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The Newtonian gravitational constant: recent measurements and related studies

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Abstract

Improvements in our knowledge of the absolute value of the Newtonian gravitational constant, G, have come very slowly over the years. Most other constants of nature are known (and some even predictable) to parts per billion, or parts per million at worst. However, G stands mysteriously alone, its history being that of a quantity which is extremely difficult to measure and which remains virtually isolated from the theoretical structure of the rest of physics. Several attempts aimed at changing this situation are now underway, but the most recent experimental results have once again produced conflicting values of G and, in spite of some progress and much interest, there remains to date no universally accepted way of predicting its absolute value. The review will assess the role of G in physics, examine the status of attempts to derive its value and provide an overview of the experimental efforts that are directed at increasing the accuracy of its determination. Regarding the latter, emphasis will be placed on describing the instrumentational aspects of the experimental work. Related topics that are also discussed include the search for temporal variation of G and recent investigations of possible anomalous gravitational effects that lie outside of presently accepted theories.

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Contents

1. Introduction

The year 1998 will mark the 200th anniversary of the publication in *The Philosophical Transactions of the Royal Society of London* of the paper by Henry Cavendish entitled, 'Experiments to determine the density of the Earth' (Cavendish 1798). In his celebrated experiment, Cavendish used a torsion balance to investigate the gravitational attraction between laboratory test masses. This is now seen as one of the first quantitative examinations ever made of the description of gravity embodied in Newton's inverse-square law,

$$F = GMm/r^2$$

where F is the gravitational force acting between masses M and m, the centres of mass of which are separated by the distance r. The quantity G is most often referred to as the Newtonian constant of gravitation, the universal gravitational constant or, simply, the gravitational constant. There is perhaps no law of physics more familiar than this one, encompassing as it does virtually all of the gravitational phenomenon intrinsic to matter at terrestrial densities. The corrections to this law arising from general relativity produce only feeble effects until the scale of the masses and densities involved far exceeds those available in the laboratory.

The appearance of the paper by Cavendish marked an important point of transition in the study of the force of gravity. Prior to it, experiments in this field typically involved the Earth as one of the test masses, whereas thereafter (with several notable exceptions), 'benchtop' experiments carried out with relatively small-scale test masses became the focus of effort. The movement in this direction was driven in part by the long series of experiments performed throughout the 19th century by several investigators who were attempting to redetermine the mean density of the Earth using the approach of Cavendish. The results of those studies have since been reinterpreted as measurements of the absolute value of G, even though this quantity was unfamiliar to the earliest workers in this field (Clotfelter 1987). A large number of experiments aimed specifically at measuring G were then carried out during the 20th century, most of them using one form or another of either the torsion balance or the torsion pendulum. In spite of these many strenuous efforts, though, the past 200 years have left us with a value of the gravitational constant that has improved in accuracy by only about one order of magnitude per century.

The reasons for this slow rate of progress are well known (Speake and Gillies 1987a). First, gravity is by far the weakest of the four fundamental forces. The gravitational interaction between two baryons is roughly 10^{40} times smaller than, for example, the electromagnetic interaction between them, thus making it relatively easy for the gravitational signal in an experiment to be masked by competing effects. Second, the gravitational force cannot be screened. Because of this, it is virtually impossible to isolate the gravitational interaction between two masses from the perturbative effects created by surrounding mass distributions. Third, G has no known, confirmed dependence on any other fundamental constant. Hence, its value cannot be estimated in terms of other quantities (see below, however, for a description of efforts aimed at changing this situation). Finally, the instrument of choice for measuring G, viz, the torsion pendulum in one of its various forms, is subject to a variety of parasitic couplings and systematic effects which ultimately limit its utility as a transducer of the gravitational force. Beam balances, vertical and horizontal pendula, and other sensitive mechanical devices are also pressed to the limits of their performance capabilities when employed for this purpose. In spite of these difficulties, nearly 300 different measurements of G have been made over the years, including several in which the objective was either to search for some type of variation in G or to reveal dependences it might have on other physical variables.

With the advent of general relativity, a much clearer picture of the nature of the force of gravity began to develop. The simple descriptive character of Newton's law gave way to the deeper philosophical connections made by Einstein between the physical structure of space and time and the behaviour of matter and energy in it. G is still there, as a scale factor appearing in the field equations, metrics and solutions. The last few decades have seen a great thrust towards unification of the forces, and this has led to truly significant theoretical advances which may yet result in a consolidated description of the interactions, including gravity. If the resulting coupling constants incorporate G (and it is hard to see how they could not), then the overall uncertainty in them would almost certainly be governed by that of the determination of G itself, and an improved value of it would likely be quite welcome. As discussed below, there are other motivations for measuring G as well, some of them far more practical and imminent in need than those arising from the arguably speculative suggestions about any part that G might play in the unification process.

The goals of the review are to assess the present situation with respect to the role of G in physics, to survey the attempts to determine its absolute value by experiment and to catalogue the most recent searches for variations in G, including measurements and/or phenomenological inferences of its temporal constancy. The focus of the review will be on the work of the last 10 to 15 years. Hence, only a brief synopsis of the historical background for the modern studies will be presented, although several references to earlier detailed reviews will be provided. With that as a foundation, the status of G within the structure of modern physics will be examined, including a survey of recent attempts to arrive at a theoretical estimate of G. From there, the perspective will change to that of the experimental situation, and the results of recent determinations of the absolute value of G will be examined. Works in progress, as well as experiments proposed for both terrestrial and space-based laboratories will also be discussed, as will some details of the instrumentation and mass distributions that have either been used or proposed for use in measurements of G. The next section concentrates on the attempts to investigate the temporal constancy of G and on other experimental explorations of variability in or anomaly of the gravitational force. The review closes with a brief discussion of how the field stands relative to the rest of physics, and of where it is likely to go as the technological basis for precise experiments continues to improve.

2. Historical background

The earliest quantitative attempts to arrive at a value for the mean density of the Earth, Δ, involved measurements of the deflection from vertical of a plumb line held in the vicinity of a mountain. Although the mountain under study was surveyed carefully in each such effort, the geology-related inaccuracies intrinsic to this general approach led to its virtual abandonment following the introduction of the torsion-balance method by Cavendish. However, a geophysical role in the history of the measurement of Δ and, subsequently, G was maintained by those who inferred the values of these quantities from measurements of the acceleration of gravity, g, as a function of depth in mineshafts. These three quantities are interrelated by the expression

$$g = (4\pi R_\oplus/3)(G\Delta)$$

where $R_\oplus$ is the mean radius of the Earth. An additional geophysical role for the gravitational field of the Earth is in all beam-balance determinations of G, where it forms the dominant background against which such measurements are made. While the Earth's field is the principle feature in those cases, one of the great advantages of the method introduced by Cavendish is that it places the gravitational interaction between the test masses in a plane orthogonal to the direction of the Earth's field, thus essentially eliminating its effect from the experimental arrangement.

Practically all of the early work on this subject has been catalogued in the monographs of Poynting (1894) and Mackenzie (1900), who describe the results obtained with each of the different experimental approaches used throughout the 18th and 19th centuries. (The experiments carried out by both of these authors are of considerable interest in their own right, and an historical narrative describing that of Poynting has been prepared recently by Falconer (1991).) A little known but very interesting review of the experimental situation at the start of the 20th century was published by Burgess (1902a), who introduced the liquid mercury bearing into torsion-balance measurements of G (Burgess 1902b). This technique has been used again recently, and the resulting experiment has yielded a value of G which (along with those from other experiments) is central to much of the revived interest in this field.

The value of G obtained by Boys (1895) was held by most as the accepted value until it was displaced by the result of Heyl and Chrzanowski (1942), who concluded that

$$G = (6.673 \pm 0.003) \times 10^{-11}\ \mathrm{m^3\ s^{-2}\ kg^{-1}}.$$

Heyl, who had been a co-inventor of the 'Earth induction compass' used by Lindbergh on the first solo transatlantic flight (Heyl 1954), had a lifelong interest in the measurement of the gravitational force, having made several previous determinations of both G and g. The beginning of the modern era of measurements of the absolute value of G is usually associated with the appearance of his results, a slight modification of which stood as the accepted value until the 1986 adjustment of the fundamental constants, which brings us to recent times.

The second half of the 20th century has seen more experimental work on the measurement of G than has any other period in the history of science. Not only have there been many diligent efforts aimed at making new determinations of the absolute value of G, but there have also been a number of major studies that focused on questions of the variability of G in space and time, and on searches for forms of anomalous behaviour in the gravitational force. Several workers chronicled the investigations that have been carried out in these areas, and some representative reviews are those of Cook (1971), Beams (1971), Gaskell *et al* (1972) and Sagitov (1976), who provided useful contemporary surveys of the status of the field. A virtually complete listing of the values obtained in each of the measurements of the absolute value of G made prior to the 1980s was published by Mills (1979).

The 1970s also saw a surge of interest in tests of the exactness of the inverse square law of gravity, potential breakdowns in which were then typically modelled in terms of a spatially-dependent gravitational constant, i.e. as $G(r)$. This work, carried out on both the laboratory and geophysical scales of distance, was motivated in part simply by empirical interest in the behaviour of G, but also on a more fundamental level by suggestions that there may be a non-Newtonian component to the gravitational potential. At about the same time, the first astronomical tests of the time variation of G were beginning to yield preliminary results, and sensitive laboratory experiments for detection of a non-zero $(\mathrm{d}G/\mathrm{d}t)/G \equiv \dot{G}/G$ were proposed and, in some cases, undertaken by a few groups. The original motivation for investigating the temporal constancy of G arose from the 'large numbers hypothesis' of Dirac but was further enhanced by the predictions of the Brans–Dicke (scalar tensor) and other contemporary theories of gravity. The experimental status of both lines of work at the end of the 1970s was evaluated by Ritter (1982) and Gillies and Ritter (1984).

Much of the experimental work on the measurement of the absolute value of G that was carried out during the 1960s and 1970s came to fruition with the 1986 adjustment of the fundamental constants, when the recommended value of G became

$$G = (6.672\,59 \pm 0.000\,85) \times 10^{-11}\ \mathrm{m}^3\ \mathrm{s}^{-2}\ \mathrm{kg}^{-1}$$

(Cohen and Taylor 1987, Taylor and Cohen 1990). In that same year Fischbach *et al* (1986) announced the possible discovery of a new weak force in nature that might manifest itself as a non-null result in searches for a composition dependence of the gravitational force. The great excitement created by this interesting conjecture led to massive efforts on both the theoretical and experimental fronts, all aimed at exploring the possibility that such a force might indeed exist. For reviews that tracked the evolution of that search, see Fischbach *et al* (1988), Adelberger (1990a) and Fischbach and Talmadge (1992). While no compelling evidence in support of this ‘fifth force’ was ultimately uncovered, it nevertheless played a very important role in stimulating a re-examination of the scientific basis of the universality of free fall and the weak equivalence principle within the structure of modern physics. Thorough discussions of the status of experimentation in these areas have been presented by Adelberger *et al* (1991) and Adelberger (1994). Superimposed on all of this were the ongoing advances in string theory that have led (among many other things) to new mechanisms by which a non-zero value of $\dot{G}/G$ might arise. A general overview of the arguments that lead to such an effect has been presented by Hellings (1988).

Several rather sweeping reviews that cover the field of Newtonian gravity as a whole were published from the mid 1980s onward, beginning with the rapporteurial works of de Boer (1984), Stacey (1984) and Cook (1987). Perhaps the most thorough survey of this period was that of Cook (1988), who examined most of the then-available results of laboratory tests of the inverse square law and the weak equivalence principle, and who went on to review the status of the gravitational constant, as well. Another comprehensive review is that of Stacey *et al* (1987), who discussed the many different geophysical measurements of Newtonian gravity, including those performed in mines, in the oceans and in pumped-storage reservoirs. A research bibliography of all known measurements relating directly to G, including those searching for variations in it with respect to the temperature of the test bodies, their electromagnetic state, the orientation of their crystalline axes, etc was assembled by Gillies (1987). The experiments that made up these different categories of measurements were subsequently discussed in greater detail in articles published thereafter (Gillies 1988, Gillies 1990). Representative papers on a variety of these topics were selected and issued as a reprint book, too (Gillies 1992). An updated, companion bibliography to that of Gillies (1987) was prepared by Fischbach *et al* (1992) for the purpose of documenting the historical record of the work done on the fifth force through that point in time. It included citations to virtually all of the many different tests of composition-dependent gravity and of the inverse square law that were carried out during the course of experimentation on the fifth force. Franklin (1993) went a step further and wrote a monograph on the whole episode, emphasizing those aspects of interest to the historian of science.

A few other recent reviews should also be mentioned here, as they contain commentary of interest to those pursuing measurements of G. These include the detailed critique of all aspects of contemporary experimental gravitation published by Will (1992), the extensive discussion of laboratory techniques employed in measurements of Newtonian gravity prepared by Chen and Cook (1993), the tutorial articles on the absolute value of G by Gillies and Sanders (1993a, 1993b), and the textbook survey of the field by Ohanian and Ruffini (1994).

3. Relationship of G to modern physics

The classical reasons usually advanced for seeking a more accurate value of G include the simple metrological challenge of reducing the uncertainty of its determination, the concomitant potential for improvement in our knowledge of the inverse square law and the ensuing obtainment of an improved numerical estimate of the mass of the Earth in kilograms. As mentioned above, a new motivation may be forthcoming if a *bona fide* prediction of the value of G is made, and especially if this value then enters into scale factors or other quantities that arise within theories of the unification of the forces. While these might be reasons enough to strive for an improved determination of G, there are a number of other physical, geophysical and astronomical arguments that can be made in favour of finding a more accurate value for this constant. In what follows, we catalogue several of them and then review the status of attempts to derive theoretical estimates for the value of G.

3.1. G as an entity in physical science

The value of G governs the scale of the gravitational interaction and the smallness of it makes gravity the weakest of the four known forces. Because the force of gravity is intrinsically attractive and cannot be shielded, however, the collective action of large amounts of matter makes it the dominant interaction between ponderable bodies at large distances. The motion of the Earth around the Sun is the prototypical example: Kepler's laws of planetary motion are one consequence of the inverse square law and the conservation of angular momentum. Not surprisingly, then, either G or (more typically) a factor that plays implicitly the same role is found in virtually all of the equations of motion in orbital dynamics and celestial mechanics that are pertinent to the central force problem. The specific formulations of the various types of 'gravitational constants' that are used in these fields are presented in a number of textbook expositions, for example that of Herrick (1971).

One such entity is $GM_\oplus$, which is called the geocentric gravitational constant where $M_\oplus$ is the mass of the Earth. It has been determined by Ries *et al* (1992) to have the value

$$GM_\oplus = (398\,600.4415 \pm 0.0008)\ \mathrm{km}^3\ \mathrm{s}^{-2}.$$

The effects of the mass of the Earth's atmosphere were taken into account in arriving at this value, which was based on laser ranging to the LAGEOS satellite. The 1-σ error quoted in the result represents an uncertainty of only 0.002 parts per million (ppm). Their result is in good agreement with that obtained by lunar laser ranging (Dickey *et al* 1994). Feng *et al* (1993) also report the results of a recent measurement which, in their case, yielded a value that was approximately 0.007 ppm smaller than that of Ries *et al*. Tabulated listings of the results of many of the earlier determinations of $GM_\oplus$ are presented by Lerch *et al* (1978) and Ries *et al* (1989). It is through this quantity that a decreased uncertainty in the value of G will lead directly to an improved value of the mass of the Earth in kilograms.

Likewise, the heliocentric gravitational constant is $GM_\odot$ where $M_\odot$ is the mass of the Sun. It is designated in the International Astronomical Union's 1976 *System of Astronomical Constants* as a derived constant. The most recently adopted value of it, in customary units, is (US Navy Nautical Almanac Office 1995)

$$GM_\odot = 1.327\,124\,38 \times 10^{20}\ \mathrm{m}^3\ \mathrm{s}^{-2}.$$

Of special interest is the Gaussian gravitational constant, k, which is also referred to as the defining constant. It arises via the application of Kepler's third law to the motion of the

Earth around the Sun. The operative expression is (Herrik 1971)

$$k = (2\pi a^{3/2})/(P[1+m]^{1/2})$$

where, using the astronomical system of units, P is the orbital period of the Earth in days (d) of exactly 86 400 s each and m is measured in solar masses. Under these conditions a is the value of the Earth–Sun distance (i.e. the astronomical unit or AU) for which k has the fixed value

$$k \equiv 0.017\,202\,098\,95$$

(US Navy Nautical Almanac Office 1995). G and k^2 have the same dimensions and the relationship between them is

$$G = k^2 \ \mathrm{AU}^3 \ \mathrm{M}_\odot^{-1} \ \mathrm{d}^{-2}.$$

Because of its scaling, however, k^2 is the quantity that is used as the gravitational constant in expressions employing the astronomical system of units. The values of these units in terms of their SI counterparts and listings of the relevant astronomical constants have been prepared by Abalakin *et al* (1987) and Cohen (1996).

In spite of the universality of k^2 in celestial mechanics, G can always be retained explicitly in a particular expression, as is the case, for instance, in the equation for a parameter used to scale relativistic effects in the modelling of precession in binary orbits. Gallmeier *et al* (1995) write it as

$$S = [2(3+e)/(1-e^2)][2\pi G(M_1+M_2)/Pc^3)]^{2/3}$$

where e is the eccentricity of the orbit and M_1 and M_2 are the masses of the binary's components.

Not only the motions of celestial bodies, but also models of their evolution and structure depend on the value of G. For example, the equation of hydrostatic equilibrium for stars reflects the balance between the outward forces produced at points in the star's interior by radiation pressure, $P(r)$, and the compressive gravitational forces produced by the stellar material, $M(r)$, of density ρ within a radial distance r of the star's centre (see, for example, Ezer and Cameron 1966):

$$\mathrm{d}P/\mathrm{d}r = -GM(r)\rho/r^2.$$

Moreover, the luminosity of such a star is a very strong function of G, going approximately as G^7 (Teller 1948), which has stimulated much discussion about detecting temporal variations in G by examining the paleological record of possible luminosity-dependent phenomenon. Observational validations of these models could benefit from a better value of G. Finally, Linde (1990) has noted that the rate at which the universe expands (at a given temperature) becomes slower as the value of G decreases, until a point is reached where the deviations from thermal equilibrium needed for post-inflation baryosynthesis are too small to yield the presence of matter in places where it is found in the universe. As we shall see later, many workers have used nucleosynthesis rates as a sensitive means to search for a time variation in G.

There are also important geophysical reasons for wanting reduced uncertainty in G. McQueen (1981), for instance, points out that the uncertainties with which the density and elastic parameters used in Earth models are known cannot be any less than that with which G is known. From a very practical point of view, Kolosnitsyn (1992) has observed that the calibration accuracy of the gravitational gradiometers sensitive to higher-order derivatives of the Earth's field is limited to the precision with which G is known, presumably with implications for geophysical prospecting and related studies.

G also arises in several very fundamental ways in modern theoretical studies of particles and fields, cosmology and, of course, gravitational physics. The remainder of this subsection provides a brief glimpse of a few of the many ways in which this happens. An interesting starting point is the definition of the Planck scale of length, time and mass:

$$\text{Planck length} \equiv (\hbar G/c^3)^{1/2} \approx 1.6 \times 10^{-35}\ \text{m}$$
$$\text{Planck time} \equiv (\hbar G/c^5)^{1/2} \approx 5.4 \times 10^{-44}\ \text{s}$$
$$\text{Planck mass} \equiv (\hbar c/G)^{1/2} \approx 2.2 \times 10^{-8}\ \text{kg}.$$

These particular combinations of $\hbar$, c and G establish the cut-off point at which the structure of spacetime can no longer be viewed with geometric continuity and below which quantum fluctuations produce a non-stationary background. A Planck energy, temperature and charge can be similarly constructed. For a discussion of the role of the Planck units within the hierarchy of the elementary constants, see Hantzsche (1990).

The recent development of string theory has brought forth a new tool that is being used to explore the unification of gravity with the other interactions at this scale of events. De Sabbata and Sivaram (1995) began their investigations in this area by noting that the string tension in the Planck scale is given by

$$T_{\text{Pl}} = c^2/G$$

and they go on to propose a way in which charge and mass can arise from torsion-induced string tension. Elizalde and Odintsov (1995) studied a theoretical approach to string-inspired dilatonic gravity in which the renormalization group β-functions for G and for the dilatonic couplings were synthesized to first order in G.

