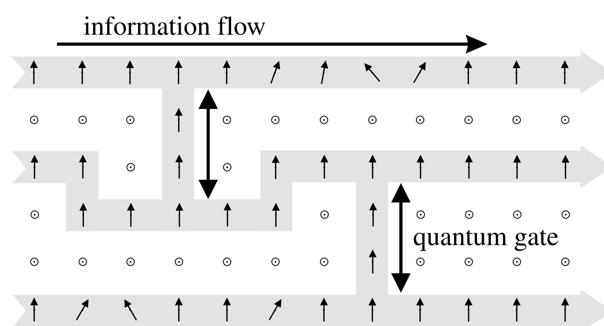

FIG. 1. Quantum computation by measuring two-state particles on a lattice. Before the measurements the qubits are in the cluster state $|\Phi\rangle_C$ of (1). Circles $\odot$ symbolize measurements of σ_z, vertical arrows are measurements of σ_x, while tilted arrows refer to measurements in the x-y plane.

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The state $|\Phi\rangle_C$ is thereby projected into a tensor product $|\mu\rangle_{C\setminus\mathcal{N}} \otimes |\tilde{\Phi}\rangle_{\mathcal{N}}$ consisting of the state $|\mu\rangle_{C\setminus\mathcal{N}}$ of all measured particles (subset $C\setminus\mathcal{N}$) on one side and an entangled state $|\tilde{\Phi}\rangle_{\mathcal{N}}$ of yet unmeasured particles (subset $\mathcal{N} \subset C$), on the other side. These unmeasured particles define a "network" $\mathcal{N}$ corresponding to the shaded structure in Fig. 1. The state $|\tilde{\Phi}\rangle_{\mathcal{N}}$ of the network is related to a cluster state $|\Phi\rangle_{\mathcal{N}}$ on $\mathcal{N}$ by a local unitary transformation which depends on the set of measurement results μ. More specifically, $|\tilde{\Phi}\rangle_{\mathcal{N}}$ satisfies Eqs. (1)—with C replaced by the subcluster $\mathcal{N}$—except for a possible difference in the sign factors, which are determined by the measurement results μ.

To process quantum information with this network, it suffices to measure its particles in a certain order and in a certain basis. Quantum information is thereby propagated horizontally through the cluster by measuring the qubits on the wire while qubits on vertical connections are used to realize two-bit quantum gates. The basis in which a certain qubit is measured depends in general on the results of preceding measurements. The processing is finished once all qubits except the last one on each wire have been measured. At this point, the results of previous measurements determine in which basis these "output" qubits need to be measured for the final readout. We note that, in the entire process, only one-qubit measurements are required. The amount of entanglement therefore decreases with every measurement [8] and all entanglement involved in the process is provided by the initial resource, the cluster state. This is different from the scheme of Ref. [11], which uses Bell measurements (capable of producing entanglement) to realize quantum gates.

In the following, we show that any quantum logic circuit can be implemented on a cluster state. The purpose of this is twofold. First, it serves as an illustration of how to implement a particular quantum circuit in practice. Second, in showing that any quantum circuit can be implemented on a sufficiently large cluster we demonstrate the universality of the proposed scheme. For pedagogical reasons we first explain a scheme with one essential modification with respect to the proposed scheme: before the entanglement operation S, certain qubits are selected as input qubits and the input information is written onto them, while the remaining qubits are prepared in $|+\rangle$. This step weakens the scheme since it affects the character of the cluster state as a genuine resource. It can, however, be avoided [see (e)]. Points (a) to (c) are concerned with the basic elements of a quantum circuit, quantum gates, and wires, point (d) with the composition of gates to circuits.

(a) Information propagation in a wire for qubits. A qubit can be teleported from one site of a cluster to any other site. In particular, consider a chain of an odd number of qubits 1 to n prepared in the state $|\psi_{\text{in}}\rangle_1 \otimes |+\rangle_2 \otimes \cdots \otimes |+\rangle_n$ and subsequently entangled by S. The state that was originally encoded in qubit 1, $|\psi_{\text{in}}\rangle$, is now delocalized and can be transferred to site n by performing σ_x measurements (basis $\{|+\rangle_j = |0\rangle_{x,j}, |-\rangle_j = |1\rangle_{x,j}\}$) at qubit sites $j = 1, \ldots, n-1$ with measurement outcomes $s_j \in \{0, 1\}$. The resulting state is $|s_1\rangle_{x,1} \otimes \cdots \otimes |s_{n-1}\rangle_{x,n-1} \otimes |\psi_{\text{out}}\rangle_n$. The output state $|\psi_{\text{out}}\rangle$ is related to the input state $|\psi_{\text{in}}\rangle$ by a unitary transformation $U_\Sigma \in \{1, \sigma_x, \sigma_z, \sigma_x\sigma_z\}$ which depends on the outcomes of the σ_x measurements at sites 1 to $n-1$. A similar argument can be given for an even number of qubits. The effect of U_Σ can be accounted for at the end of a computation as shown below [see (d)]. It is noteworthy that not all classical information gained by the σ_x measurements needs to be stored to identify the transformation U_Σ. Instead, U_Σ is determined by the values of only two classical bits which are updated with every measurement.