De Sabbata *et al* (1992) note that theories with torsion lead to a prediction that the spin-aligned particles in primordial matter will give rise to a magnetic field of value

$$H = (8\pi/3c)(2\alpha G)^{1/2}\sigma$$

where α is the fine structure constant and σ is the spin density. While this effect is still under study, they additionally argue that torsion-induced spin alignment also provides a theoretical foundation for Blackett's law

$$S = qU.$$

Here, S is intrinsic angular momentum, U is the magnetic moment of a star and q is a universal constant having the value

$$q \approx c(\alpha G)^{-1/2}$$

which they point out is in excellent agreement with the numerical value derived from astrophysical observations.

Terazawa (1980) has developed a simple relation between α, G and the Fermi weak-coupling coefficient, G_{F}, within the context of a unified model of the elementary particle forces that includes gravity. Using it, he arrived at the following expression for α:

$$\alpha = 9\pi/\{8N \ln[18G_{\text{F}}/5(\sqrt{}2)N\alpha G]\}$$

where N denotes the number of generations of Weinberg–Salam multiplets of leptons and quarks included in the calculation. At $N = 6$, the predicted value of α closely matches the experimental value, but the underlying physical significance of this observation requires further study. Earlier work underlying this model was carried out by Landau (1955) and others and is discussed by Terazawa (1980). Rozental' (1980) has also described a possible relationship between α and G and discusses it within the context of bounds on the stability of the proton.

G arises in the expression for the fractional difference between gravitational masses of the K^0 meson and its antimeson as derived from the associated vector–scalar interaction energy (Kenyon 1991). The smallness of the difference ($\lesssim 2 \times 10^{-13}$) has been cited as an explanation of why long-range vector–scalar couplings are not observed.

The origin of G and the nature of possible constraints on its role in physics have been discussed extensively in the literature. Fujii (1982), for example, notes that G emerges from the cosmological background value of the scalar field in scalar-tensor (i.e. Jordan–Brans–Dicke or JBD) theories of gravity, and that within a scale-invariant version of this theory G remains constant in the frame in which fundamental particle masses are constant. Zhang (1993) explores the Newtonian limit of such theories and finds that they predict that the motion of a test particle is governed by an 'effective' gravitational constant, G_{eff}, which nevertheless incorporates G:

$$G_{\mathrm{eff}} = (1 - \beta^2/4)G.$$

Here, β is a constant appearing in the Lagrangian for the generalized JBD theory, and its value is constrained by inflation models such that $\beta^2 < 0.3$.

Li and Zhang (1992) explored the nature of the constraints on G in the complicated spacetime topology of a wormhole. They conclude that when G is maximized in a parameter space where the cosmological constant approaches zero, there will be strong charge-parity conservation and implications for the quark mass ratios. Kim *et al* (1993) investigated the constraints on the time evolution of G that can be derived from models of big-bang nucleosynthesis (as have several others; see below). As a starting point, they used an expression that relates the expansion rate of the universe, H, to G:

$$H^2 = ((\mathrm{d}R/\mathrm{d}t)/R)2 = (8\pi/3)G\rho_{\mathrm{rad}}$$

where R is the cosmological scale factor and ρ_{rad} is the energy density. They went on to discuss a number of astrophysical and cosmological consequences of a variable-G scenario.

Greensite (1994) presented a transfer matrix formalism for quantum gravity wherein the Planck mass (and, hence, G) is a dynamical quantity with the result that each of the stationary physical states has a different value of G. This is equivalent to saying that there is a dispersion of G in this approach of size $\Delta G/G$, in analogy to the dispersion of energy of the non-stationary states in non-relativistic quantum mechanics. Nordtvedt (1994) discusses a number of different tests of post-Newtonian gravity and emphasizes the importance of making measurements that would reveal the size of the higher-order effects characteristic of the gravitational self-energies of the interacting bodies. G plays a role in the relevant expressions for the two-body coupling parameter, the Newtonian 'sensitivity' factor, and the non-perturbative mass of a compact body.

These are just a few of the many ways that G, through the ubiquity of the gravitational force as described by general relativity, has permeated the structures of astronomy and physics. Cook (1988) has even raised the question of whether G might not be included as one of the fundamental constants in a system of metrology, as is c (the speed of light) now by definition in the SI. He also notes that a much more precise value of G is required if that possibility is to be considered. Deeds (1993) has argued that even within the present structure of the SI units the uncertainty in the value of G impacts advancement of the knowledge of the kilogram. Taylor (1991), however, points out that a non-artifactual realization of the kilogram is desirable, and that the achievement of this will be dependent on relationships between the fundamental atomic constants, atomic masses and lattice spacings.

Some authors (e.g., Thüring 1961) have concluded that G has been introduced somewhat arbitrarily into physics and that it cannot be associated with a unique property of nature.

The tide of opinion, though, as expressed through incorporation of G into an ever-growing number of physical and astrophysical discussions points to a different conclusion, and the consensus is that an improved experimental determination of G would indeed be most welcome. The remainder of this section takes up the companion issue of the status of efforts towards a *bona fide* theoretical prediction of G.

3.2. Theoretical estimates of G

Most of the recent attempts at making a theoretical estimate of G fall into two broad classes: those conceived within the context of the 'standard model' of particles, fields and cosmology, and those that invoke new physics. The former typically work from a general relativistic starting point, using derivations based on the classical Lagrangian and the matter action expressions to examine the outcome of some scenario in which G or a factor containing it arises from the calculations. The goal in these efforts is not so much to arrive at a numerical value for G itself as it is to see if the consequences of the predictions are consistent with the observed behaviour of matter in gravitational fields, with the structure, symmetry and sizes of the existing forces, with the cosmological conditions of the early universe, etc. The latter class of approach usually seeks the *ad hoc* introduction of a new field or effect to create a situation in which a value for G can be built from ratios of other fundamental constants and numerical factors. Some examples of both types of calculations are presented in what follows, beginning with those representative of the first category.

Much of the modern work on calculation of the gravitational constant was stimulated by Sakharov (1968) who questioned the fundamentality of the gravitational force and conjectured that it might instead exist because of zero-point fluctuations of the vacuum that presumably occur when matter is present. Subsequently, Adler (1980) derived an expression for the 'induced' gravitational constant, G_{ind}, in terms of flat spacetime quantities. He started by calculating the change in the matter stress–energy tensor induced by curvature in spacetime for the case of a conformally flat spacetime of constant curvature. By using an expansion form of the metric and through subsequent manipulation of it, he arrived at an expression that yielded $(8\pi G_{\mathrm{ind}})^{-1}$ in terms of an integral (in spacetime coordinates) over the trace of the stress–energy tensor. Zee (1982, 1983) had also been working in this area and derived his own expression for G_{ind} within an infrared stable class of scale-invariant gauge theories. He noted that the specific theoretical model underlying the calculation was neither confining nor symmetry breaking, however, and thus provided an incomplete description of physical reality. Even so, a pursuit of the model's implications led to explicit expressions for the sign and magnitude of Newton's constant, thus demonstrating its potential calculability and again raising the question of just how fundamental a parameter G truly is. A somewhat related endeavour was undertaken by Krasnikov and Pivovarov (1984), who used finite energy sum rules to calculate the induced gravitational constant. Zee (1982, 1983) also discussed the previous efforts of others who had worked on either the calculation of G or related topics.

de Alfaro *et al* (1983) developed a model in which the dimensional constants that govern the behaviour of low-energy processes are related to the vacuum expectation values of the initial fields. They used it to derive relationships between the Planck mass, the cosmological constant and the original coupling constants which appear in the Lagrangian. Evaluation of the Planck mass in this way is equivalent to establishing a value for G, which is taken here to be the factor that characterizes the emission and absorption amplitudes of gravitons. Pollock (1983) developed a theory of induced gravity that incorporated a scalar field. He formulated a modified version of Einstein's equations which contained an ‘effective gravitational’ constant, G_{eff}, that could be expressed in terms of the inverse of the mean square of the field. He then added a thermal component to the potential and reformulated the Einstein equations once again. Evaluation of them and comparison with his previous result led him to the conclusion that the gravitational constant was independent of temperature, with the implication being that symmetry among the interactions is not restored at very high temperatures.

Cahill (1984a, b) noted that when G is defined in terms of the Planck mass, it can be interpreted as being a mass scale which is governed by the large-scale structure of the universe. He developed an expression for the action of a Weyl spinor interacting with a gravitational field, and rescaled the metric and the Fermi field such that the Planck mass did not appear in the expression. This led him to question whether or not a constant that could be scaled away could also be fundamental, since the coupling constants cannot be scaled away in Yang–Mills theories. He then used cosmological arguments to relate G to the magnitude of the scaled metric, and suggested that G arose in theories of gravity because the vacuum expectation values of the Robertson–Walker metrics were on the order of unity. Other scaling-related issues involving the coupling constants of the fundamental interactions and various cosmological parameters were taken up by Sivaram (1994, and references therein). He examined their behaviour in the early universe within the context of a model in which the fundamental forces (including gravity) are produced by four-fermion self-interactions in curved space with torsion. One of his expressions related the Fermi weak interaction constant to the gravitational constant, while another incorporated G into an arrangement where the macroscopic mechanical quantities of, for example, a neutron star (energy, equilibrium radius, angular momentum) were related to the inverse of the superstring tension.

Mathiazhagan and Johri (1984) investigated a modified inflationary universe within the context of the Brans–Dicke theory of gravity. Their model gave rise to an exact solution for the size of the Brans–Dicke scalar field, the inverse of which was used to replace the gravitational constant in this theory. Numerical estimates of the size of the field yielded equivalent values of G of approximately 10^{-40} GeV^{-2}. This value is within two orders of magnitude of the size of G as interpreted in terms of the Planck units, i.e. approximately 10^{-38} GeV^{-2}, an encouraging level of agreement given that their model did not include quantized gravity. Linde (1990) and García-Bellido *et al* (1994) considered how extended chaotic inflation and fluctuations in quantum gravity might affect the value of G in different parts of the universe. The models they developed yielded scenarios in which the probability distributions of finding a given region of the universe having a particular value of G could be calculated. They also discussed how the anthropic principle might play a role in determining the value of G.

Puthoff (1989) returned to Sakharov’s original conjecture and developed a zero-point-fluctuation interaction model in which gravitational mass and its behaviour were shown to arise in a manner consistent with ‘zitterbewegung’ particle motions. His model led to a prediction of the value of G given in terms of an integral over the vacuum zero-point-fluctuation spectrum, with the upper limit of the integral being the Planck cut-off frequency. He concluded that gravity could be viewed as being a form of long-range van der Waals force, not dissimilar in origin from the familiar short-range van der Waals and Casimir forces (i.e. arising because of the dynamics of the interactions between particles and the vacuum electromagnetic field’s zero-point fluctuations). Carlip (1993) disputed these results, pointing out that the force predicted by Puthoff’s model actually went as the inverse fourth power of distance, and hence was incompatible with Newtonian gravity. Puthoff (1993) responded by noting that the use of appropriate physical cut-offs would maintain the inverse-square behaviour of the predicted force, and thus preserve the utility of the model.

Damour (1996) followed the suggestion of Landau (1955) that the small dimensionless constant $Gm^2/\hbar c$, where m is the mass of a fundamental particle, might be related to the fine structure constant α by an expression of the form

$$Gm^2/\hbar c = A \exp(-B/\alpha)$$

where A and B are numbers on the order of unity. 't Hooft (1989) had also investigated this possibility. Using the values of $A = (7\pi)^2/5$ and $B = \pi/4$ motivated by instanton physics (see the discussion in 't Hooft (1989)), Damour found a numerical value for G of $G = (6.672\,345\,8\ldots) \times 10^{-11}$ m^3 s^{-2} kg^{-1}. He noted that the agreement of this estimate relative to the experimentally-derived CODATA value is quite good, viz, $G^{\text{obs}}/G^{\text{theory}} = 1.000\,04 \pm 0.000\,13$, and emphasized the importance of improving the measured value of G should theory one day allow us to predict it in terms of other quantities.

As mentioned above, there is another category of G estimates that have as their focus the introduction of new fields, other new physics or certain cosmological considerations as a means of arriving at a numerical value of G. Table 1 contains a listing of several such efforts, including the value of G produced in each case. An abstract by Long (1967) also suggests a method for arriving at a numerical value for G and provides an expression for it, but does not go on to quote a particular value. See also the work of Aspden (1989) and his related articles. While detailed discussion of the reasoning that leads to the entries in table 1 is outside the scope of this review, it is interesting to note that several of these calculated values are also in close agreement with the CODATA value.

Table 1. Some theoretical values of G as calculated by various authors. Where given, the figure in parentheses following the predicted value is the estimated error in the last digit of the value.

Reference/date	Physical basis of prediction	Estimate ($\times 10^{-11}$ m^3 s^{-2} kg^{-1})
Bleksley (1951)	Conservation-of-energy arguments applied to an expanding universe	≈ 9
Krat and Gerlovin (1974)	'Fundamental field theory' evaluation of elementary particle parameters	6.673 11(4)
Sternglass (1984)	Early universe scenario with a model of a 'charmonium-like' massive charge pair	6.6721(5)
Soldano (1986)	Invariant component of G in a model of causal reference frame dependency in α	6.7340
Gasanalizade (1992a, 1993)	Ratio of gravitational red shift of H in solar spectrum to electron Compton wavelength	6.679 197 926
Spaniol and Sutton (1992a, b, 1993)	Consideration of rest mass of electron from field self-energies	6.672 527 5(9)
Lidgren (1996)	Thermodynamic interpretation of the gravitational force	6.664(2)

Finally, as a point of historical interest, Bartoli (1886) made an early attempt at calculating the related quantity Δ, the mean density of the Earth. He divided the sum of the atomic weights of the then-known elements by the sum of the ratios of the atomic weight to the specific gravity of each element, and arrived at the value $\Delta \approx 5.78$ g cm^{-3}, a value roughly 6% larger than that obtained by Cavendish (1798).

4. Experimental determinations of the absolute value of G

4.1. Survey of modern measurements

An appropriate starting point for discussions of the modern measurements of G is the CODATA value for it, established during the 1986 adjustment of the fundamental constants (Cohen and Taylor 1987, Taylor and Cohen 1990):

$$G = (6.672\,59 \pm 0.000\,85) \times 10^{-11}\ \mathrm{m^3\ s^{-2}\ kg^{-1}}.$$

This value is essentially that obtained by Luther and Towler (1982), see also Luther (1983) and Luther and Towler (1984), in work carried out at the US National Institute of Standards and Technology (then the US National Bureau of Standards), but with the assignment of a larger uncertainty to reflect the spread between the three best results available at the time. (The other two experiments which also claimed uncertainties of approximately $1 : 10^4$ had been carried out somewhat earlier in France (Pontikis 1972a) and the Soviet Union (Sagitov *et al* 1979).) In fact, the actual experimental values obtained in these three measurements excluded one another within the limits of the errors quoted by each group. They all used torsion pendulums, although of fundamentally different designs, to make the measurements. The physics of the torsion pendulum is well known and has been reviewed recently by Gillies and Ritter (1993). Moreover, its use as a detector of weak forces in gravitation experiments has been discussed extensively in the literature, for example by Cook (1971, 1987, 1988) and Chen and Cook (1993). Hence, except where new developments are involved relative to the determination of G, the details of torsion-pendulum theory and application will not be addressed here. (Section 4.7 below, however, contains on overview of some of the factors thought to limit the precision with which G can be determined by torsion pendula and other experimental techniques.)

Similarly, the details of the experiments on which the 1986 CODATA value is based have also been discussed by many authors, so only a brief sketch of each is included here. Luther and Towler (1982) employed a torsion pendulum of classic design, measuring the time of swing of the pendulum with the attracting masses in place near the suspended attracted masses, and then again when the attracting masses were removed. G was then determined from the difference in the measured periods and the metrological properties of the apparatus. The attracting masses were spheres of tungsten, approximately 10.5 kg each, and the small masses were disks of tungsten suspended from a 12 μm diameter quartz torsion fibre that was 40 cm long. The French experiment used a resonant torsion pendulum similar to that conceived by Kunz (1927, 1930), first put in use by Zahradníček (1933), and analysed in detail by Langevin (1942). In it, the attracting masses excited oscillation of the suspended masses (Facy and Pontikis 1970, 1971), with G being derived from measurements of the resonant response. Measurements were made with attracting masses of silver, copper, bronze and lead, all of which were spheres of approximately 1.5 kg. Their suspension fibre was made of tungsten. A detailed report on the design of the apparatus and the results obtained with its use was prepared by Pontikis (1972b). The measurement in the Soviet Union was carried out at the Shternberg Astronomical Institute in Moscow using a torsion pendulum with cylindrically-shaped attracting and attracted masses, the former of which could be translated axially to facilitate the evaluation of various experimental parameters. The attracting masses were made of non-magnetic steel and were approximately 39.7 kg each. The small masses were made of copper, about 30 g each, and suspended from a 32 μm tungsten fibre. The oscillations of the suspended body were recorded photo-optically, and the resulting data were fitted to an equation of motion for the pendulum from which G was subsequently evaluated. Theirs is perhaps the most thoroughly documented experiment on

G ever undertaken: there are over 50 papers by the Shternberg group and other colleagues (Gillies 1987) describing the apparatus, the theory behind it, the assessment of uncertainties and the results of the measurements. While many of those papers are in the Russian language, Stegena and Sagitov (1979) have published a monograph which also describes this experiment extensively in English.

The values of G obtained in these three experiments (along with those of all other experiments discussed in this section) are summarized in table 2 and figure 1. The disagreement between these three results did not go unnoticed, and the existence of this discrepancy helped motivate subsequent measurements of G. As we shall see, however, history has repeated itself in that a new generation of experiments completed since then have produced a similar and perhaps even more interesting discrepancy. Before proceeding to a discussion of this latest round of experiments, though, some commentary is offered on a number of other measurements that were made between *circa* 1970 and 1990. A few special features of the apparatus and its performance are mentioned in each case, as some of these experiments are not well known outside of the gravitational physics community.

A determination of G was carried out at the Eötvös University in Budapest by Renner (1970, 1974), using a torsion pendulum operated in the time-of-swing mode. The large ('attracting') masses in this case were mercury-filled cylinders that could be positioned either in line with the balance beam or at a 90° angle relative to it. The suspension fibre was a 20 cm strand of a platinum-iridium alloy, and it supported the balance beam and the small ('attracted') masses that were attached to it. The small masses were 16 g each and made of lead. All linear measurements were carried out by cathetometer to an accuracy of 10 μm, and the difference in the period of oscillation produced by moving the mercury masses from the in-line to the quadruture positions was measured to an accuracy of 10 ms. The torsion pendulum was placed in a sealed chamber evacuated to approximately 130 Pa ($\approx$ 1 mm Hg) and with this apparatus G was measured to a precision of $1 : 10^3$.

A novel approach to the measurement was undertaken by Faller and Koldewyn (1983), who built a torsion pendulum in which the attracting masses were large, thick-walled, open-bore cylinders (gravitational 'doughnuts'). A test body moved along the axis of a mass of this type will be subject to a gravitational force that vanishes at the centre of the cylinder due to symmetry, passes through a broad and relatively flat maximum and then vanishes again at infinity. (A mathematical derivation of the gravitational force produced on a test body by such a mass was given by Hulett (1969), who attempted to use a horizontal pendulum to measure G with gravitational doughnuts serving as the attracting masses.) The use of attracting masses of this type makes it possible to relax the requirements on the accuracy needed in the measurement of the intermass spacing, since the gravitational force between the masses remains nearly constant over a small but non-negligible span of distance. Another interesting feature of this experiment was the use of a magnetic suspension system to replace the torsion fibre. The softness of the restoring torque produced by the magnetic suspension allowed the pendulum's period to be about 3.5 h, but domain realignment in the ferrite material used for the suspension components resulted in large fluctuations in the period. Even so, it proved possible to obtain an early-stage value of G with this apparatus in the time-of-swing mode (Koldewyn 1976).

J W Beams and colleagues also experimented with magnetic suspension of the balance beam in a determination of G, but ended up using a quartz fibre suspension instead. The principal innovation introduced in their experiment was rotation of the torsion balance on an air-bearing supported turntable (Rose *et al* 1969). The angle between the small and large mass systems was detected by an autocollimator. As the small mass system tended to twist towards the large masses, a feedback signal based on the autocollimator output was

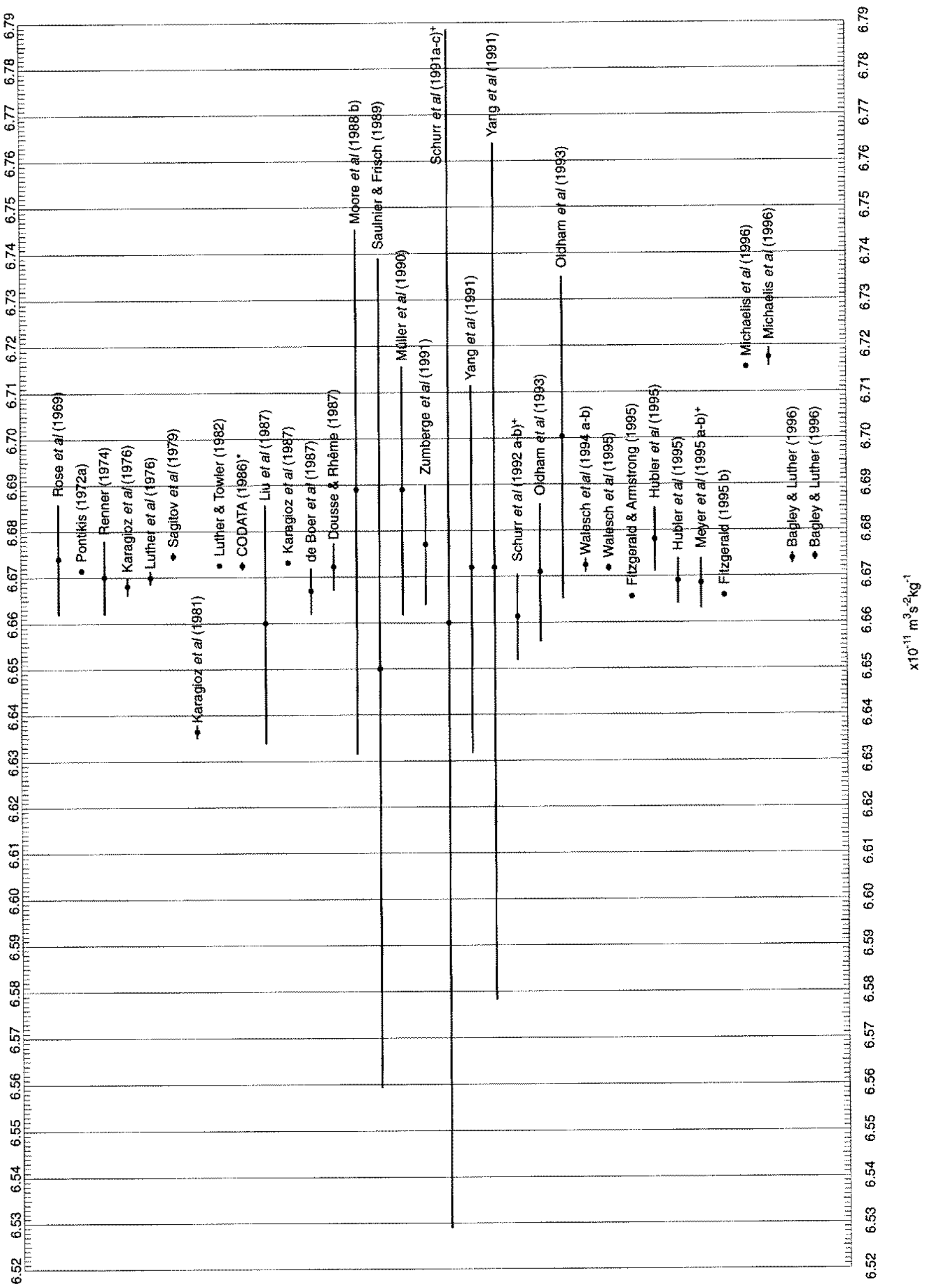

Figure 1. Several recent determinations of the Newtonian gravitational constant, G, selected from the results presented in table 2. * See Cohen and Taylor (1987); + the error bars represent the quadrated sum of the individually listed type A and type B uncertainties.

Table 2. Modern determinations of the Newtonian gravitational constant, G.

Reference/date	Method employed	G (10^{-11} m^3 s^{-2} kg^{-1})
Rose *et al* (1969)	Rotating torsion balance with servo control of mass spacing	6.674 ± 0.012
Pontikis (1972a)	Resonant torsion pendulum with various attracting masses	6.6714 ± 0.0006
Renner (1974)	Torsion pendulum in time-of-swing mode	6.670 ± 0.008
Karagioz *et al* (1976)	Evacuated torsion pendulum using a single, spherical steel attracting mass	6.668 ± 0.002
Koldewyn (1976)	Magnetically suspended torsion pendulum in time-of-swing mode	6.57 ± 0.17
Luther *et al* (1976)	Rotating torsion balance with servo control of mass spacing	6.6699 ± 0.0014
Mikkelsen and Newman (1977)[a]	$G_c \equiv$ value of G found from geophysical and astronomical considerations in the range from 10^3–10^8 km	$0.6 < G_c/G_0 < 1.3$
Yu *et al* (1978, 1979)	Worden gravimeter used to measure the field of an oil tank when full and empty; range of experiment, 15 m	6.67 ± 1.20
Sagitov *et al* (1979)	Torsion pendulum with cylindrically shaped attracting and attracted masses	6.6745 ± 0.0008
Page and Geilker (1981)	Torsion pendulum with a measurement strategy governed by a quantum decision process	6.1 ± 0.4
Karagioz *et al* (1981)	Refinement of the results reported by Karagioz *et al* (1976)	6.6364 ± 0.0015
Luther and Towler (1982)	Torsion pendulum in time-of-swing mode	6.6726 ± 0.0005
Oelfke (1984b)	Torsion balance with small intermass spacing	6.7 ± 0.2
Cohen and Taylor (1987)	CODATA value for G from the 1986 adjustment of fundamental constants	6.67259 ± 0.00085
Speake and Gillies (1987b)	Evacuated beam balance with servo control over the beam motion	6.65 ± 0.23
Liu *et al* (1987)	Rotationally driven two-body interaction with suspended-coil sensing system	6.660 ± 0.026
Goldblum (1987)	Relative measurement of G using spin-polarized test masses	$6.67(1.09 \pm 0.07)$
Karagioz *et al* (1987)	Evacuated torsion balance with magnetic damper and fibre rotation mechanism	6.6731 ± 0.0004
de Boer *et al* (1987)	Mercury-bearing-supported torsion balance with restoring torque supplied by quadrant electrometer	6.667 ± 0.005
Dousse and Rhême (1987)	Offset-mass torsion pendulum with servo-tracking optical lever (last series of deflection-mode data reported for apparatus)	6.6722 ± 0.0051
Moore *et al* (1988b)	Evacuated beam balance used to measure attraction of layers of water in a reservoir at an effective distance of 22 m	6.689 ± 0.057
Saulnier and Frisch (1989)	Ballistic motion of test masses on a torsion balance in accelerative field of depleted uranium pseudospheres	6.65 ± 0.09

Table 2. (Continued)

Reference/date	Method employed	G (10^{-11} m^3 s^{-2} kg^{-1})
Müller *et al* (1990)	Gravimetry at a pumped-storage hydroelectric reservoir at effective distances of 40–70 m	6.689 ± 0.027
Zumberge *et al* (1991)	Submarine-based geophysical measurement using 5 km deep gravimetric profiles	6.677 ± 0.013
Schurr *et al* (1991a, b, c)	Fabry–Perot microwave resonator with external attracting mass (type A uncertainty listed first, then type B uncertainty)	$6.66 \pm 0.06 \pm 0.12$
Yang *et al* (1991)	Gravimetric measurement of a large cylindrical oil tank at interaction distances of 30 m (low) and 60 m (high)	6.672 ± 0.040 (low) 6.672 ± 0.093 (high)
Taylor *et al* (1992)[a]	Estimation of a strong-field value of $G \equiv G_e$ using PPK formalism and binary pulsar timing data	$0.89 < G_e/G_0 < 1.14$
Schurr *et al* (1992a, b)	Fabry–Perot microwave resonator with external attracting mass (type A uncertainty listed first, then type B uncertainty)	6.6613 ± 0.0011 ± 0.0093
Oldham *et al* (1993)	Gravimetry at a pumped-storage hydroelectric reservoir at effective distances of 26–94 m	6.671 ± 0.015 (low) 6.703 ± 0.035 (high)
Walesch *et al* (1994a, b)	Fabry–Perot microwave resonator with external attracting mass (type A uncertainty = 100 ppm, type B = 200 ppm))	6.6724 ± 0.0015
Walesch *et al* (1995)	Fabry–Perot microwave resonator with external attracting mass (type A uncertainty = 74 ppm, type B = 83 ppm)	6.6719 ± 0.0008
Fitzgerald and Armstrong (1995)	Electrostatically nulled torsion balance; (type A uncertainty = 56 ppm, type B = 77 ppm)	6.6656 ± 0.0006
Hubler *et al* (1995)	Electromagnetic balance used at a pumped storage reservoir; with intermass spacings of 88 m (low) and 112 m (high)	6.678 ± 0.007 (low) 6.669 ± 0.005 (high)
Meyer *et al* (1995a, b)	Fabry–Perot microwave resonator with external attracting mass (type A uncertainty listed first, then type B uncertainty)	6.6685 ± 0.0007 ± 0.0050
Fitzgerald (1995b)	Electrostatically nulled torsion balance; (type A uncertainty = 56 ppm, type B = 77 ppm)	6.6659 ± 0.0006
Michaelis *et al* (1995/96)	Mercury-bearing-supported torsion balance with restoring torque supplied by quadrant electrometer (tungsten attracting masses)	6.71540 ± 0.00056
Michaelis *et al* (1995/96)	Mercury-bearing-supported torsion balance with restoring torque supplied by quadrant electrometer (Zerodur® attracting masses)	6.7174 ± 0.0020
Bagley and Luther (1996)	Torsion pendulum in time-of-swing mode, with Kuroda anelasticity correction for fibres with a Q of 950 (a) and 490 (b)	6.6739 ± 0.0011 (a) 6.6741 ± 0.0008 (b)

[a] G_0 here is the normal laboratory value of the Newtonian gravitational constant.

used to drive the turntable at a constant angular acceleration that kept the angle between the mass systems fixed. The value of G was then calculated from a knowledge of the angular acceleration and the metrological constants of the experiment. In one particular set of measurements (Luther *et al* 1976), this approach produced a statistical uncertainty of about 2 : 10^4 in the result for G. An anticipated virtue of their experimental arrangement was that the rotation of the apparatus would lead to the cancellation of the torques produced on the balance beam by external gravity gradients. In practice, however, the initial angular accelerations were so small that 'mechanical filtering' of this type had virtually no effect on the balance response during a significant fraction of any given run.

Oelfke (1984a, 1984b) designed a torsion balance for measuring G at intermass spacings from 3 to 50 mm. His work was motivated by interest in searching for the possible existence of a short-range, Yukawa-like component in the gravitational potential which, if found, might signal the presence of a massive gravitational exchange particle. To measure G at such a short range, he used an evacuated torsion balance that had flat brass disks as the interacting bodies. The position of the balance beam was monitored by a differential capacitance bridge, and an electrostatic feedback torque was applied to the beam to damp the torsional oscillations and electromechanically balance the system. The size of the feedback signal served as the measure of the gravitational force, as well. Although microseisms limited the accuracy with which G could be determined by this technique to $\lesssim 3\%$, the results nevertheless confirmed the validity of the inverse square law within this range of distances.

Speake (1983) developed a very high-sensitivity beam balance for the measurement of G. It used a bifilar suspension for the beam, consisting of two tungsten wires that were 88 μm in diameter and 4.5 cm long. The beam itself was 80 cm long and made of titanium. The 0.5 kg attracted masses were machined from high-purity copper and fixed one each at opposite ends of the beam. The balance was operated in a vacuum chamber that was maintained at 1.3×10^{-5} Pa (10^{-7} mm Hg) by an ion pump. Gravitational torque was applied to the beam by moving an external attracting mass close to one end of the balance. The resulting motion of the beam was sensed by a parallel-plate capacitive transducer, and a feedback signal was synthesized and used to return the balance to its null position. The apparatus could detect gravitational accelerations of as little as 2×10^{-10} m s^{-2}, but various systematic effects limited the accuracy with which G was determined to approximately 3% (Speake and Gillies 1987b).

Liu *et al* (1987) developed a novel device in which G was measured from a two-body interaction. They put a 8.7 kg brass cylinder on a turntable and let it revolve around a 0.24 kg brass sphere that was hung from one end of the beam of a torsion balance. The vertical axis of the suspended sphere was collinear with that of the rotating turntable. Also hanging from the same end of the balance beam was a two-turn coil that carried a stable current in it. This 'moving coil' was geometrically centred between two larger coils that were fixed in the laboratory. An optical lever was used to monitor the motion of the beam, as influenced by the torque applied to it by the gravitational interaction between the revolving cylinder and the suspended sphere. By driving an electrical current through the fixed coils, a counterbalancing torque could be produced on the beam via the magnetic coupling with the moving coil and, with measurement of all the relevant electromechanical quantities, G could thus be determined to an accuracy of approximately 0.4%.

Stacey and colleagues have made many contributions to the development of modern geophysical methods for determining G (see Stacey *et al* (1987) for a review of geophysical tests of gravitational theories and a discussion of their own measurements of G). In one of their experiments, a vacuum beam balance was used to investigate the force of gravity between 10 kg stainless steel test masses and a 10 m deep layer of water in a hydroelectric reservoir. The deflection of the beam was detected by capacitance transducers that monitored its movement relative to a mercury level tiltmeter. An important advantage of this arrangement was the presence of an absolute horizontal reference provided by the tiltmeter, especially since the platform on which the whole apparatus was placed could itself tilt appreciably. An automated mechanism was used to place the test masses onto (and remove them from) cradles hung from the arms of the balance beam by flexure suspensions and crossed knife-edge supports. Additional automated features allowed weighings to be made over periods of up to a week without the observer being present, and with the apparatus sensing (via accelerometer) when the balance should be clamped because of large-amplitude, wind-driven vibrations. A full discussion of the special features of this apparatus is available elsewhere (Moore *et al* 1988a), as is a report of the results of its use to measure G to about 0.8% at an effective mass spacing of 22 m (Moore *et al* 1988b).

Ritter and colleagues made a measurement of G as a precursor to their search for an anomalous spin–spin interaction in gravity. The experiment was carried out using spin-polarized test masses made from Dy_6Fe_{23} powder that was housed in cylindrical containers which also served as magnetic shields. Two of these masses were fixed to an aluminum bar that was suspended by a 38 μm tungsten fibre inside an evacuated housing, and the period of this torsion pendulum was measured with the attracting masses (also of Dy_6Fe_{23}) alternately in place at and then removed from their positions near the suspended beam. The period of the pendulum was nominally 4.4 min and the oscillations were monitored by an optical lever, with the resulting periodic waveforms stored by computer for off-line analysis. This consisted of a fast Fourier transform (FFT) which, following some windowing corrections, was carried out on 4096 consecutive samples obtained at intervals of 1.9 s each during a given run. While this approach permitted a precise determination of the pendulum's period, it was known in advance that the overall magnetic susceptibility of the masses and other considerations would work to make this a relative rather than an absolute measurement of G. In fact, the point of this effort had not been to deal with the absolute metrology of the apparatus, but to determine the sensitivity of the pendulum and detection system in preparation for its use in a long series of runs to search for a possible spin–spin anomaly. Even so, the result with the spin-polarized test masses had a relative uncertainty of around 1% due to statistical fluctuations of the data about the mean (Goldblum 1987, Goldblum *et al* 1987), although the absolute value of G as measured in this way was roughly 9% higher than the accepted value.

Zumberge *et al* (1991) reported a very interesting submarine-based measurement of G that was made in the northeast Pacific ocean. The experiment consisted of a detailed gravimetric survey carried out along vertical profiles through the water, as well as measurements of gravity on the seafloor, the ocean surface and on horizontal planes in between. From measurements of gravitational acceleration made as a function of vertical position within a medium of known density (in this case, the water) and an estimate of the gravitational gradient, it is possible to compute the value of the gravitational constant. (This is the 'Airy' method; see Stacey *et al* 1987 for details.) The gravity surveys were carried out with LaCoste-Romberg S-110 and S-38 gravity meters and a Bell Aerospace BGM-3 gravity meter. The latter had a resolution of less than 0.1 mGal and was used in the submersible to measure the vertical profiles during dives that were 5000 m deep, with data being taken every 12 s. The seawater density was determined to $1:10^4$ from an equation of state that incorporated measured values of the water's conductivity and temperature. Their result for G agreed with the laboratory value of Luther and Towler (1982) to within $1:10^3$, and had an uncertainty of about 0.2%. (Land-based gravimeters have also been used to determine the value of G. Yu *et al* (1978, 1979) have done so by measuring the field of an oil tank when full and empty using a Worden gravimeter located 15 m from the tank. Their results are presented in table 2. See also section 5.1 for a brief discussion of related experiments by others.)

All of the determinations of G discussed above have involved either benchtop experimentation or geophysical gravimetry. Some constraints on the value of G have also been established by various types of astronomical observations as well, and Mikkelsen and Newman (1977) discuss the relevant results. One of the strongest such constraints arises from analysis of the orbits of the planets in the solar system. Citing the results of lunar laser-ranging and Mariner 10 tracking studies, they place limits on the spatial constancy of G (i.e. the size of $G(r)$) over distances that range from approximately 10^3 to 10^8 km. Specifically, letting G_c denote the gravitational constant in that distance range, they find that $[G(r) - G_c]/G_c \leqslant 0.03\%$ from 10^4 to 3×10^8 km, and that it is tenfold larger than this down to 3×10^3 km. While these results point to the constancy of G_c in this range, they do not address the question of its absolute value. Taking G_0 to be the nominal laboratory value of G, Mikkelsen and Newman argue that a model of the Earth that makes some reasonable assumptions about the Earth's density distribution and moment of inertia leads to limits on the absolute value of G_c of $0.50 < G_c/G_0 < 1.32$. Similarly, they find that a model of the solar structure that uses the present helium abundance on the surface of the Sun leads to a value for G_c of $0.90 < G_c/G_0 < 1.36$. After examining other possibilities they conclude that the most reliable, likely-allowable range for G_c is $0.6 < G_c/G_0 < 1.3$, which includes the value $G_c = 0.75G_0$ arrived at independently by Fujii (1972). This wide span of possible values highlights the imprecision with which G is known on this scale of distances. Resolution of the problem may well require some form of sophisticated satellite-based determination of G (see section 4.6 below) or the development of a new approach to measurements that involve celestial bodies (Fujii (1972) suggested one such possibility).

Finally, Taylor *et al* (1992) have pointed out that while solar system and terrestrial tests of relativistic gravity examine only the weak-field limit of the interaction, strong-field phenomenon can be investigated through observations of binary pulsars. They use a 'parametrized post-Keplerian' (PPK) formalism to examine binary pulsar data, and within that framework define a strong-field, effective gravitational constant, G_e. It is given by

$$G_e = G_o[1 + \tfrac{1}{2}\beta'(c_1^2 + c_2^2)]$$

where G_0 is the nominal laboratory value of the Newtonian constant, β' is a PPK parameter that describes possible strong-field deviations from general relativity, and c_1 and c_2 are compactness factors that scale the fractional gravitational binding energy of the interacting bodies. Limits on β' and related parameters obtained from binary pulsar observations allowed them to make the estimate of G_e presented in table 2, thus providing quantitative evidence that the strong-field value of G is not significantly different from that measured in the laboratory.