(b) An arbitrary rotation $U_R \in \mathrm{SU}(2)$ can be achieved in a chain of five qubits. Consider a rotation in its Euler representation $U_R(\xi, \eta, \zeta) = U_x(\zeta)U_z(\eta)U_x(\xi)$, where $U_x(\alpha) = \exp(-i\alpha\frac{\sigma_x}{2}), U_z(\alpha) = \exp(-i\alpha\frac{\sigma_z}{2})$. Initially, the first qubit is in some state $|\psi_{\text{in}}\rangle$, which is to be rotated, and the other qubits are in $|+\rangle$; i.e., their common state reads $|\Psi\rangle_{1,\ldots,5} = |\psi_{\text{in}}\rangle_1 \otimes |+\rangle_2 \otimes |+\rangle_3 \otimes |+\rangle_4 \otimes |+\rangle_5$. After the five qubits are entangled by S they are in the state $S|\Psi\rangle_{1,\ldots,5} = 1/2|\psi_{\text{in}}\rangle_1|0\rangle_2|-\rangle_3|0\rangle_4|-\rangle_5 - 1/2|\psi_{\text{in}}\rangle_1|0\rangle_2|+\rangle_3|1\rangle_4|+\rangle_5 - 1/2|\psi^*_{\text{in}}\rangle_1|1\rangle_2|+\rangle_3|0\rangle_4|-\rangle_5 + 1/2|\psi^*_{\text{in}}\rangle_1|1\rangle_2|-\rangle_3|1\rangle_4|+\rangle_5$, where $|\psi^*_{\text{in}}\rangle = \sigma_z|\psi_{\text{in}}\rangle$. Now, the state $|\psi_{\text{in}}\rangle$ can be rotated by measuring qubits 1 to 4, while it is teleported to site 5 at the same time. The qubits $1, \ldots, 4$ are measured in appropriately chosen bases $\mathcal{B}_j(\alpha_j) = \{\frac{|0\rangle_j + e^{i\alpha_j}|1\rangle_j}{\sqrt{2}}, \frac{|0\rangle_j - e^{i\alpha_j}|1\rangle_j}{\sqrt{2}}\}$ whereby the measurement outcomes $s_j \in \{0, 1\}$ for $j = 1, \ldots, 4$ are obtained. Here, $s_j = 0$ means that qubit j is projected into the first state of $\mathcal{B}_j(\alpha_j)$. The resulting state is $|s_1\rangle_{\alpha_1,1} \otimes |s_2\rangle_{\alpha_2,2} \otimes |s_3\rangle_{\alpha_3,3} \otimes |s_4\rangle_{\alpha_4,4} \otimes |\psi_{\text{out}}\rangle_5$ with $|\psi_{\text{out}}\rangle = U|\psi_{\text{in}}\rangle$. For the choice $\alpha_1 = 0$ (measuring σ_x of qubit 1) the rotation U has the form $U = \sigma_x^{s_2+s_4}\sigma_z^{s_1+s_3}U_R[(-1)^{s_1+1}\alpha_2, (-1)^{s_2}\alpha_3, (-1)^{s_1+s_3}\alpha_4]$. In summary, the procedure to implement an arbitrary rotation $U_R(\xi, \eta, \zeta)$, specified by its Euler angles ξ, η, ζ is (i) measure qubit 1 in $\mathcal{B}_1(0)$; (ii) measure qubit 2 in $\mathcal{B}_2((-1)^{s_1+1}\xi)$; (iii) measure qubit 3 in $\mathcal{B}_3((-1)^{s_2}\eta)$; (iv) measure qubit 4 in $\mathcal{B}_4((-1)^{s_1+s_3}\zeta)$. In this way the rotation U'_R is realized: $U'_R(\xi, \eta, \zeta) = \sigma_x^{s_2+s_4}\sigma_z^{s_1+s_3}U_R(\xi, \eta, \zeta)$. The extra rotation $U_\Sigma = \sigma_x^{s_2+s_4}\sigma_z^{s_1+s_3}$ can be accounted for at the end of the computation, as is described below in (d).