4.2. Recent high-precision experiments

There have been four experiments within the last ten years that have reported absolute values of G with uncertainties on the order of or less than 100 ppm, all using variants of a torsion pendulum. A fifth recent measurement (still ongoing) employed a Fabry–Perot resonator as the detector. It and a sixth experiment, while not quite reaching this level of accuracy, have made important contributions. The latter experiment, for example, has produced the most accurate result to date for G at intermass spacings on the order of 100 m by use of a mass comparator balance. As mentioned above, there are disagreements between the results produced by these experiments and we shall examine this situation in what follows. In most cases the work has been ongoing for several years and, as discussed in section 4.4 below,

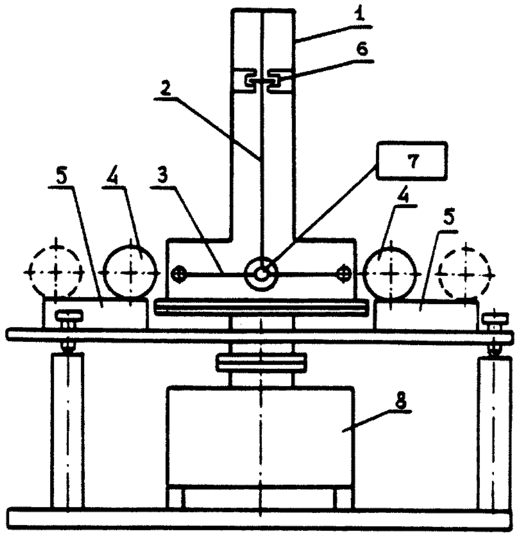

Figure 2. The essential features of the generic form of torsion-pendulum apparatus used by Karagioz and colleagues for their laboratory measurement of *G* in Russia. Key: (1) vacuum chamber, (2) suspension fibre, (3) balance beam, (4) attracting masses, (5) mechanism for adjusting the position of the attracting masses, (6) magnetic damper, (7) angular motion sensing system, and (8) vacuum pump (courtesy of O V Karagioz and Tribotech Research and Development Company).

some of these efforts are still in progress.

From a historical perspective, the first of these experiments is that of Karagioz and colleagues in Moscow who designed an evacuated torsion pendulum that was used in the time-of-swing mode. A unique feature of the first version of their apparatus was that they used only a single attracting mass to create the gravitational couple on the attracted masses on the balance beam. (They termed this the 'asymmetric' mode of operation.) The large mass could be placed at one of ten locations along a titanium rail, each point separated from the next by 12 mm, with the closest being about 13.5 cm from the suspension axis of the beam. This arrangement permitted not only a measurement of the absolute value of *G* but also a test of the exactness of the inverse square law over intermass spacings from roughly 5 to 16 cm, since the beryllium balance beam had a lever arm of about 8 cm. A tungsten fibre that was 5 μm in diameter and 25 cm long supported the beam. The small masses were copper spheres, 300 g each. Large masses of non-magnetic copper, brass and aluminum were tested, but most of the data were taken with a steel sphere having a mass of 4.28 kg. The pendulum was interrogated by an optical lever, and the measured periods were on the order of 1800 s. As noted in table 2, the first value of *G* obtained with this apparatus (using the steel sphere) had an uncertainty of approximately 300 ppm (Karagioz *et al* 1976). A re-analysis of the experiment (Karagioz *et al* 1981) discussed various sources of systematic error, and the possibility of a magnetic coupling caused by the field of the steel attracting mass in particular. Subsequent corrections to the intermass spacings used in the calculations reduced the uncertainty in the measurement of *G* to 226 ppm but also resulted in a 0.4% reduction in its absolute value compared to the first result. They concluded that improvements to the apparatus were needed and went forward with redesign and reconstruction. A schematic of the more recent, improved system is shown in figure 2. One of the principal changes to the apparatus was the inclusion of a version of magnetic damper of the type used by Luther and Towler (1982). With it, all of the swinging modes could be made to decay in less than 5 s (a modification to the moment of inertia of the beam put the highest-frequency such mode at $\approx$ 6 Hz). The suspension fibre was increased in diameter to 25 μm but, due to the increase in its loading, the period of the pendulum with the attracting mass removed remained at about 1800 s. Also added was a mechanism for the *in situ* vacuum annealing of the suspension fibre via the application of a 10 MHz current. This reduced the internal friction in the fibre and decreased the logarithmic decrement of the beam's motion to 5×10^{-4}. A 1 mm thick Permalloy magnetic shield was placed outside the vacuum chamber to reduce to a negligible level any external magnetic couplings to the paramagnetic components of the balance beam. The data taking process was fully automated, and a motor-driven system was installed to move the attracting mass from one position to the next on the titanium rail. As suggested in figure 2, the new arrangement was also modified to permit data to be taken with two attracting masses (i.e. in the 'symmetric' mode of operation). The chamber was evacuated to approximately 10^{-6} Pa ($\approx 7.5 \times 10^{-9}$ mm Hg) by an ion pump while experiments were being carried out. Finally, an improved mathematical model of the beam oscillations was developed, and this permitted useful data to be taken even with swing amplitudes of up to 5°. All of these modifications are discussed more extensively by Karagioz *et al* (1987) and Izmaylov *et al* (1993). The lowest-uncertainty value of G that has been reported by this group is

$$G = (6.6731 \pm 0.0004) \times 10^{-11} \text{ m}^3 \text{ s}^{-2} \text{ kg}^{-1}$$

(Karagioz *et al* 1987). In a subsequent series of experiments carried out in 1991, however, a wider spread in the data was observed (Izmaylov *et al* 1993). Testing done at that time indicated that microseisms were responsible for at least part of the increased uncertainty, and they concluded that additional work on vibration isolation of the apparatus would be needed to eliminate destabilizing factors of this kind.

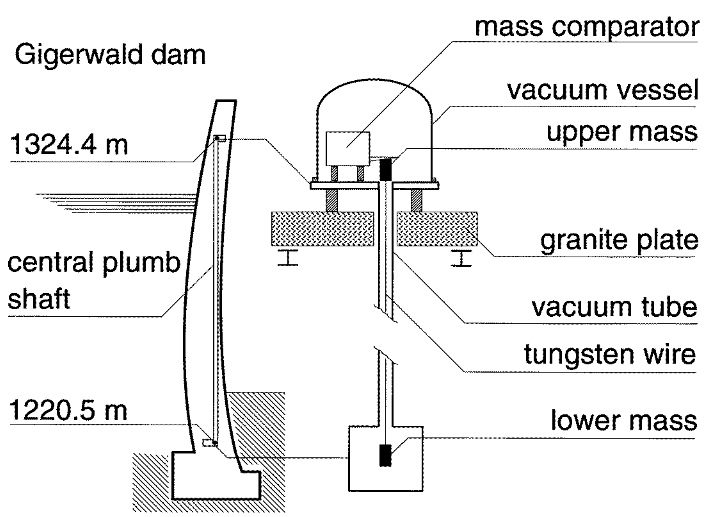

Figure 3. The apparatus used by Cornaz and colleagues for their determination of G at the Gigerwald dam in Switzerland (from Cornaz *et al* 1994, copyright of the American Institute of Physics).

Researchers at the University of Zürich have carried out a very interesting absolute measurement of G at effective intermass spacings on the order of 100 m (Cornaz *et al*

1991, 1994, Hubler *et al* 1994, 1995). As with the experiment of Moore *et al* (1988b), the goal here was to use a precision mass comparator to measure the difference in weight between two widely separated test masses as a function of the height of the water level in a pumped-storage reservoir. One of the test masses was suspended from the comparator by a long wire which let it hang down inside a plumbshaft inside the wall of the dam, while the other one was suspended by a short wire keeping it near the top of the dam. As the water level in the reservoir would change, the Newtonian attraction acting on the test masses would change with it, and G was derived from the resulting data. The work was carried out at the Gigerwald lake in Eastern Switzerland using a Mettler-Toledo flexure-strip, single-pan balance in the arrangement shown in figure 3. The resolution of the balance was better than 1 μg and it was located inside of a vacuum vessel held at a pressure of about 0.1 Pa ($\approx 7.5 \times 10^{-4}$ mm Hg) by a turbomolecular pump. This vessel was located in a room at the top of the dam in which the temperature was held constant to within 0.2 °K. Moreover, there were two stages of thermostatically controlled water circulated through thermal regulation shields that kept the temperature of the balance itself stable to within 1 mK of its set point. The overall stability of the balance was such that the weighings had variations no greater than approximately 0.5 μg per day. The upper test mass was an open-bore cylinder while the lower one was a solid cylinder, both of stainless steel, approximately 1.1 kg each. They were suspended from the balance arm by 100 μm diameter tungsten fibres hanging in an evacuated tube that extended from the balance vessel down through the plumbshaft. The vertical positions of the masses were determined to within 3 mm by survey relative to fixed reference points in the plumbshaft. The suspension system for the masses incorporated a novel gimbal that consisted of sequential linkages between the pan and the upper and lower mass support points. The contacting surfaces in the gimbal had a 5 μm thick, low-friction coating of tungsten-carbide/carbon which helped insure that lateral displacements of the bearing point were no more than 50 μm off centre. This, and the overall design of the suspension, were aimed at making the size of any parasitic torques that might act on the balance beam as small as possible. The compliance of the lower mass suspension fibre gave rise to a mode of longitudinal oscillation at approximately 0.77 Hz that was damped by the control loop of the balance, while low-frequency pendulum and torsional oscillations were removed by allowing a snubber plate to contact the lower mass between weighings. The masses were weighed one at a time, for a period of 3 min each, after which an automated interchange procedure would enable one mass to be removed from the balance while the other was simultaneously placed in suspension. The computer-based system which accomplished this held the load on the balance constant to within 1 g at all times, thus circumventing relaxation of the flexure strips and the introduction of any systematic error that might result. The height of the water level in the lake was calculated from measurements of the subsurface water pressure and determinations of the air density, water density and the local acceleration of gravity. A complete description of the technical details of the experimental arrangement is available elsewhere (Hubler *et al* 1995). The data consisted of measurements of weight difference as a function of water level in the lake, and they were collected at various intervals over a three-year period. From a knowledge of the shape of the lake, the geology of the shoreline and the geometry of the experiment, a gravity model was developed and used to calculate the gravitational attraction between the slab of water and the test masses, and the effective interaction distance between them. Analysis of the results within the context of this model made use of the fact that the measured weight differences at a particular water level must always be the same. A least-squares linear regression of the coefficients in the resulting expression for the weight differences yielded the experimental values of G at effective intermass spacings of 88 m,

$$G = (6.678 \pm 0.007) \times 10^{-11} \text{ m}^3 \text{ s}^{-2} \text{ kg}^{-1}$$

and 112 m,

$$G = (6.669 \pm 0.005) \times 10^{-11} \text{ m}^3 \text{ s}^{-2} \text{ kg}^{-1}$$

(Hubler *et al* 1995). The error budget for the experiment included estimates of the uncertainties in the contours of the dam and shoreline, the porosity of the scree surrounding the shoreline, the positions of the test masses and the density and level of the water. The authors also interpreted their results in terms of a Yukawa-modified form of the gravitational potential, and were able to set new constraints on the (α, λ) parametrization of a composition-independent fifth force.

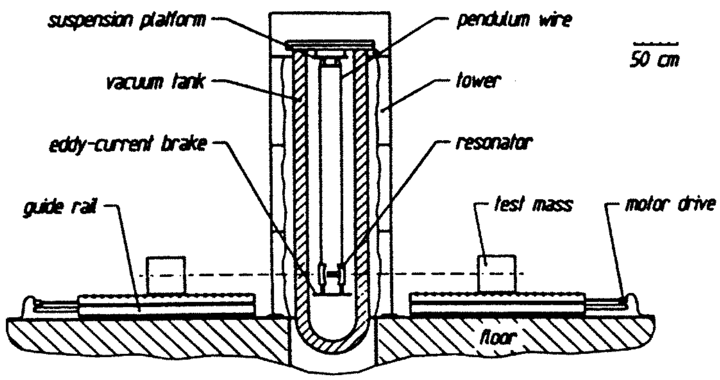

Figure 4. The apparatus used by Meyer and colleagues for their microwave resonator-based measurement of G at the Bergische Universität in Wuppertal, Germany (from Walesch *et al* 1995, copyright 1995 IEEE).

A very novel type of experiment has been carried by a team at the Bergische Universität in Wuppertal, Germany. Started in the 1980s (Klein 1987, 1989, Schurr 1988), the focus of the project has been the development of a gravimeter that uses a Fabry–Perot microwave resonator as the sensor. As shown in figure 4, the reflectors of the resonator are suspended as pendula and the Newtonian attraction of either one or two large perturbing masses is used to change the spacing between them. The physical characteristics and operating parameters of this apparatus are as follows. The mirrors are made of OFHC copper, 192 mm in diameter, 9 mm thick at the centre, and approximately 5.4 kg each. Their radius of curvature is 580 mm, and they are separated by 241 mm. There is a 2.6 m long bifilar suspension for each mirror consisting of 0.2 mm diameter tungsten wire. The spacing between the wires is fixed at their attachment points to the suspension platform by a quartz spacer, thus holding the thermal expansion-related drift in the mirror separations to about 60 nm $^\circ\text{K}^{-1}$. A cylindrical shield is positioned between the mirrors. It contains an absorber ring that damps spurious higher-order modes of the microwave field in the cavity. The natural frequency of the pendula are about 0.3 Hz, and oscillations of them can be excited by natural Earth microseisms. Therefore, an eddy-current brake consisting of a tessellated pattern of permanent magnets mounted on iron plates was designed to fit around each of the suspended masses. It imposes damping (with a 2 s time constant) on any motions of the mirrors. This entire apparatus is contained inside a vacuum tank supported by an outer framework of steel girders. The working pressure of the experiment is 2 Pa, and pressure fluctuations are kept below 0.01 Pa to minimize variations in the resonator frequency. The resonator frequency, ω_0, is typically centred in the range from 20 GHz $< \omega_0 <$ 26 GHz. The quality factor, Q, of the cavity is limited by the reflection losses of the mirrors and has a value of $Q \approx 2 \times 10^5$. A small coupling hole in one of the mirrors serves as the inlet port for the microwaves, which are produced by a synthesized sweep generator, and a similar hole in the other mirror allows the resonator power to be measured by a detector diode connected by a waveguide. The field profile is that of TEM-mode standing waves in a Gaussian beam with a waist of 3 cm. In the early work with this apparatus only one attracting mass, M, was used to gravitationally perturb the positions of the resonator's mirrors. It was a cylinder made from a low-susceptibility brass alloy, 44 cm in diameter and 43 cm long, with a mass of approximately 576 kg. There are now two such masses used in the experiment. They rest on slide rails (on either side of the vacuum tank) that allow the positions of the masses to be adjusted by stepping motor drives such that spacings over the range from 0.6 to 2.1 m can be established between the centres of mass of a given mirror and the nearest attracting mass, thus making it possible to test the exactness of the inverse square law over this range. The positioning errors are known to within 80 μm per metre of distance relative to the smallest intermass spacing. When both attracting masses are in use, one of them is located about 20 cm closer to the resonator than the other one, and they are moved forward and backward together (with a cycle time of approximately 20 min per measurement of G). A null experiment can also be performed by moving the masses in opposite directions. Detailed accounts of the design and performance of the apparatus are available elsewhere (Walesch 1991, Schurr 1992, Schurr *et al* 1992b, Langensiepen 1992). The rate of change of resonator frequency, f, with intermirror distance, b, establishes the fundamental sensitivity of the measurement, typically $\mathrm{d}f/\mathrm{d}b \approx 100$ Hz nm^{-1}. This quantity is determined in each particular experiment to an uncertainty of less than 1.4×10^{-5}. The expression governing the relationship between G and the quantities measured experimentally is

$$\Delta f(r) = (\mathrm{d}f/\mathrm{d}b)\omega_0^{-2} GM\{[1/r^2 - 1/(r+b)^2]K(r) - [1/r_{\mathrm{ref}}^2 - 1/(r_{\mathrm{ref}}+b)^2]K(r_{\mathrm{ref}})\}$$

where Δf is the shift in the resonance frequency created by the differential displacement of the mirrors that results when the attracting masses are moved. The reference position of the closer attracting mass is r_{ref} and the point to which it is moved along the rail is denoted by r. $K(r)$ is a correction function that accounts for the departure from point-source-like behaviour of the interacting masses. The overall sensitivity of the apparatus is such that the resolution in the measurement of the mirror spacing, b, is on the order of one picometre. (As an interesting aside, the apparatus can be excited by earthquakes of 4.7 on the Richter scale that occur anywhere on Earth.) The determinations of G resulting from the use of this apparatus are listed in table 2. The first value obtained was an average over six measurements made with a single attracting mass, and it had a statistical (type A) uncertainty of 0.9% and a systematic (type B) uncertainty of approximately 1.8% (Schurr *et al* 1991a, b, c). (Those unfamiliar with the accepted convention of assigning type A and type B uncertainties to a measurement should consult the US National Institute of Standards and Technology Guidelines (Taylor and Kuyatt 1994) or the International Standards Organization Guidelines (ISO 1995) for the appropriate definitions and background.) Following additional work, still with a single attracting mass, a more accurate value was subsequently reported (Schurr *et al* 1992a, b), and eventually the statistical component of the uncertainty in the measurements was reduced to 100 ppm and the systematic component to 200 ppm (Walesch *et al* 1994a, b). The first results reported in a paper describing the two-attracting-mass system were somewhat better, with the statistical uncertainty cited as being 74 ppm and the systematic uncertainty 83 ppm (Walesch *et al* 1995). The experiment has been discussed most recently by Meyer (1995a, b) and the latest value of G it has produced, for measurements made over the range from 40 cm $< r <$ 100 cm, is

$$G = (6.6685 \pm 0.0007 \pm 0.0050) \times 10^{-11}\ \mathrm{m^3\ s^{-2}\ kg^{-1}}$$

where the first uncertainty represents the type A uncertainty and the second is the type B uncertainty. (The type B uncertainty cited here is 5 times larger than that presented by Meyer (1995a, b) and factored into the uncertainty quoted for their experiment in *Physics Today* (1995 **48**(6) 9), with the correction arising from a previously undetected source of error they found recently in the determination of the distances of the field masses.) This value is lower than the CODATA value, although they do overlap each other within the limits of the errors cited. The most recently published error budget for the experiment is that of Walesch *et al* (1995). The limiting factors, present status and future plans for this experiment are discussed briefly in section 4.4.

A new type of torsion-balance measurement of G has been introduced by Fitzgerald and colleagues at the Measurement Standards Laboratory of Industrial Research Ltd, in New Zealand (Fitzgerald *et al* 1994). Shown schematically in figures 5(a) and 5(b), it consists of a thin, cylindrical small mass suspended within the centre of a parallel-plate electrometer by an electrically grounded torsion fibre. A pair of vertically-oriented, cylindrical attracting masses are located on opposite sides of the electrometer, with the suspension axis of the small mass situated along the axial midline between them. The large masses can be rotated about the suspension axis to fixed measurement positions where the gravitational torque, Γ_G, they produce on the small mass is maximized. The angular position of the small mass is sensed by an autocollimator that monitors a mirror fixed to the suspension fibre. The resulting signal is used to synthesize a feedback voltage, V_G, that is applied to the appropriate pair of capacitor plates in the electrometer to keep the small mass from turning under the influence of the torque. During an experiment, the sequential movement of the attracting masses produces a repetitive reversal of Γ_G, thus generating a periodic signal in the electrometer, which can be averaged to reduce the effects of drift and background torques. The relationship between Γ_G, V_G and G is given by

$$\Gamma_G = GK = (\mathrm{d}C/\mathrm{d}\theta)V_G^2/2$$

where K is the calculated gravitational torque divided by G, and $\mathrm{d}C/\mathrm{d}\theta$ is the rate of change of the electrometer capacitance with the angular displacement of the small mass. To determine $\mathrm{d}C/\mathrm{d}\theta$, a variation of the technique used by Beams and colleagues (Rose *et al* 1969, Luther *et al* 1976) to measure G is employed. A constant voltage V_A is applied to the electrometer to create a torque on the suspended mass. With the large masses removed and the feedback that normally nulls the position of the small mass deactivated, the output signal from the autocollimator is then used to drive a different feedback loop which accelerates a turntable supporting the whole apparatus. The angular acceleration of the turntable $\mathrm{d}^2\alpha/\mathrm{d}t^2$, (where α is the rotation angle of the apparatus), is just sufficient to keep the angular position of the small mass fixed relative to the electrometer. By making $V_A > V_G$, the level of acceleration can be made large enough to allow for rotational filtering of the torques produced by static gravitational gradients that would otherwise parasitically couple to the small mass. If the moment of inertia of the small mass system is I, then

$$\mathrm{d}C/\mathrm{d}\theta = 2(I\mathrm{d}^2\alpha/\mathrm{d}t^2)/V_A^2$$

and, therefore, G can be determined from the expression

$$G = V_G^2[(\mathrm{d}^2\alpha/\mathrm{d}t^2)/V_A^2](I/K).$$

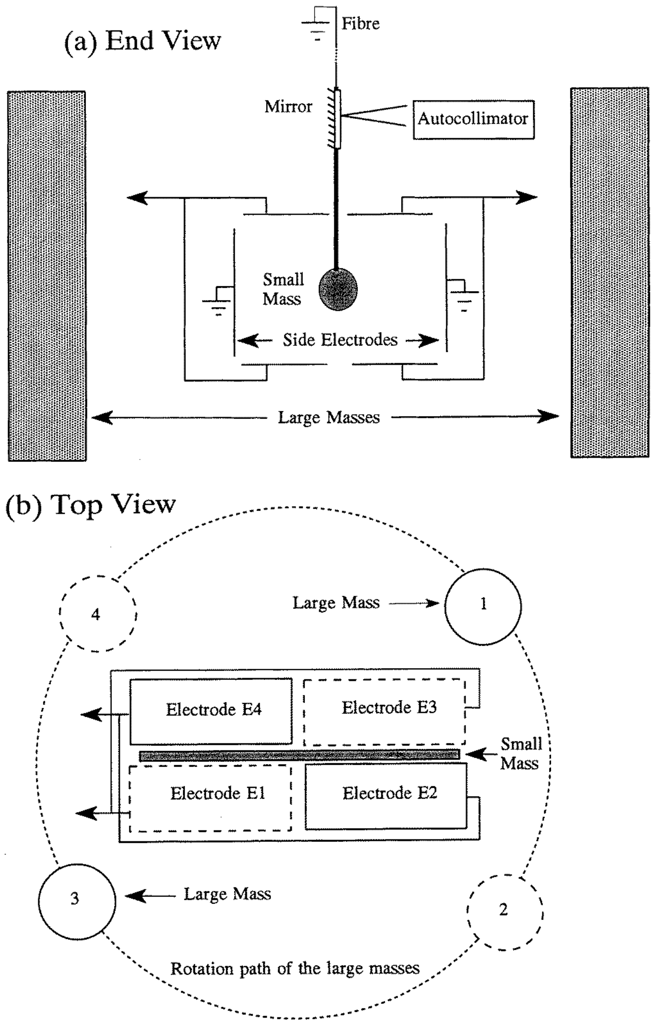

Figure 5. Illustrations of the experimental arrangement used by Fitzgerald and colleagues for their torsion-balance determination of G at Industrial Research Ltd in New Zealand. (a) Schematic diagram, end view. (b) Schematic diagram, top view. (c) Full-view photo of the apparatus. (d) Close-up photo of the suspension chamber (schematic diagrams from Fitzgerald *et al* (1994); photographs courtesy of M P Fitzgerald).

The goals underlying this design were the elimination of the movement of the small mass and the subsequent twisting of the suspension fibre, the simplification of the dimensional metrology, the minimization of drifts in the signal, and the reduction of the sensitivity of the measurement to density gradients in the masses. The apparatus built to realize this design is shown in full view in figure 5(c) and a close-up of the torsion balance components is provided in figure 5(d). The suspension fibre is made of tungsten, 50 μm in diameter and slightly over 1 m long. The small mass is an approximately 97 g cylinder of high-purity copper, 22 cm long and approximately 7.9 mm in diameter. The two stainless steel attracting masses are 43.8 cm long, 10.1 cm in diameter and approximately 27.9 kg each. The spacing of the electrometer plates is 80 mm between the two pairs on the top and

(c)

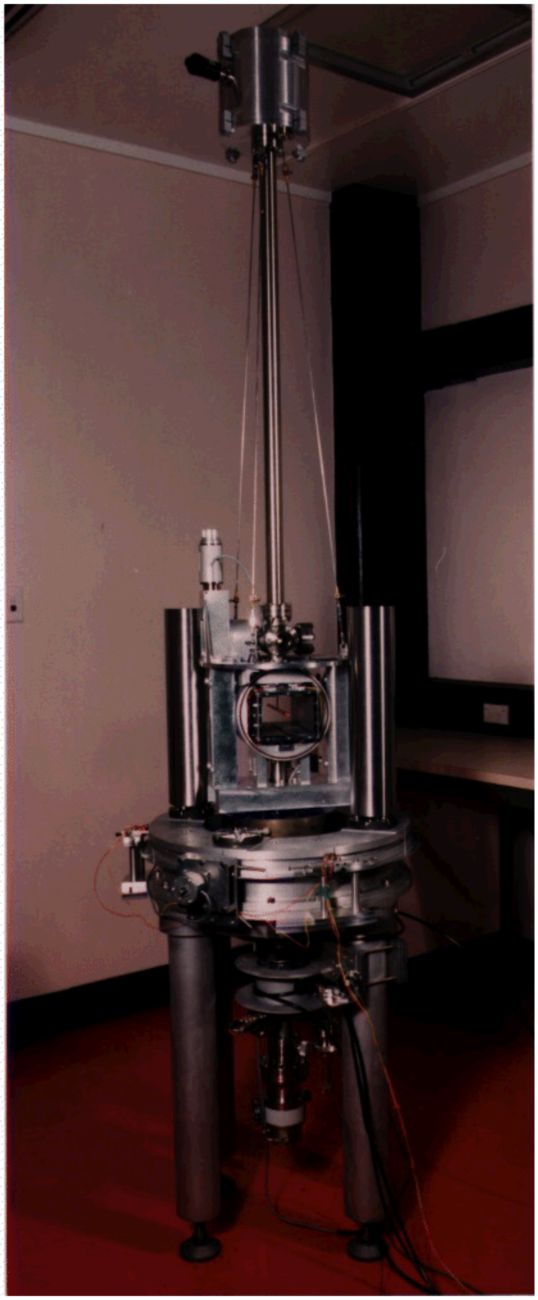

Figure 5. (Continued)

bottom, and 10 mm between the (grounded) pair on either side of the small mass. At this spacing, the torque constant is about 1 pF rad^{-1}. The experiment is operated inside a vacuum chamber that holds the pressure at less than 10^{-4} Pa. The large masses are kinematically mounted on a rotatable ring outside the vacuum chamber, and can thus be repositioned from one measurement point to the next, as described above. The vacuum chamber and

(d)

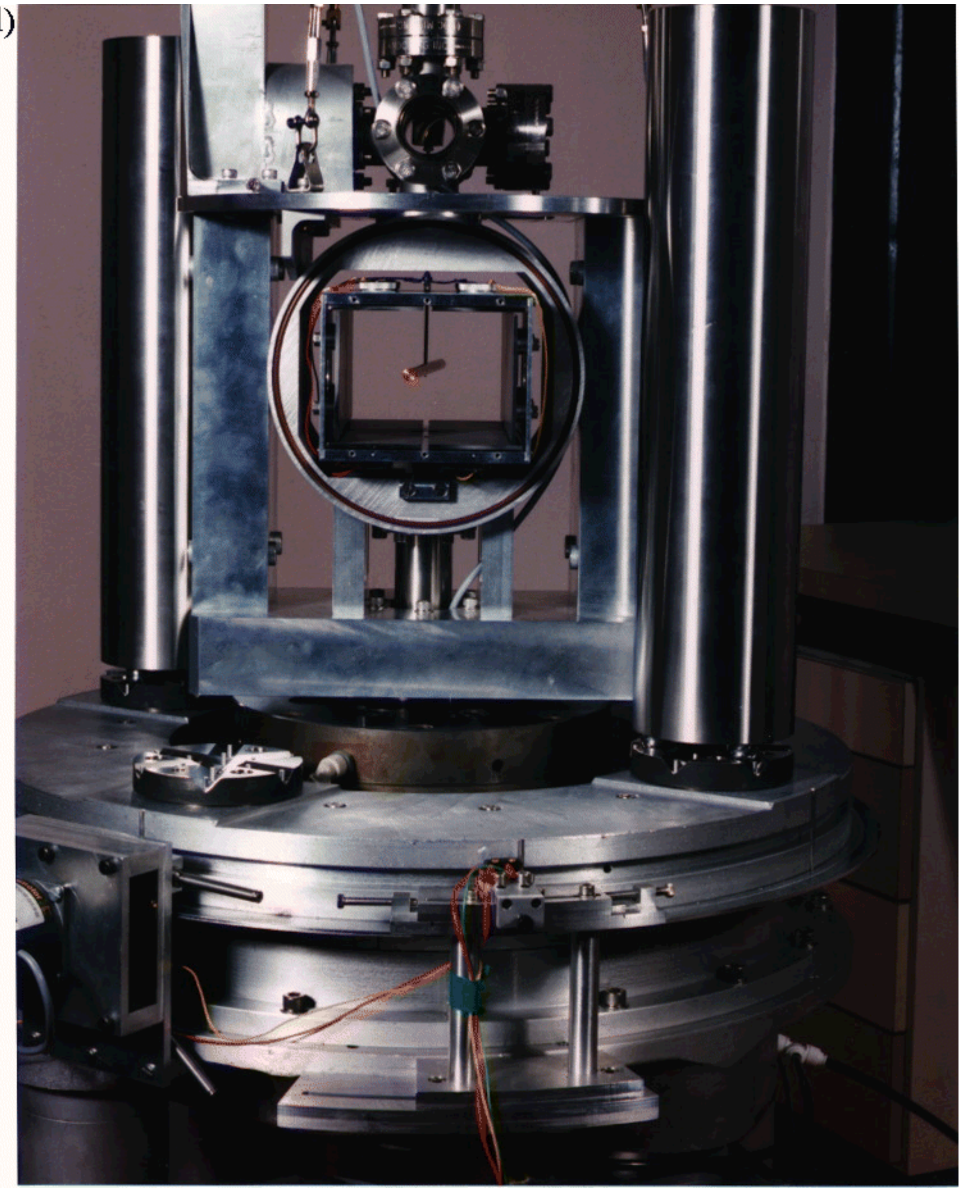

Figure 5. (Continued)

the support structure for the suspension system are mounted on an air bearing that is driven by a microstepping motor during the accelerative calibration of the electrometer, a process that requires roughly two revolutions of the apparatus. Timing of the angular motion is carried out by using a second autocollimator to monitor a polygonal reflector fixed to the rotating system. The entire experiment is under computer control, with a proportional-integral-derivative (PID) algorithm used to regulate the position of the small mass. Full details of the design, construction and operation of the apparatus have been published by Fitzgerald *et al* (1994) and Fitzgerald and Armstrong (1994). The first value of G reported for this experiment was (Fitzgerald and Armstrong 1995, Fitzgerald 1995a)

$$G = (6.6656 \pm 0.0006) \times 10^{-11}\ \mathrm{m}^3\ \mathrm{s}^{-2}\ \mathrm{kg}^{-1}$$

which included a 56 ppm type A uncertainty and a 77 ppm type B uncertainty. The article by Fitzgerald and Armstrong (1995) contains a very thorough assessment of all the sources of error that contribute more than 1 ppm of uncertainty to the final result. A slight correction to this value was subsequently reported by Fitzgerald (1995b), although the number itself did not appear in print (the revised value was $G = (6.6659 \pm 0.0006) \times 10^{-11}\ \mathrm{m}^3\ \mathrm{s}^{-2}\ \mathrm{kg}^{-1}$).

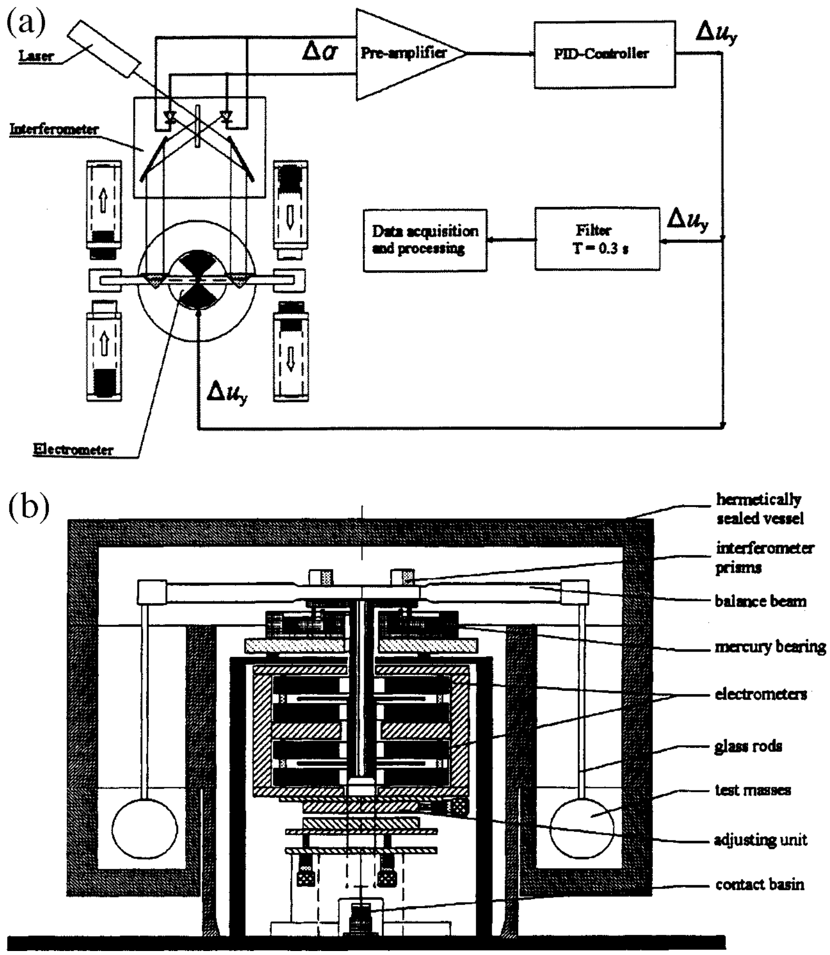

Figure 6. (a) Schematic diagram of the experimental arrangement of the torsion balance used to measure G at the PTB in Braunschweig, Germany. (b) Cross sectional view of the mercury bearing, electrometers and experimental chamber housing the apparatus. (c) Photograph showing an external view of the apparatus (schematic diagrams from Michaelis *et al* (1995/96); photo courtesy of W Michaelis).

Perhaps the most enigmatic result to emerge from any of the modern experimental determinations of G is that which has been found by workers at the Physikalisch-Technische Bundesanstalt (PTB) in Germany. Planning for the experiment there began in 1976 (de Boer 1977) and focused on the development of a torsion balance that would eliminate

(c)

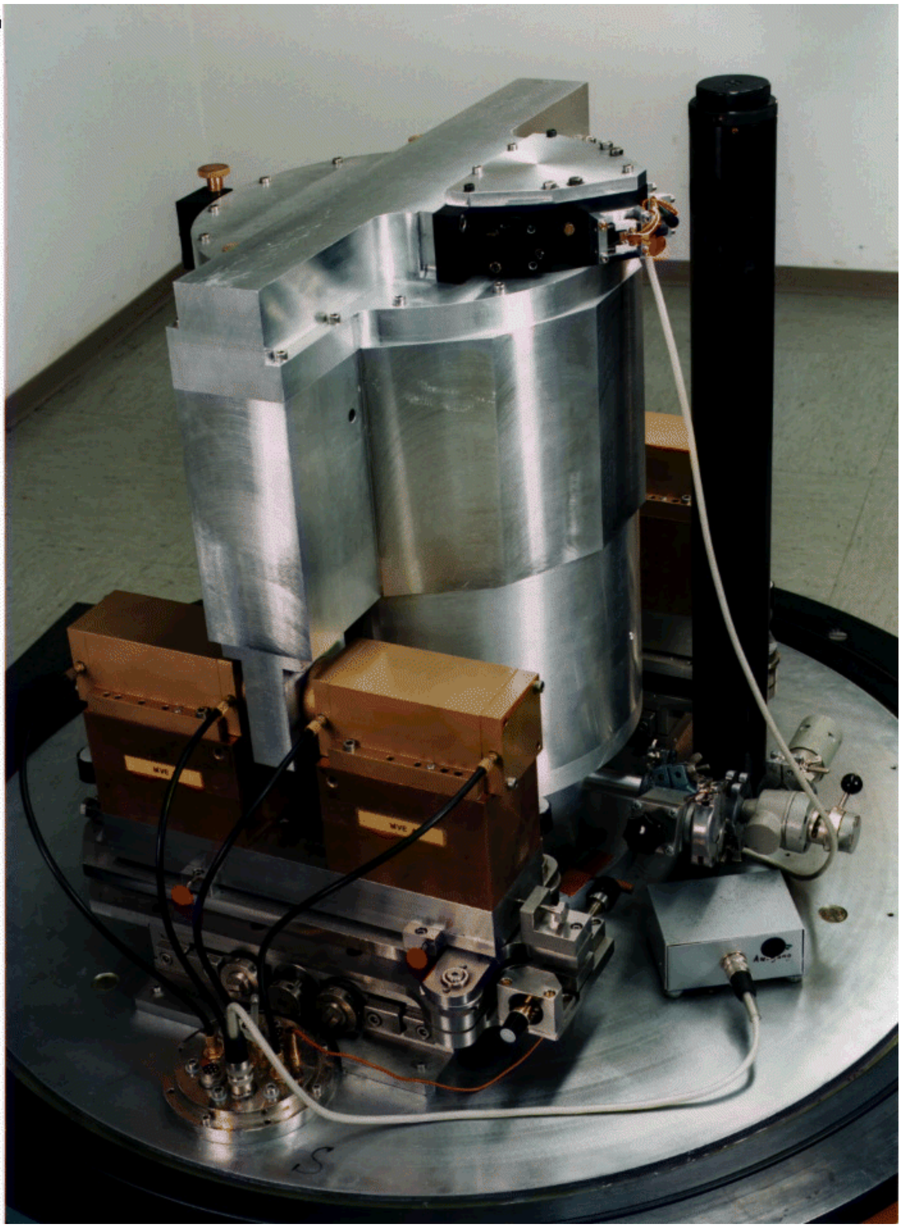

Figure 6. (Continued)

the usual suspension fibre by employing a mercury bearing to support the beam and a quadrant electrometer to provide the restoring force. As mentioned in section 2, the use of a mercury bearing in this manner had been pioneered by Burgess (1902b). He realized that the relatively large forces of buoyancy acting on a float submerged in a volume of mercury would make it possible for a beam attached to the float to bear attracted masses that were much larger than those that could be suspended from a delicate torsion fibre, thus increasing the size of the gravitational signal. The PTB experiment incorporated this concept and took advantage of the fundamental nature of the design change to introduce several other features aimed at refining torsion-balance measurements of G. (Others have used liquid-based suspension systems in gravity experiments, as well. See Gillies and Ritter (1993) for a discussion.) The resulting apparatus is shown schematically in figures 6(a) and 6(b). In essence, the experiment is a servo-controlled torsion balance, in which the motion of the beam is sensed by a differential laser interferometer. Moveable attracting masses can be used to apply a calculable alternating torque, $G\mu$, to the beam via their gravitational interaction with the attracted masses fixed to it. Control signals applied to the vanes of a quadrant electrometer provide the electrostatic restoring torque, M_e, used to counteract the gravitational torque and null the motion of the balance. The difference in the electrostatic torque, ΔM_e, measured when the attracting masses are moved from one of their test positions to the other, will be equal to the associated change in the gravitational torque, $G\Delta\mu$, acting on the beam. In principle, then, G is determined by assessment of the quantity

$$G = \Delta M_e/\Delta\mu$$

with the challenge being one of identifying and accurately evaluating all of the constituent terms that make up ΔM_e and $\Delta\mu$. Early versions of the apparatus built to make this measurement were placed on a granite surface plate the levelling of which was pneumatically servo-controlled (de Boer *et al* 1980a). The final form of the experiment, however, was located on a pillar that extended 3 m into the ground. The vacuum chamber housing the experiment is shown in figure 6(c). It held a helium atmosphere at a pressure of 10^4 Pa. Also shown in the foreground of the photo are the brass housings for one pair of the movable attracting masses. The masses were moved from the near to the far positions (a distance of 10.5 cm), relative to the attracted masses on the beam, by compressed air fed into ports in the cylindrical housings. The orientation of the attracting masses relative to those on the beam was adjusted by the three-axis position controller on which the housings rested (Beuke *et al* 1983). The masses on the beam were made of Zerodur® and were approximately 120 g each. The attracting masses were of two types: 40 mm diameter cylinders of tungsten, 900 g each, and replacement masses of Zerodur®, about 118 g each. The mercury bearing that supported the beam was designed to have only negligible rotational friction (Augustin *et al* 1981) and to be self-centring (Augustin *et al* 1982). The volume of the floater component of the bearing was approximately 100 cm^3 and had a buoyancy in the mercury that was sufficient to sustain a load of 13 N. Rigorous component cleaning and mercury purification processes were carried out to minimize contamination of the bearing. Also, a layer of sulphuric acid was allowed to stand on the free surface of the mercury to dissolve any films that might have otherwise produced torques on the balance beam. A two-stage quadrant electrometer was used to generate the restoring torque that held the beam at a constant position under the influence of the gravitational torques applied to it by the attracting masses. The design principles, electrical characteristics and sensitivity of the single-stage version of this device have been discussed by de Boer *et al* (1980b). Although only one of these electrometers was actually needed for the measurement of G, the incorporation of a second made it possible to null out any voltages that developed on the needle of the first, even though the needles of both were electrically grounded. Since the torque developed by a quadrant electrometer is governed by the rate at which its electric field energy changes with the angular position of the needle between the plates, careful measurements of the change in electrometer capacitance with needle angle, $dC/d\alpha$, were therefore needed and made. The sensor used to monitor the angular location of the beam and, hence, that of the electrometer needle, was a modified Michelson interferometer. A one-fringe signal in this interferometer corresponded to a beam rotation of $\pm 10^{-6}$ rad. The interferometer signal was processed by a computer-based PID controller that sent control voltages of up to ± 2.5 V to the electrometer electrodes, with a noise level on the signal of about 10 mV. A report documenting all of the design and performance features of this apparatus has been prepared by Michaelis *et al* (1995), and it should be consulted by anyone interested in either the details of the metrology or the measurement processes employed in this work. The first result obtained with this apparatus was (de Boer *et al* 1987)

$$G = (6.667 \pm 0.005) \times 10^{-11}\ \mathrm{m^3\ s^{-2}\ kg^{-1}}$$

consistent with the 1986 CODATA value. A thorough reassessment of the instrumentation, however, led to the identification of various of sources of systematic error. Subsequent improvements to the apparatus and extensive acquisition of data during 1992 and 1993 led to a new set of results which, surprisingly, are 0.6% (50 standard deviations) above the CODATA value (Michaelis *et al* 1994, 1995/96, Michaelis 1995a, b):

$$G = (6.715\,40 \pm 0.000\,56) \times 10^{-11}\ \mathrm{m^3\ s^{-2}\ kg^{-1}}$$

for measurements with the tungsten attracting masses, and

$$G = (6.7174 \pm 0.0020) \times 10^{-11}\ \mathrm{m^3\ s^{-2}\ kg^{-1}}$$

for measurements with the Zerodur® attracting masses. An exhaustive search for undiscovered systematic errors (Michaelis *et al* 1995) has failed to reveal any explanation for the large size of the discrepancy between this result and the presently accepted value. See section 4.7 for some further discussion of the questions raised by this result.