(c) To perform the gate $\text{CNOT}(c, t_{\text{in}} \to t_{\text{out}}) = |0\rangle_{cc}\langle 0| \otimes 1^{(t_{\text{in}}\to t_{\text{out}})} + |1\rangle_{cc}\langle 1| \otimes \sigma_x^{(t_{\text{in}}\to t_{\text{out}})}$ between a control qubit c and a target qubit t, four qubits, arranged as depicted Fig. 2a, are required. During the action of the gate, the target qubit t is transferred from t_{in} to t_{out}. The following procedure has to be implemented. Let qubit 4 be the control qubit. First, the state $|i_1\rangle_{z,1} \otimes |i_4\rangle_{z,4} \otimes |+\rangle_2 \otimes |+\rangle_3$ is prepared and then the entanglement operation S is performed. Second, σ_x of qubits 1 and 2 is measured. The measurement results

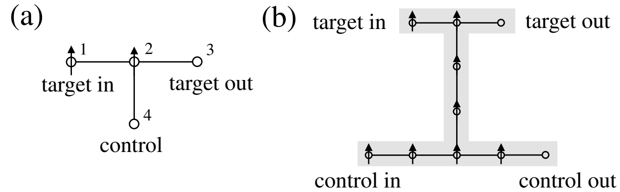

FIG. 2. Realization of a CNOT gate by one-particle measurements. See text.

$s_j \in \{0,1\}$ correspond to projections of the qubits j into $|s_j\rangle_{x,j}$, $j = 1, 2$. The quantum state created by this procedure is $|s_1\rangle_{x,1} \otimes |s_2\rangle_{x,2} \otimes U_\Sigma^{(34)}|i_4\rangle_{z,4} \otimes |i_1 + i_4 \,\mathrm{mod}2\rangle_{z,3}$, where $U_\Sigma^{(34)} = \sigma_z^{(3)^{s_1+1}} \sigma_x^{(3)^{s_2}} \sigma_z^{(4)^{s_1}}$. The input state is thus acted upon by the CNOT and successive σ_x and σ_z rotations $U_\Sigma^{(34)}$, depending on the measurement results s_1, s_2. These unwanted extra rotations can again be accounted for as described in (d). For practical purposes it is more convenient if the control qubit is, as the target qubit, transferred to another site during the action of the gate. When a CNOT is combined with other gates to form a quantum circuit it will be used in the form shown in Fig. 2b.

To explain the working principle of the CNOT gate we, for simplicity, refer to the minimal implementation with four qubits. The minimal CNOT can be viewed as a wire from qubit 1 to qubit 3 with an additional qubit, No. 4, attached. From the eigenvalue equations (1) it can now be derived that, if qubit 4 is in an eigenstate $|i_4\rangle_{z,4}$ of σ_z, then the value of $i_4 \in \{0,1\}$ determines whether a unit wire or a spin flip σ_x (modulo the same correction $U_\Sigma^{(3)}$ for both values of i_4) is being implemented. In other words, once σ_x of qubits 1 and 2 have been measured, the value i_4 of qubit 4 controls whether the target qubit is flipped or not.