The most recently completed high-precision experiment is that of Bagley and Luther (1996), who used a torsion pendulum in the time-of-swing mode to measure G at the Los Alamos National Laboratory in New Mexico. Their experiment was a redesigned version of that of Luther and Towler (1982), and it incorporated some of the same components (e.g., the 10.5 kg tungsten attracting masses). The apparatus was located in the inner room of an isolated bunker situated remotely from the main site of the Los Alamos National Laboratory. The thermal insulation surrounding this room kept the daily temperature constant to within 1 °C, with some long-term upward drift noted as the season changed from Winter to Spring. The attracting masses were positioned on a Cervit™ table that was fixed to the top of a non-magnetic air bearing, the rotary drive for which was regulated by a computer. The suspension head for the pendulum's fibre was mounted on a second air bearing, this one located axially above the centre of geometry of the Cervit™ table and attracting masses below it. A tubular vacuum chamber surrounded the fibre and suspended masses, and a rotary vacuum feedthrough connected the stationary lower part of the chamber to the upper part of it which could be rotated via the upper air bearing to position the fibre's abutment. An eddy-current damper using four Sm–Co magnets was installed just below the rotary feedthrough, at the point where the vacuum pump was attached. The position of the upper air bearing was sensed interferometrically and also computer-controlled. An autocollimator with a 2048-element CCD array monitored the motion of a mirror fixed to the pendulum, with a sensitivity of (464.9 ± 0.2) pixels per degree of angular motion. The dumbbell-shaped pendulum fob was made of tungsten discs, 2.5472 mm thick by 7.1660 mm in diameter, mounted on the ends of a 28.5472 mm long tungsten rod.

A very interesting feature of the experiment was that two series of measurements were made, each with a different tungsten suspension fibre. Both fibres had a diameter of about 13 μm, and the first yielded a quality factor, Q, for the torsion pendulum of 950, while with the second fibre (which had a gold coating) the Q was 490. By making measurements at two different quality factors, they were able to test the arguments advanced recently by Kuroda (1995) and others (see section 4.7) that anelasticity in the suspension fibre produces an upward bias in torsion-pendulum determinations of G, with the magnitude of the effect being $1/(\pi Q)$. The values of G obtained with the Los Alamos apparatus using these two different fibres were

$$G = (6.6761 \pm 0.0011) \times 10^{-11}\ \mathrm{m}^3\ \mathrm{s}^{-2}\ \mathrm{kg}^{-1}$$

and

$$G = (6.6784 \pm 0.0008) \times 10^{-11}\ \mathrm{m}^3\ \mathrm{s}^{-2}\ \mathrm{kg}^{-1}$$

for the suspension quality factors of 950 and 490, respectively. Indeed, the anelasticity of the suspension fibres was found to be the dominant damping mechanism in the experiments, and so these values were corrected by subtracting the anelasticity shift, with the final values being $[G - G/(\pi Q)]$, or

$$G = (6.6739 \pm 0.0011) \times 10^{-11}\ \mathrm{m}^3\ \mathrm{s}^{-2}\ \mathrm{kg}^{-1}$$

and

$$G = (6.6741 \pm 0.0011) \times 10^{-11}\ \mathrm{m}^3\ \mathrm{s}^{-2}\ \mathrm{kg}^{-1}.$$

Figure 7 is a schematic diagram of an early version of the apparatus. Full details of the experiment, more recent illustrations of it and an uncertainty analysis for it are available elsewhere (Bagley 1996). As discussed below in section 4.4, the possibility of designing an hybrid version of the apparatus that incorporates the features of both the static and dynamic modes of the torsion pendulum is under consideration at Los Alamos.

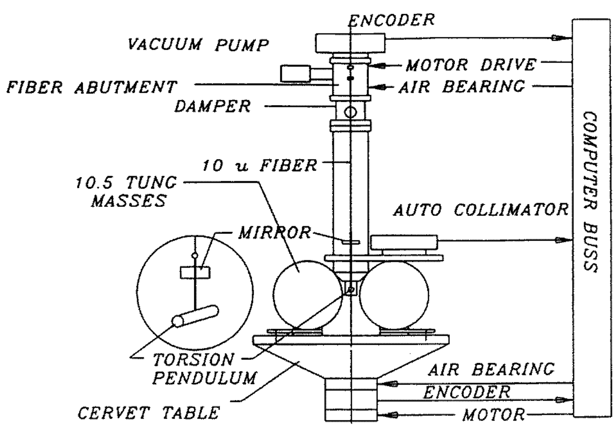

Figure 7. Schematic diagram showing the general features of the torsion pendulum used in the time-of-swing measurement of G carried out at the Los Alamos National Laboratory in Los Alamos, New Mexico, USA (courtesy of G G Luther).

4.3. Pedagogic studies

Before going on to discuss experiments that are either in progress or are being planned, it is worthwhile to consider still another class of recent measurements of G, viz, those carried out primarily for teaching rather than research purposes. As a point of background on this topic, we note that an early commercial version of a Cavendish balance designed for use in the undergraduate laboratory was originally made available by the Leybold Company, and its set-up and operation was discussed by Purcell (1957) and, more recently, by Leybold (1994) itself. Similar types of apparatus are also commercially available from various other vendors. Even so, some physics instructors and others interested in classroom/laboratory demonstrations of either the gravitational force or the design of mechanical instruments have chosen to build their own such devices, and several of them are described in the literature. The goal in most cases has been to provide students with an apparatus that will let them measure G to accuracies of 5 to 10% over periods of time commensurate with a single laboratory session (a couple hours or less).

Meissner (1957), for instance, sought to shorten the observation period needed to sense the deflection of the attracted masses in a Cavendish balance in which two 10 kg lead spheres were used as the attracting masses. To accomplish this, he introduced into the measurement an optical lever that used photoelectric cells to monitor the motion of a mirror attached to the small mass system, making him the first person to employ photoelectronic sensing in a determination of G (Heyl and Chrzanowski (1942) had used an elaborate photographic technique to record their data). Block *et al* (1965) described a torsion balance that could be constructed by the students themselves, a process that confronted the class with several interesting and instructive design decisions, and which led them to a $\pm 10\%$ determination of G. Stong (1967) described an experiment carried out by N E Lindenblad, who attempted to gravitationally induce oscillations in a simple pendulum hanging at rest, by making a second simple pendulum oscillate in close proximity to the first one. Lindenblad realized that this approach might form the basis for a determination of G. Shortly thereafter Southwell (1967) worked out a mathematical description of the gravitational interaction between the two pendula and described the physical characteristics of a laboratory apparatus that might actually be used for this purpose (although no one seems to have gone forward with such a measurement). Crandall (1983) developed a servo-controlled torsion balance in which the movement of the beam was sensed photo-optically. The sensor signal served as the input to a feedback circuit that drove current through a small coil that exerted a restoring force on a permanent magnet fixed to the beam. This feedback loop could quickly null the motion of the beam when external attracting masses were used to create a gravitational torque on it. The principal advantage of this arrangement was that it reduced the time required to make a deflection-mode measurement of G from roughly an hour to a minute, thus significantly improving the utility of the Cavendish balance in lecture demonstrations. Karim and Toohey (1986) applied a modified version of Crandall's approach to the standard Leybold Cavendish balance. Specifically, they used eddy currents to produce the restoring torque thus eliminating the need to install a permanent magnet on the balance beam. Electronic damping in the control circuit was used to shorten the settling time of the beam's response to movement of the attracting masses. Dunlap (1987) replaced the torsion fibre in the Leybold apparatus with a melt-spun ribbon of amorphous $C_{50}Zr_{50}$ that was 16 μm thick, 0.89 μm wide and 25 cm long. The increased mechanical strength of this material reduced the difficulty of handling the suspension fibre and the associated risk of breaking it. Dousse and Rhême (1987) designed a very robust instrument that could be used in either the deflection or the time-of-swing mode. They used a 60 cm length of 30 μm diameter Nicotine® fibre to suspend the beam, which was specially designed to carry the attracted masses at different vertical positions to avoid gravitational cross-talk with the nearby attracting masses. An optical lever with a computer-servoed tracking mirror was used to monitor the position of the beam, and a magnetic damper was attached to the top of the torsion fibre to eliminate the non-torsional components of the beam's motion. Sufficient attention was paid to the metrology of the apparatus to allow measurements of G made in the deflection mode to have an uncertainty of 717 ppm and those made in the time-of-swing mode to have an uncertainty of 1016 ppm (see table 2 for some representative results). D'Anci and Armentrout (1988) installed a phototransistor on the pen pointer of a strip-chart recorder, thus allowing it to serve simultaneously as the sensor of an optical lever and as the means for continuously tracing the oscillations of the torsion balance used by their students to measure G. With this arrangement, they were able to observe anomalous oscillations of the balance beam that probably arose because of thermal effects. Saulnier and Frisch (1989) built a torsion balance in which they sought to make the restoring torque small enough for the suspended test masses to exhibit essentially linear motion over very small distances ($\approx$ 100 μm). They were then able to 'launch' the test masses relative to the attracting masses at a speed of about 10^{-3} cm s^{-1} and observe their 'trajectory' through the point of maximum displacement and then back towards the attracting masses. Knowledge of the local gravitational acceleration produced by the attracting masses and measurements of the accelerations of the test masses thus allowed them to obtain a value of G that had an uncertainty of approximately 1.4% (see table 2). An interesting feature of the apparatus was that it used depleted uranium 'pseudospheres' as the attracting masses. They were about 3 kg constructs of cylinders and truncated cones that were easy to manufacture and had the point-source field of a sphere to within 0.5%. The authors also demonstrated that this apparatus could be used to test the inverse square law, finding the value of the exponent governing the distance dependence to be 2.1 ± 0.1. MacInnes (1974), too, was interested in the inverse square law, and he described a simple levered balance that let students feel the pull of an inverse square force that was created by reaction of some of the balance components to the linear extension of others. The details of construction for this device are presented in his paper, as is a calculation clarifying its principles of operation.

4.4. Experiments in progress

Karagioz and colleagues have recently taken a substantial amount of new data with their apparatus and are in the process of analysing it. They have taken advantage of the precision positioning capabilities of the devices designed to move their large masses, running experiments that will allow them to search for a variety of systematic effects, establish a new absolute value for G and search for the presence of a $G(r)$ effect at short distances. The data were taken in 30 parts, each consisting typically of 100 or more individual measurements of G. The results will be forthcoming.

In Europe, Kündig *et al* (1996, see also Nolting *et al* (1996)) of the Universität Zürich are using a modified version of the Mettler-Toledo AT1006 kilogram intercomparator with a resolution of 10^{-10} kg in a new measurement of G. This balance serves as the detector in an arrangement designed to sense the change in the gravitational force acting on gold-plated copper 1 kg test masses when large tanks of mercury (500 l) are alternately positioned on top of and underneath the test masses. The goal of the new effort, being carried out at the Paul-Scherrer-Institute in Villigen, is to determine G with an uncertainty of 10 ppm.

The group at the Bergische Universität Wuppertal continues to refine their existing apparatus and search for sources of systematic uncertainty. Technical issues that are being addressed include the evaluation of temperature-related sensitivities in the apparatus, improvement of the metrology of the field masses, development of a more precise geometric correction factor, and the reinstallation of the experiment at a new site.

The other recent measurement of G in Germany, that of the PTB in Braunschweig, has been discontinued. There are no present plans to re-initiate it.

Elsewhere within Europe, Quinn *et al* (1996a, b) are undertaking a measurement of G at the Bureau International des Poids et Mesures in Sèvres, France, using test and source masses in the hexadecupole arrangement shown in figure 8. The four test masses (0.47 kg each)

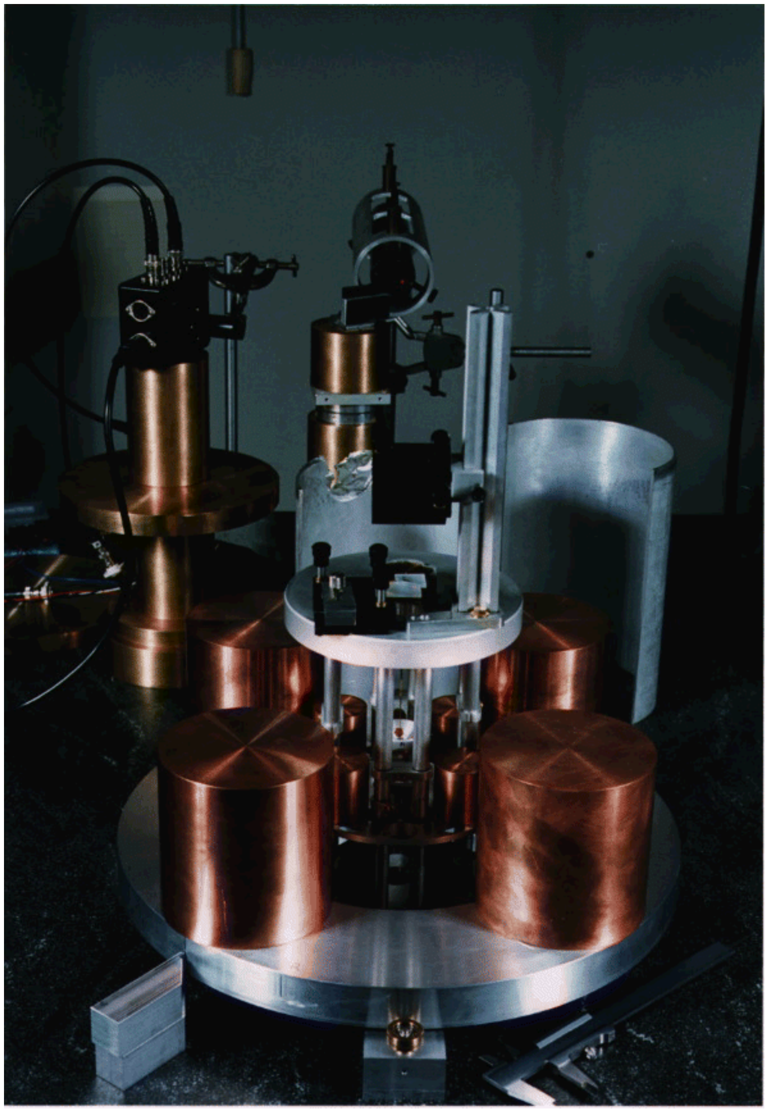

Figure 8. Photograph of the flexure-strip torsion balance under development for the measurement of G at the BIPM in Sèvres, France (courtesy of T J Quinn).

are supported on a circular plate that is suspended by a torsion strip suspension. The strip is 80 mm long, 1.25 mm wide and 30 μm thick. The whole balance assembly, including the four source masses (10 kg each), is mounted on a coordinate measuring machine to enable precise determination of the relative positions. The pendulum's period is about 80 s and it has a very low level of zero-drift, which is consistent with the expected behaviour of such a suspension. Moreover, preliminary measurements made with this system confirm that the gravitational restoring torque dominates that due to the elastic response of the torsion strip. Preliminary results obtained with this apparatus are encouraging and further development of it is underway.

In Italy, De Marchi and colleagues (1996a, b) have begun an experiment using a simple pendulum, and it has the goal of determining G to within a fractional uncertainty of 10^{-5}, providing that the frequency of the 2 m long pendulum is stable to within 10^{-11} of the nominal value. The apparatus is carefully vibration isolated and consists of an approximately 1 g test mass undergoing pendular oscillations (in vacuum) between two large perturbing masses.

Other ongoing efforts elsewhere in the world include that of the workers at Industrial Research Ltd in New Zealand, who continue to refine their apparatus (Armstrong and Fitzgerald, 1996a, b). In particular, they have installed a new tungsten torsion fibre of rectangular cross section (17 μm $\times$ 340 μm) that now suspends a 500 g test mass, improved the electrometer calibration to better resolve the gravitational potential, and added viscous damping to the suspension fibre's attachment point. While these changes have significantly extended the amount of time each week that the experiment can be operated, there has also been an as-yet unexplained increase in the size of the noise on the signal, and an increase in the drift of the rest point of the suspension. Attempts to resolve these problems are presently underway.

In Taiwan, Ni (1995) has begun a measurement similar to that of the Bergische Universität in Wuppertal, except that his resonant cavity is a high-finesse Fabry–Perot optical interferometer. It will use two lasers that are frequency locked such that an intermirror displacement of just under 1 fm, when converted to a frequency measurement, will correspond to approximately 1.5 μHz (under certain assumptions for cavity finesse and linewidth resolution). The resulting mirror displacements produced by the gravitational attraction of 100 kg source masses could thus be measured with enough sensitivity to permit a determination of G that is accurate in principle to 1 ppm.

Another new absolute measurement of G in the orient is that under consideration by Luo Jun in the People's Republic China, the second such experiment carried out there in recent times. (The first was that of Liu *et al* (1987), discussed above.)

There are also several interesting redeterminations of G underway within the United States. For instance, as part of the ongoing experimental effort at Los Alamos National Laboratory, Luther and Bagley (1994) have discussed the possibility of modifying their measurement of G in such a way that the apparatus would incorporate features of both the deflection and time-of-swing modes of the torsion pendulum. In the modification, a pair of spherical attracting masses would gravitationally alter the oscillation period of the suspended body in the usual way, but a servo system would rotate the suspension point of the torsion fibre to keep the average position of the suspended body constant when the large masses are moved. The natural frequency of the pendulum would be measured at an intermass angle of approximately 45°, where there is an inflection point in the potential, thus yielding a value of the torsion constant of the fibre at a point where it is unaffected by the gravitational attraction of the large masses. The angular displacement through which the suspension's support point would have to be turned by the servo system times the torsion constant of the fibre is then a measure of the gravitational torque produced on the balance by the attracting masses. A knowledge of this torque would then allow G to be determined once the mass distributions are known satisfactorily.