(d) Quantum circuits. The gates described—the CNOT and arbitrary one-qubit rotations—form a universal set [5]. In the implementation of a quantum circuit on a cluster state the site of every output qubit of a gate overlaps with the site of an input qubit of a subsequent gate. Because of this, the entire entanglement operation can be performed at the beginning. To see this, compare the following two strategies. Given a quantum circuit implemented on a network $\mathcal{N}$ of qubits which is divided into two consecutive circuits, circuit 1 is implemented on network $\mathcal{N}_1$ and circuit 2 is implemented on network $\mathcal{N}_2$, and $\mathcal{N} = \mathcal{N}_1 \cup \mathcal{N}_2$. There is an overlap $\mathcal{O} = \mathcal{N}_1 \cap \mathcal{N}_2$ which contains the sites of the output qubits of circuit 1 (these are identical to the sites of the input qubits of circuit 2). The sites of the readout qubits form a set $\mathcal{R} \subset \mathcal{N}_2$. Strategy (i) consists of the following steps: (1) write input and entangle all qubits on $\mathcal{N}$; (2) measure qubits $\in \mathcal{N}\backslash\mathcal{R}$ to implement the circuit. Strategy (ii) consists of (1) write input and entangle the qubits on $\mathcal{N}_1$, (2) measure the qubits in $\mathcal{N}_1\backslash\mathcal{O}$. This implements the circuit on $\mathcal{N}_1$ and writes the intermediate output to $\mathcal{O}$; (3) entangle the qubits on $\mathcal{N}_2$; (4) measure all qubits in $\mathcal{N}_2\backslash\mathcal{R}$. Steps 3 and 4 implement the circuit 2 on $\mathcal{N}_2$. The measurements on $\mathcal{N}_1\backslash\mathcal{O}$ commute with the entanglement operation restricted to $\mathcal{N}_2$, since they act on different subsets of particles. Therefore the two strategies are mathematically equivalent and yield the same results. It is therefore consistent to entangle in a single step at the beginning and perform all measurements afterwards.

Two further points should be addressed in connection with circuits. First, the randomness of the measurement results does not jeopardize the function of the circuit. Depending on the measurement results, extra rotations σ_x and σ_z act on the output qubit of every implemented gate. By use of the relations $U_R(\xi,\eta,\zeta)\sigma_z^s\sigma_x^{s'} = \sigma_z^s\sigma_x^{s'}U_R\big((-1)^s\xi,(-1)^{s'}\eta,(-1)^s\zeta\big)$, and $\mathrm{CNOT}(c,t)\sigma_z^{(t)^{s_t}}\sigma_z^{(c)^{s_c}}\sigma_x^{(t)^{s_t'}}\sigma_x^{(c)^{s_c'}} = \sigma_z^{(t)^{s_t}}\sigma_z^{(c)^{s_c+s_t}}\sigma_x^{(t)^{s_c'+s_t'}}\sigma_x^{(c)^{s_c'}}\mathrm{CNOT}(c,t)$, these extra rotations can be pulled through the network to act upon the output state. There they can be accounted for by adjusting the measurement basis for the final readout. The above relations imply that for a rotation $U_R(\xi,\eta,\zeta)$—different from the CNOT gate—the accumulated extra rotations U_Σ at the input side of U_R need to be determined before the measurement bases that realize U_R can be specified. This introduces a partial temporal ordering of the measurements on the whole cluster. Second, quantum circuits can also be implemented on irregular clusters. In that case, qubits may be missing which are required for the standard implementation of the circuit. This can be compensated by a large flexibility in shape of the gates and wires. The components can be bent and stretched to fit to the cluster structure as long as the topology of the circuit implementation does not change. Irregular clusters are found in lattices with a finite site occupation probability $0 < p < 1$. In such a situation, the possibility of *universal* quantum computation is closely linked to the phenomenon of percolation. For p above a certain critical value p_c, which depends on the dimension of the lattice, an infinitely extended cluster exists that may be used as the carrier of the quantum circuit. In two dimensions, for example, exactly one such cluster C exists. Suppose this cluster is divided into two subclusters C_1 and C_2 by a one-dimensional cut $\mathcal{O} = C_1 \cap C_2$. It can be shown, e.g., by using Russo's formula [12] from percolation theory that, for any cut $\mathcal{O}$, $|\mathcal{O}| = \infty$. Therefore there is no upper bound, in principle, to the "capacity" of the cluster, i.e., to the number of qubits that can be processed across such a cut.

(e) Full scheme. It is important to note that the step of writing the input information onto the qubits before the cluster is entangled was introduced only for pedagogical reasons. For illustration of this point consider a chain of five qubits in the state $S|+\rangle_1 \otimes |+\rangle_2 \otimes \cdots \otimes |+\rangle_5$. Clearly, there is no local information on any of the qubits. However, by measuring qubits 1 to 4 along suitable directions, qubit 5 can be projected into any desired state (modulo U_Σ). What is used here is the knowledge that the resource has been prepared with qubit 1 in the state $|+\rangle_1$ before the entanglement operation. By the four measurements, this qubit is then rotated as described in (b). In order to use qubit 5 for further processing, the five-qubit chain considered here should, of course, be part of a larger cluster such that particle 5 is still entangled with the remaining network, after particles 1 to 4 have been measured. The method of preparing the input state remains the same, in this case, as explained in (d). In a similar manner any desired input state can be prepared if the rotations are replaced by a circuit preceding the proper circuit for computation. In summary, no input information needs to be written to the qubits before they are entangled. Cluster states are thus a genuine resource for quantum computation via measurements only.

For a cluster of a given *finite* size, the number of computational steps may be too large to fit on the cluster. In this case, the computation can be split into consecutive parts, for each of which there is sufficient space on the cluster. The modified procedure consists then of repeatedly (re)entangling the cluster and imprinting the actual part of the circuit—by measuring all of the lattice qubits except the ones carrying the intermediate quantum output—until the whole calculation is performed. This procedure has also the virtue that qubits involved in the later part of a calculation need not be protected from decoherence for a long time while the calculation is still being performed at a remote place of the cluster. Standard error-correction techniques [13,14] may then be used on each part of the circuit to stabilize the computation against decoherence.

A possible implementation of such a quantum computer uses neutral atoms stored in periodic micropotentials [15–18] where Ising-type interactions can be realized by controlled collisions between atoms in neighboring potential wells [16,18]. This system combines small decoherence rates with a high scalability. The question of scalability is linked to the percolation phenomenon, as mentioned earlier. For a site occupation probability above the percolation threshold, there exists a cluster which is bounded in size only by the trap dimensions. For optical lattices in three dimensions, single-atom site occupation with a filling factor of 0.44 has been reported [19] which is significantly above the percolation threshold of 0.31 [20]. As in other proposed implementations for quantum computing, the addressability of single qubits in the lattice is, however, still a problem. (For recent progress, see Ref. [21]). Recently, it has also been shown that implementations based on arrays of capacitively coupled quantum dots may be used to realize an Ising-type interaction [22].

In conclusion, we have described a new scheme of quantum computation that consists entirely of one-qubit measurements on a particular class of entangled states, the cluster states. The measurements are used to imprint a quantum circuit on the state, thereby destroying its entanglement at the same time. Cluster states are thus one-way quantum computers and the measurements form the program.

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[1] C. H. Bennett and D. P. DiVincenzo, Nature (London) **404**, 247 (2000).
[2] See Ref. [1] for a recent review.
[3] J. I. Cirac and P. Zoller, Nature (London) **404**, 579 (2000).
[4] D. Deutsch, Proc. R. Soc. London **425**, 73 (1989).
[5] A. Barenco *et al.,* Phys. Rev. A **52**, 3457 (1995).
[6] H.-J. Briegel and R. Raussendorf, Phys. Rev. Lett. **86**, 910 (2001).
[7] The second Hamiltonian is of the standard Ising form. The symbol "$\cong$" means that the states generated from a given initial state, under the action of these Hamiltonians, are identical up to a local rotation on certain qubits. We use the first Hamiltonian to make the computational scheme more transparent. The conclusions drawn in the paper are, however, the same for both Hamiltonians.
[8] By the "amount of entanglement" contained in the resource, we mean any measure that satisfies the criteria of an entanglement monotone [9]. For cluster states, the entanglement can be calculated, e.g., in terms of the Schmidt measure of Ref. [10].
[9] G. Vidal, J. Mod. Opt. **47**, 355 (2000).
[10] J. Eisert and H.-J. Briegel, quant-ph/0007081.
[11] D. Gottesman and I. L. Chuang, Nature (London) **402**, 390 (1999).
[12] See, e.g., G. Grimmett, *Percolation* (Springer-Verlag, New York, 1989).
[13] A. Calderbank and P. W. Shor, Phys. Rev. A **54**, 1098 (1996).
[14] A. Steane, Phys. Rev. Lett. **77**, 793 (1996).
[15] G. K. Brennen *et al.,* Phys. Rev. Lett. **82**, 1060 (1999).
[16] D. Jaksch *et al.,* Phys. Rev. Lett. **82**, 1975 (1999).
[17] T. Calarco *et al.,* Phys. Rev. A **61**, 022304 (2000).
[18] H.-J. Briegel *et al.,* J. Mod. Opt. **47**, 415 (2000).
[19] M. T. DePue *et al.,* Phys. Rev. Lett. **82**, 2262 (1999).
[20] J. M. Ziman, *Models of Disorder* (Cambridge University Press, Cambridge, United Kingdom, 1979).
[21] R. Scheunemann *et al.,* Phys. Rev. A **62**, 051801(R) (2000).
[22] T. Tanamoto, quant-ph/0009030.