A new measurement of G is being made at the Joint Institute for Laboratory Astrophysics in Boulder, Colorado, by Faller and colleagues (1996). The apparatus is shown in figure 9. They are using a falling-corner-cube, model FG5 absolute gravimeter (Faller and Marson 1988) as the sensor. By positioning a 100 kg bronze ring alternately above and below the gravimeter's drop chamber, the Newtonian attraction of the ring either subtracts from or adds to the local gravitational acceleration, g, acting on the test mass and G can be determined from the resulting change in acceleration, Δg. The bronze ring used in this case

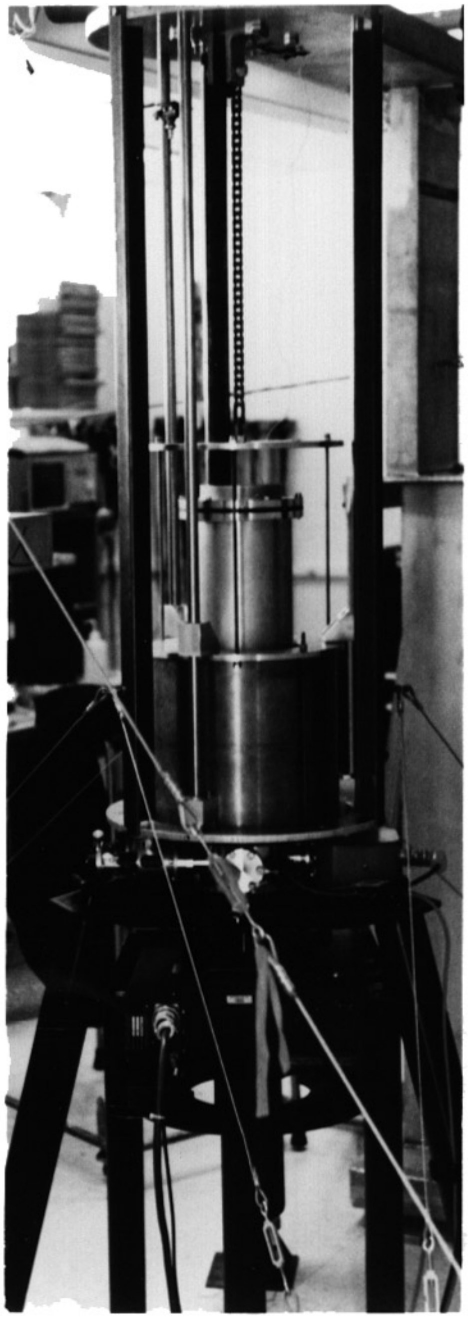

Figure 9. Photograph of the model FG5 absolute gravimeter and the moveable cylindrical source mass used to determine *G* at the Joint Institute for Laboratory Astrophysics in Boulder, Colorado, USA. The source mass is shown positioned around the dropping chamber of the gravimeter (courtesy of J E Faller).

is one of the same sources masses that had been used by Hulett (1969) in his horizontal pendulum measurement of *G* at Wesleyan University. It has a 30 cm outer diameter, a

20 cm inner diameter and a 15 cm thickness. To date, there have been approximately 100 000 drops made with this apparatus, in groups of 100 drops per run, first with the ring above the gravimeter and then with the ring repositioned below it for another 100 drops. Each 100 drop run requires about 20 min to complete. The preliminary value obtained for Δg is approximately 20 μgal, which may allow G to be determined with an uncertainty of a few parts per thousand once the experiment is completed. By changing to a tungsten source mass and implementing other improvements, the projected uncertainty might possibly be decreased by a factor of 10. It is interesting to note that experimental arrangements somewhat similar to this one have been used by others to carry out Newtonian-attraction calibrations of LaCoste–Romberg gravimeters (Varga *et al* 1995, Csapó and Szatmári 1995) and superconducting gravimeters (Achilli *et al* 1995), although without subsequent use of the data to infer a published value of G.

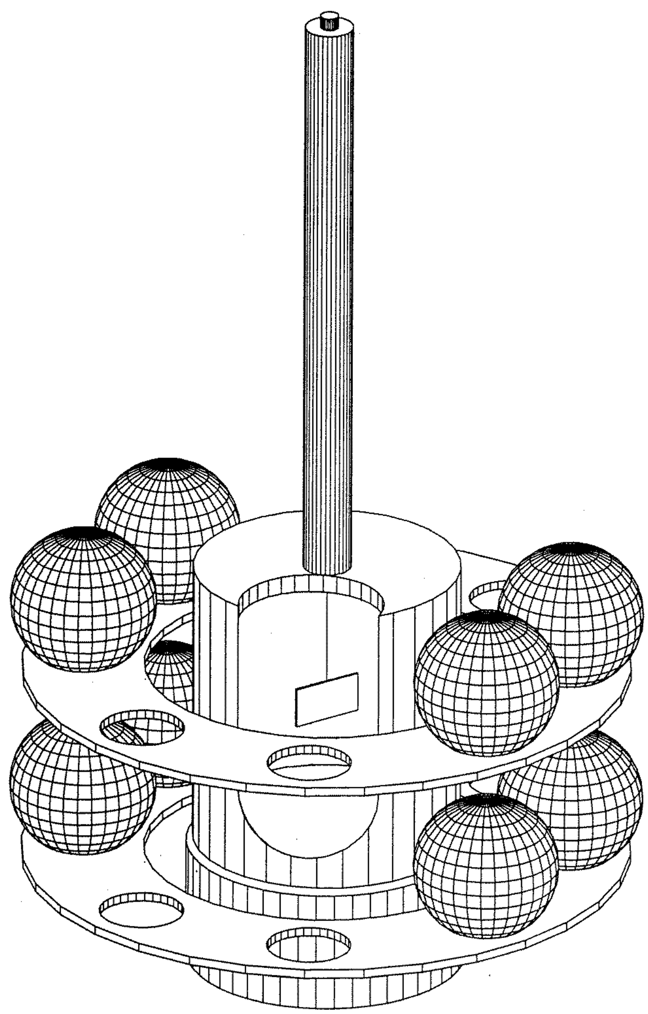

Figure 10. Schematic diagram of the symmetrized torsion pendulum proposed for use in the measurement of G at the University of Washington in Seattle, Washington, USA (from Gundlach *et al* 1996, copyright of the American Insitute of Physics).

Workers at the University of Washington have embarked on a torsion balance determination of G (Gundlach *et al* 1996) that incorporates several unique and interesting features. The layout of the apparatus is shown in figure 10. An underlying principle of this design is the selection of mass geometries that produce a gravitational acceleration on the test body that does not depend on its mass and dimensions, or on any density gradients it may have in its structure. As a step towards achieving this in a working instrument, they analysed the dynamics of a torsion balance in terms of the mass multipoles of the interacting bodies. This led them to an experimental arrangement in which the ‘attractor’ would consist of eight large spheres centred around the suspended body, itself a thin, rectangular quartz plate. The torsion balance would be rotated relative to the attractor, with a resulting time-dependent gravitational torque then acting on the suspended body. By introducing feedback into the experiment, they keep the suspension fibre from twisting during the measurement, thus preventing variations in the fibre’s elastic properties and subsequently minimizing that source of systematic uncertainty in the value of G. This is accomplished by using an autocollimator to monitor the angular displacement of the pendulum, processing the measurements with a PID digital control algorithm, and using the resulting signal to modulate the turntable’s angular speed so as to prevent the fibre from twisting and thus null the output of the autocollimator. (The masses constituting the attractor are also be rotated, at a different nominal angular speed, to eliminate the gravitational coupling to background masses.) Numerical simulations of the experiment suggest that an uncertainty in G of 10^{-5} could be achieved over short run times ($\approx$ 1 day), and a preliminary test of the principle using the existing Eöt-Wash apparatus (Adelberger *et al* 1990, Su *et al* 1994) yielded a value of G within 2% of the accepted value. Gundlach *et al* (1996) provide a discussion of the design principles, a possible experimental arrangement, the permissible component tolerances and the advantages of this method relative to other approaches.

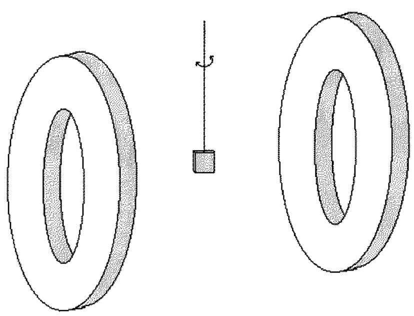

Figure 11. Elements of the torsion pendulum to be used by Newman and colleagues at the University of California in Irvine, California, USA for the measurement of G. The attracting masses have the shape of open-bore cylinders, while the attracted mass is a thin rectangular plate suspended from the torsion fibre (courtesy of R D Newman).

There is one other new measurement of G currently underway, and it is being carried out by Newman and colleagues at the University of California, Irvine (Newman 1995a, Wang *et al* 1996, Bantel *et al* 1996). They are using a torsion pendulum operated in the time-of-swing mode, but at a temperature of 4.2 °K. Like Gundlach *et al* (1996), they independently analysed the gravitational coupling in a torsion pendulum in terms of the multipole moments of the interacting masses and this led them, too, to select a rectangular, fused silica plate for use as the suspended body. As shown in the (not-to-scale) illustration in figure 11, their attracting masses are relatively thin cylindrical rings. They produce a pure quadrupole field gradient at the location of the suspended body, and this makes the measurement virtually autonomous to uncertainties in the location, size and mass distribution of this body. The determination of G results from measurements of the shift in the pendulum's oscillation frequency produced by changing the angular orientation of the rings by 90° about the z-axis of the apparatus. Many interesting and useful benefits accrue from the use of this geometry, including the possibility of operating the experiment at certain large oscillation amplitudes at which the frequency shift is an extremum and the signal-to-noise ratio is maximized. Expressions describing the pendulum dynamics, the dependency of the frequency shift on the various experimental parameters, and other characteristics of the system are given by Newman (1995a). The experiment is being done in two phases, with the first being carried out at the University of California, Irvine. It will use copper attracting masses that are approximately 50 cm outer diameter, 24 cm inner diameter, 5 cm thick and 70 kg each. The test mass will be a rectangular plate of fused silica, having dimensions of $4 \times 4 \times 0.4$ cm and suspended by a 30 cm length of 25 μm diameter type 5056 aluminum fibre. In phase II, the apparatus will be moved to the US Department of Energy's Hanford, Washington Site, where it will be located in a seismically-quiet, former defensive missile bunker. At that point, fused silica rings may replace the copper rings as the source masses. The dimensions of the silica rings are to be approximately a 52 cm outer diameter, 26 cm inner diameter and 10 cm thick, with a mass of 34 kg each. In both cases, the source masses will be located outside of the cryostat containing the torsion pendulum. The target accuracy for G in the phase I experiment is 20 to 50 ppm and for phase II it is 1 to 5 ppm (presuming that the fused silica rings are used and that the experiment is operated at 2 °K).

4.5. Proposed terrestrial experiments

In addition to the many recent measurements of G and the experiments in progress discussed above, there have also been several proposals for measurement of G that have appeared in the literature since about 1960. A number of them are listed in table 3. The efforts associated with these proposals range from planning-stage *Gedankenexperimenten* (e.g., Kolosnitsyn 1993a) to those in which substantial design work and apparatus construction was done but no result obtained (e.g. Marussi, 1972). Some of the proposals incorporate superconducting technology into the measurement of G (Bleyer and John 1976, 1977a, b, Bleyer *et al* 1977, 1984, Fialovsky 1981) or describe novel alternatives to the torsion balance (Rudenko 1979, Speake and Quinn 1988). Of course, there are several others that discuss a variety of interesting torsion-balance-based possibilities, as well (Berman 1968, Cook 1968, Chen 1984a). The prospect of determining G in ways that might shed some light on the quantization of gravity has been discussed by Nieto and Goldman (1980) who propose using the superconducting ac Josephson effect to measure G/h, and by Opat *et al* (1989) who mention that there is interest in determining G via neutron interferometry. All of the proposed measurements listed in table 3 call for laboratory experiments that would be done on the surface of the Earth. In the next subsection, we comment briefly on that class of experiments proposed for flight on board satellites in Earth orbit.

4.6. Proposed satellite experiments

Sanders and Gillies (1996) have recently written a thorough review of the many proposals for the measurement of G on board a spacecraft. The possibility of doing an experiment in space has attracted the attention of several researchers, since this method would seem to offer the opportunity to greatly alleviate at least two of the central problems encountered in

Table 3. Several of the recent proposals for making terrestrial measurements of G that have been discussed in the literature. In some of these cases, substantial experimental work was carried out on a prototype of the apparatus, but no final results were reported.

Reference/date	Proposed technique	Special features
Mark *et al* (1963)	Modified vertical field gradiometer with test masses passed through an axial hole in the cylindrical source mass	Source mass dimensions to be 30 cm OD and made of depleted uranium
Berman (1968)	Excitation of resonant oscillations in a suspended rectangular plate by a second one rotated at constant speed below it	Positions of the interacting masses to be determined by interferometry
Cook (1968); Marussi (1972)	Large-scale torsion pendulum used in the time-of-swing mode; gravitational torques dominate measurement	Special 500 kg attracting masses had the point source field of a sphere
Bleyer and John (1976, 1977a)	Superconductively levitated ring undergoing two-dimensional oscillations between a pair of symmetric source masses	G is proportional to the beat frequency of the two superimposed normal modes
Rudenko (1979)	Dynamic measurement via source and receiver gravity wave antennae; two gradiometer measurements also proposed	$\Delta G/G$ perhaps 0.1% at 50 m for gravity wave experiment, 3% for gradiometers at 10 m
Nieto and Goldman (1980)	G/h to be measured via superconducting ac Josephson effect (equivalent to rotating a battery in an external gravitational field)	Gravitational force creates an additional EMF that changes Josephson effect frequency
Chen (1984a)	Excitation of a resonant torsion pendulum that has one mass suspended below the level of the other one on the beam	A source mass on a turntable orbits around the lower mass producing a sinusoidal torque
Gillies and Marussi (1986)	Attraction of a large source mass produces an apparent increase in weight of a test mass magnetically suspended above it	Test mass position nulled; change in suspension coil current is signal of interest
Speake and Quinn (1988)	Vertical dumb-bell balance with the upper mass subjected to a sinusoidal torque by a rotating, external source mass	Balance position nulled by a servoed magnetic coupling to the dumb-bell's lower mass
Opat *et al* (1989)	Neutron interferometry with cold neutron beams; measured phase shift is proportional to G and other quantities	No specific experimental arrangement discussed; overview of work of others
Kolosnitsyn (1993a)	Torsion pendulum in a spherical cavity cut from an uniaxial ellipsoid; gravitational field homogeneous in second derivatives[a]	Free oscillation frequency of pendulum shifts as ellipsoid is rotated about vertical axis
De Marchi *et al* (1996a, b)[b]	Evacuated simple pendulum undergoing oscillations in the presence of two high-density perturbing masses	Target accuracy of 10^{-5} G expected with the use of tungsten or uranium perturbing masses

[a] An apparatus similar to this has been built by Tarbeyev *et al* (1994) and used for the calibration of accelerometers. Also, Klimchitskaya *et al* (1996) have proposed that it be used in experimental searches for composition-dependent gravity.
[b] The experiment of De Marchi *et al* is presently in progress.

terrestrial measurements of G, viz, isolation of the gravitational interaction between the test masses from the couplings produced by any other masses, and an escape from seismic noise and other sources of disturbance. Their review categorized all of the different proposals into two broad classes: those that are largely conceptual in nature and those that have not only introduced a concept but focused on the practicalities of implementing it as well. The first category includes roughly 15 different experiments that typically involve test masses in either artificial binary, weakly-coupled oscillator or Lagrange-point configurations. The second category, however, consists of only three experiments. One of them is the G/ISL test proposed for incorporation into the STEP ('Satellite Test of the Equivalence Principle') mission (Blaser *et al* 1993), another is Project NEWTON which is proposed as a stand-alone measurement of G (Nobili *et al* 1990), and the third is the SEE ('Satellite Energy Exchange') method suggested by Sanders and Deeds (1992). All three of these proposals have received scrutiny from the scientific community, but none of them have yet advanced to the point of detailed engineering design studies. The G/ISL experiment would measure G and test the inverse square law through the use of high-precision accelerometers in which all of the source and test masses are suspended by magnetic bearings. Project NEWTON would determine G by monitoring the kinematics of an artificial binary, while Project SEE would do so by monitoring the encounter between a 'shepherd' mass and a test particle moving in Darwinian horseshoe-shaped orbits. A schematic diagram of the experimental arrangement foreseen for the latter is shown in figure 12; illustrative diagrams and full discussions of all these experiments are available elsewhere (Sanders and Gillies 1996; see also Alexeev *et al* 1994, Baker 1996). It is not clear at present which, if any, of the proposed experiments might someday be selected for engineering development and subsequent flight preparation. Any such undertaking is extremely expensive and would clearly require a large-scale collective effort, very likely one that is multinational in scope. Attempts to build a consensus in favour of such an effort are underway (Spaniol and Sutton 1995).

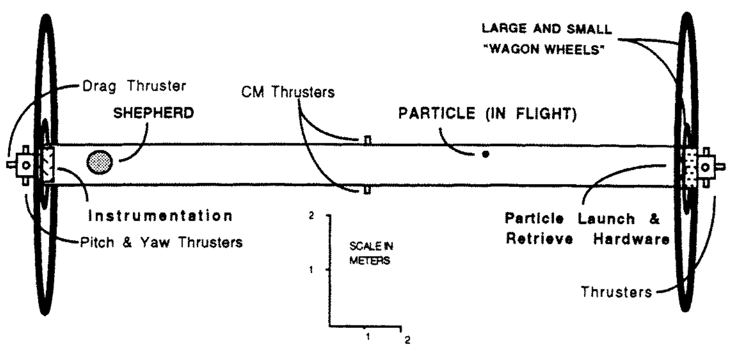

Figure 12. Schematic diagram of the 'satellite energy exchange' (SEE) method proposed by the University of Tennessee, Knoxville, Tennessee, USA, for the determination of G and other precise gravitational measurements. The 'particle' and 'shepherd' masses would undergo close-proximity encounters while revolving around the Earth in Darwinian horseshoe-shaped orbits. G would then be determined from an analysis of their motions (from Sanders and Deeds 1992, copyright of the American Institute of Physics).

4.7. Instrumentation issues

Each of the recent measurements of G has been made with instrumentation designed to advance the state of the art in experimental gravitational physics. Two broad classes of apparatus include most of the devices used for this purpose: torsion pendulums and beam balances, both of which have been studied for many years. The operational principles and performance limits of many types of these devices are well understood. Reviews of torsion pendulum instrumentation have been published by Gillies and Ritter (1993) and Chen and Cook (1993), while reviews of beam balances have been written by Speake (1987a) and Quinn (1992). The large spread in the experimentally determined values of G, however, continues to suggest that the understanding of these devices is incomplete and that there are yet-to-be-uncovered dynamical effects producing systematic errors in the measurements. The torsion pendulum, in particular, has been the focus of much attention for this reason, and the experiments that are now in progress (see section 4.4) incorporate versions of it that have been designed with this situation in mind.

An example of one such effect that has surfaced recently is that discussed by Kuroda (1995). Drawing on the work of Quinn *et al* (1992, 1995), he noted that anelasticity in the materials of the fibres used to suspend the attracted masses in time-of-swing determinations of G can introduce an upward bias into the results perhaps explaining part of the spread in the values of G obtained in recent measurements. He began by taking the model of Quinn *et al* where the energy loss in the fibre is due to a spectrum of relaxation processes of equal strength but with a distribution of relaxation times, and he derived an expression for the increase in the resonant angular frequency of a torsion pendulum over that predicted for the case where the damping mechanism in the fibre is presumed to be independent of frequency. Kuroda then went on to derive an expression for the increase in the resonant frequency which was valid for a wide range of energy loss mechanisms (Saulsen 1990, Saulsen *et al* 1994) (note, however, that the limitations discussed by Ritter and Gillies (1985) must be taken into account when attempting to interpret the applicability of a given dynamical model). His work generated a great deal of interest in the behaviour of the fibre-based suspension systems used in the measurement of G (Maddox 1995), and several others have since begun assessing the nature of any impact that this effect might have on their own apparatus. As seen above, Bagley and Luther (1996) have already applied this correction to their new measurement of G. Moreover, some of the most recent experimental designs have sought to eliminate the twist in the fibre completely (e.g., Fitzgerald *et al* 1994, Gundlach *et al* 1996), thus avoiding the anelasticity problem altogether. Newman (1995b) has generalized the anelasticity analysis to a continuum Maxwell model with any distribution of relaxation strengths and shown that the measurement bias in G must then be $\leqslant 1/(2Q)$. (Newman (1996) also contends that an anelasticity-based measurement bias also exists in static, deflection-mode measurements of G.) Bagley (1996) has noted that applying this limit to the Los Alamos experiment yields a lower bound on the value of G of

$$G \geqslant (6.6726 \pm 0.0011) \times 10^{-11}\ \mathrm{m^3\ s^{-2}\ kg^{-1}}$$

and

$$G \geqslant (6.6716 \pm 0.0001) \times 10^{-11}\ \mathrm{m^3\ s^{-2}\ kg^{-1}}$$

for the fibres with $Q = 950$ and 490, respectively. It is interesting that the first of these two numbers coincides exactly with the value obtained by Luther and Towler (1982), although with slightly more than twice the quoted uncertainty.

The question of frequency-dependent loss mechanisms in fibre-based suspensions is not a new one, of course, and it has also received attention within other contexts in gravitational physics. Speake and Quinn (1988) first reported observations that Q varied inversely as the square of the frequency for a variety of pivots. They further pointed out that this should lead to $1/f$ mechanical force noise. Then, in a seminal paper, Quinn *et al* (1992) reported a study of the anelasticity in flexure pivots in a vertical dumb-bell pendulum at strain frequencies of 10^{-2} to 10^{-3} Hz. The flexure pivots consisted of Cu–1.8%Be and were of the type developed for kilogram comparators and the beam-balance test of the fifth force. They showed that a model of fibre anelasticity comprising a spectrum of relaxation processes as discussed above explained this anomalous behaviour of Q and concluded that such suspensions may best be constructed from single crystal silicon. Kovalik and Saulson (1993) measured the vibrational-mode quality factor, Q, as a function of frequency for tungsten, sapphire, silicon and fused silica fibres of the type that might be used to suspend the test masses in the LIGO gravity wave detectors. The experimental results were compared against thermoelastic damping predictions, with tungsten showing much lower Q than expected. A later study aimed at evaluating the loss mechanisms in the suspension clamps designed for the VIRGO detector masses was carried out by Cagnoli *et al* (1996). In a further study of Be–Cu flexure and torsional suspensions operating at stresses up to 1.1 GPa, Quinn *et al* (1995) identified the main sources of damping at work in the suspensions. They also demonstrated that the modulus defect of this material was independent of stress up to a level of 95% that of the yield stress. On a related note, Brodt *et al* (1995) have designed an instrument that can determine the viscoelastic properties of various solids over ten decades of time and frequency (10^{-6}–10^{4} Hz), and they have described the results of its use with different types of metallic alloys. All of these studies should help point the way towards high Q mechanical suspensions that will be stable and reliable in the measurement of G and that will also find use in a variety of other gravitational physics experiments.

Another issue in torsion balance design that has received substantial attention is that of the sensitivity of the apparatus to seismic disturbance. Horizontal and vertical vibrations of the suspension point will excite the non-torsional modes of the apparatus and, through mode-coupling mechanisms, cause the period of the balance to vary, as well. Analytical models of the effects of vibrations on torsion fibre instruments have been formulated by a number of workers in the Russian gravitational physics community who were interested in using torsion pendulums for gravity gradiometry as well as the measurement of G (see Gillies (1987) for bibliographic references). Similar studies have been carried out by others, for instance Speake and Gillies (1987a), who developed a derivation of the torque on a torsion balance due to horizontal ground movements and assessed the severity of the problem within the context of the limiting uncertainty that it introduces into measurements of G. A useful technique for dealing with the parasitic modes of vibration in a torsion pendulum was developed by Luther and Towler (1982), who integrated a small aluminum disk into the assembly at the top of the suspension fibre. The disk was positioned between the poles of small horseshoe-shaped permanent magnets which produce eddy-current damping of the seismically-excited vibrational modes of the pendulum without perturbing its torsional modes. This arrangement has subsequently been used in a number of other measurements of G (e.g., Karagioz *et al* 1987) and in searches for weak gravitational effects (e.g., Ritter *et al* 1990). Investigations of a different experimental approach to this problem were initiated by Ritter and Gillies (1991), who proposed that precision imaging technology be employed to monitor the lateral motions of the suspended beam, thus providing kinematic information that could be used to dynamically null the seismically-induced fluctuations in the torsional mode of the beam. In a preliminary laboratory investigation of this possibility, Zeller (1992) used a computer-controlled translation stage to drive horizontal vibrations of the suspension point of a simple torsion pendulum, the motion of which was monitored by an optical lever. The results were encouraging in that a theoretical model of this method predicted the observed motion of the driven pendulum, thus suggesting that the concept of video-based cancellation of the seismic disturbances should be workable. The technique has not yet been fully realized in the laboratory, however. Earlier work at the University of Birmingham (Tingle 1991) had already demonstrated that the beam of the torsion balance would rotate when the suspension point was vibrated, and that the beam would tend to align itself at right angles to the direction of the driving motion. Subsequent work there that further explored the model of Speake and Gillies (1987a) was undertaken by Way (1994). A number of alternative suspension systems that eliminate the torsion fibre in favour of fluid or magnetic suspensions have been described in the literature (Gillies and Ritter 1993). While such devices would generally still be subject to the effects of seismic disturbance, the dynamic response of certain such systems (e.g., the purely spherical body of Karen *et al* (1990) levitated by the Meissner effect) might offer less complicated analysis and correction possibilities.

The limitations posed by thermal noise in torsion pendulum measurements of G have been discussed by several authors and reviewed recently by Chen and Cook (1990) and Gillies and Ritter (1993). Questions of arriving at meaningful estimates of signal-to-noise ratios, analysing the various experimental strategies, understanding the effects of feedback, etc can be quite complex and typically require careful assessment within the context of a given experiment. Therefore, no attempt will be made here to reiterate the generalized discussions that are already available in the two reviews mentioned above. Additional useful background information can be found in articles by McCombie (1953) who systematically applied fluctuation theory to analyse the precision limits of physical measurements, and by Hooge and Poulis (1977) and Poulis and Hooge (1978) who looked specifically at the role of thermal noise in measurements of G. Electronic cooling, a strategy aimed at lowering the effective noise temperature of an apparatus by suitable use of feedback, has been introduced into gravity gradiometers (Forward 1979) and gravity wave detectors (Oide *et al* 1978). It has been discussed for use in torsion balances (Chen and Tan 1991) as well, but feedback of other types (e.g., servoed rotation to null the intermass angle) has received much more study in determinations of G. The limits of mechanical thermal noise reduction by feedback have been explored by Ritter and Gillies (1985).

As the sensitivity and stability of precision mechanical devices like torsion balances and beam balances continues to improve, not only will the noise limits of the apparatus be confronted (as in the case of Adelberger *et al* (1990), where the torsion balance exhibited fluctuating errors only seven times larger than the thermal noise floor), but other weak phenomenon may arise as competing effects. One such possibility that is now beginning to receive attention in gravity experiments is the Casimir force (i.e. the retarded van der Waals force) which, although feeble, could conceivably compete with the gravitational force in experiments where the interacting masses are extremely close ($\lesssim 1\ \mu$m) to each other. The magnitude and sometimes even the sign of this force are generally difficult to calculate for anything other than simple geometries. Onofrio and Carugno (1995) noted that the Casimir force per unit area, P_c, between two flat, parallel, conducting (but neutral) plates is

$$P_c = K_c/d^4$$

where d is the interplate spacing and K_c is a 'kinematic constant' having a value of approximately 10^{-27} N m^2, a small number which scales the size of this force. In spite of its weakness, Mostepanenko and Sokolov (1987, 1988, 1990) have found that existing measurements of the Casimir effect allow them to place restrictions on the parameters in models of new, hypothetical long-range interactions that obey r^{-n} force laws. They have also discussed the design of some special experimental geometries that would place strong limits on the Yukawa constant α in new forces acting in the range from 10^{-8} m $< \lambda < 10^{-4}$ m (Mostepanenko and Sokolov 1993, Bordag *et al* 1995). Of course, the examination of the Casimir force as a competing effect in measurements of G would deserve attention only in those experiments where the test bodies or, perhaps, some other critical components in the balance are in very close proximity to each other. At present there is only one experiment in which this might be a potentially interesting possibility, that of the PTB in Germany (Michaelis *et al* 1995/96), where the rotor and stator of the mercury bearing are separated by distances on the order of hundreds of micrometres. No conclusive analysis of this prospect has been carried to date, though.

Most of the new torsion balance and torsion pendulum designs that have been used in recent measurements of the absolute value of G were discussed in section 4.1 through section 4.3. Some additional instrumentation-related papers on the subject include that of Chen (1984b) who analysed how nonlinearities in the time-of-swing method might contribute to the systematic error in the uncertainty of G. His analysis omitted the effects of damping, but presumed that the attracting masses were of arbitrary shape. Luther *et al* (1984) described the details of their angle-sensing autocollimators which had resolutions of less than 10^{-3} arcsec, while Goldblum *et al* (1988) discussed a fast Fourier transform technique they used to determine the period of their torsion pendulum (Ritter *et al* 1990). Zhang and Newman (1992) proposed a torsion balance design in which appropriate positioning of the attracting masses would make the overall energy balance of the system produce an effective torsion constant of the suspension fibre that could be made arbitrarily small. Finally, a number of new developments in beam balance technology as applied to the measurement of G have also been reported, including the work of Moore *et al* (1988a) and Hubler *et al* (1995), which was described above. The details of the flexure-strip beam balance designed and built at the International Bureau of Weights and Measures, and used in various gravity experiments there, have appeared in print elsewhere (Quinn *et al* 1986/87).

4.8. Field sources

Virtually all of the determinations of G that have ever been either carried out or proposed call for observation of the motion of a test mass (or masses) in the field of a source mass (or masses). A thorough knowledge of the dimensions, absolute mass and density distribution of the source mass is typically needed in order to generate an accurate model of its gravitational field. The sphere is an obvious candidate for the source mass geometry because its field is ideally that of a simple point source. The difficulty of manufacturing large dense spheres that can produce relatively strong source fields led to the consideration of cylindrical geometries, with one of the premiere examples being the right circular cylinders used as attracting masses by Heyl (1930) and Heyl and Chrzanowski (1942). The almost legendary analytical calculation of the gravitational attraction of a finite cylinder provided in those papers contains a host of terms, but even so, the convergency of the expressions was studied carefully. Hulett (1969) and Koldewyn (1976), working with Faller, introduced the open-bore cylinder, which has a rather broad, flat maximum in its axial field, thus reducing the stringency with which the spacing between the source and test masses must be measured. Cook and Parker (1962) conceived the use of composite cylinders as source masses. In their scheme, the large cylindrical structures would be designed and assembled in such a way that all of the even zonal harmonics of the field expansion vanish, leaving a field equivalent to that of a point source (as if produced by a massive sphere). A number of other such innovations have been introduced since these early efforts, and each subsequent determination of G has had its own particular approach to the selection of source masses.

As discussed in several of the previous subsections, substantial work has also been done on the development of methods for optimizing the spatial distribution of the masses in torsion balance experiments. The goal is typically one of either maximizing the gravitational torque on the suspended beam in experiments where the test masses must move, or nulling the torque in those cases where the fibre is not supposed to twist. In either case, the analysis of the gravitational interaction between the test and source masses in terms of their multipole moment couplings (e.g., Ritter *et al* 1976, Adelberger *et al* 1990, Newman 1995a, Gundlach *et al* 1996) was a significant step forward in that it provided a well understood formalism for working out the mathematical structure of these problems. Other approaches to such optimizations have been used as well. Speake (1987b), for instance, found an alternative 'near' position for the masses in a time-of-swing torsion-pendulum measurement of G by a geometric analysis of the dependence of the gravitational torque constants on the other parameters of the apparatus. Also, Winkler and Goldblum (1992) used a Monte Carlo numerical integration technique to optimize the aspect ratio of cylindrically-shaped test masses and to determine an orientation of them that would maximize the sensitivity of a torsion pendulum measurement.

There is a vast literature on the calculation of the gravitational potentials, fields and forces produced by masses of various geometries. As an information resource to those interested in the measurement of G and related topics, citations to a cross section of the calculations that are available have been listed in table 4. While not meant to be a comprehensive catalogue of the derivations and/or numerical computations that have been done to date, these entries might at least serve as a useful starting point for those seeking to analyse the field of a particular mass distribution.

5. Searches for variations in G

5.1. Spatial dependence of G

Searches for a change in G with intermass spacing have constituted a compelling quest in laboratory gravitation, especially during the past 25 years. The motivations for carrying out this kind of study were originally empirical, with the results of various benchtop experiments being interpreted in terms of either a value for or limit on some distance-dependent form of the gravitational constant (i.e. a $G(r)$ effect), or in terms of a breakdown in the inverse square law (i.e. a modification to it of the form $1/r^{2+\delta}$, where δ is the departure parameter). Then, in the 1980s, observations that seemingly revealed evidence for non-Newtonian gravity at larger distance scales (Stacey *et al* 1987) fuelled much additional interest in this line of work. The contemporaneous suggestion by Fischbach *et al* (1986) that there may be previously undiscovered, weak, long-range forces in nature provided further impetus for investigating the composition- and distance-dependence of gravity, since the presence of any such effect might reveal the existence of a new force. During this time, a theoretical framework for admitting non-Newtonian effects into discussions of the experimental results was emerging. It led to the practice of using the laboratory data to set limits on the size of the strength-range parameters in a Yukawa term added onto the Newtonian potential, and this has become a standard method for intercomparing the results of this class of experiments. Even though convincing evidence in favour of such new weak forces was never found, the many resulting experiments, when viewed as tests of the universality of free-fall, did much to improve the experimental underpinnings of the weak equivalence principle (WEP) of general relativity. In fact, searches for departures from the inverse square behaviour of Newtonian gravity have now come to be interpreted as attempts to uncover violations of the WEP. Reviews of the modern physical motivations for studying the distance dependence of gravity and discussions of the many different pertinent experiments have been written by Fischbach *et al* (1988), Adelberger (1990a), Adelberger *et al* (1991), Fischbach and Talmadge (1992) and Adelberger (1994). Given the comprehensive nature of these articles, the discussion in the remainder of this

Table 4. A listing of some calculations of the gravitational potentials, fields or forces produced by source masses of various geometries.

Reference/date	Source mass geometry	Calculation
Duska (1958)	Right circular cone, inverted right circular cone and frustrums of cones	Gravitational attraction
Duska (1958)	Spherical caps	Gravitational attraction
Duska (1958)	Oblate spheroid	Gravitational attraction
Parasnis (1961)	Circular lamina	Gravitational attraction at all points in space
Chandrasekhar and Lebovitz (1962)	Homogeneous ellipsoids	Potentials and superpotentials
Aguilar Sahagun (1965)	Rod rotating about an axis parallel to itself	Gravitational field
Nagy (1966)	Right rectangular prism	Gravitational attraction
Rao and Radhakrishnamurthy (1966)	Horizontal circular plate	Gravitational attraction
Holshevnikov (1968)	Body of arbitrary shape	Coefficients of tesseral harmonics
Levie (1971)	Non-homogeneous oblate spheroid	Potential expansion
Long and Ogden (1974)	Two coaxial thin rings	Axial force produced by superposition of the fields
Qureshi (1976)	Cylindrical sections	Gravitational field
Banerjee and Das Gupta (1977)	Rectangular parallelepiped	Gravitational attraction
Singh (1977a)	Circular disc	Vertical component of the gravitational attraction at an arbitrary point
Singh (1977b)	Right circular cylinder	Vertical component of the gravitational attraction at an arbitrary point
Chen (1982)	Long hollow cylinder of finite length	Force on a particle inside the cylinder
Cook and Chen (1982)	Finite right circular cylinder	Radial force on a particle at an arbitrary position
Chen and Wang (1983)	Finite right circular cylinder	Axial force on a particle at an arbitrary position
Metherell *et al* (1984)	Rectangular slabs	Gravitational field of rectangular slabs with rectangular holes in them
Metherell and Quinn (1986)	111 Tetrahedron	Gravitational field
Carré *et al* (1986)	111 Tetrahedron	Behaviour of the field near the apices and vertices
Chang (1988)	Finite right circular cylinder	Radial and axial forces of attraction on a particle of unit mass
Francisco and Matsas (1989)	Infinite straight string of uniform mass density	Gravitational field and force on a particle
Chen and Cook (1989)	Thick rings	Gravitational field and axial saddle points

subsection is therefore limited to descriptions of a few recent measurements and some special experimental situations.

The historical paucity of data for determining G at intermass spacings of roughly 1 to 100 m or more (except, in part, for the very old plumb-line and Airy-type experiments) was addressed in the 1970s by workers who foresaw the possibilities of making measurements at pumped-storage reservoirs and using marine gravity surveys to determine G and search for a $G(r)$ effect at these relatively large distances. As we have seen in section 4, a number of interesting measurements of G were carried out by these methods. Some of the more recently reported studies include that of Müller *et al* (1990) who made gravimetric measurements of the Hornberg reservoir in Germany. Their experiment set a limit on the departure of G from the laboratory value of no more than (0.25 ± 0.4)% over the range 40–70 m, i.e. a result consistent with the laboratory value in this range. At about the same time Hipkin and Steinberger (1990) carried out a similar experiment at the Megget reservoir in Scotland. Data were taken at seven sites within a tower submerged in this reservoir and at 14 sites on the embankment. Although the residuals of the measurement had a scatter that implied that a better model of the density distribution of the tower's structure was needed, the principle of the method they employed suggested that a precision for G of $1 : 10^3$ should be possible over distances on the order of 300 m. Also in the UK, Edge and Oldham (1990) made gravimetric measurements at the Marchlyn Mawr reservoir in North Wales. Water seepage in the rock underlying the gravimeter introduced substantial systematic errors, however, so the authors carried out a new experiment at the Stwlan reservoir of the Ffestiniog power station, also in North Wales, where the subsurface seepage was not deemed to be problematic (Oldham *et al* 1993). Their data were reduced to yield values of G at effective distances of 26, 36 and 94 m. The reported departures from the laboratory value of G were (-0.13 ± 0.22), (-0.03 ± 0.22) and (0.46 ± 0.53), respectively, all consistent with the CODATA value within the errors quoted. An experiment similar to these has been planned by Achilli *et al* (1990, 1991) for the Lago Brasimone in the Italian Apennines, with the new feature in this case being the proposed use of a superconducting gravimeter.

Other recent experimental searches for a breakdown in Newtonian gravity at large distances include a second set of tower gravity measurements made by Romaides *et al* (1994). Their data, taken at five points over a nearly 500 m vertical rise, reconfirmed the exactness of the inverse square law. A similar result over a vertical distance of approximately 320 m was obtained at a meteorological tower in China by Liu *et al* (1992). This same group had also made gravimetric measurements on a large cylindrical oil tank and found the resulting value of G to be within ± 0.6% and ± 1.4% of the laboratory value at interaction distances of 30 and 60 m, respectively (Yang *et al* 1991). (Proposals for investigating $G(r)$ at *very* large distances, i.e. from terrestrial to planetary scales, have been advanced by Kislik (1983), Collins *et al* (1990) and others who have discussed the possibility of ranging to spacecraft and objects in the solar system for this purpose.)

Work also continued on dynamical laboratory techniques for verifying the inverse square law by using moveable spinning rotors to excite gravity wave detectors. These experiments typically tested the $1/r^2$ dependence of the gravitational force at shorter distance scales, approximately 1 to 10 m (i.e. the 'intermediate' range). Ogawa *et al* (1989), for example, continued the earlier work that had been undertaken originally at the University of Tokyo and used an 18 kg rotor to excite a 100 kg cryogenic antenna. In a preliminary experiment, they set a limit of 0.02 ± 0.04 on the value of the departure parameter δ (defined earlier in this section) over the range 1–2 m. Astone *et al* (1991) worked with the much larger (2270 kg) 'Explorer' antenna in Italy and found that they were able to calibrate its sensitivity by monitoring the antenna's response to the ac gravitational field produced by a 8.75 kg rotor spinning at a distance of about 3.5 m from it. While the vibrations induced in the antenna had an amplitude of about 20 times that of the Brownian motion, the authors noted that a rotor roughly 100 times bigger would be needed to quantitatively investigate the distance dependence of the gravitational force. Other intermediate-range laboratory tests of the distance dependence of gravity that have been discussed in the literature of the last few years include superconducting gravimeter experiments (Goodkind *et al* 1993), three-axis gravity gradiometer experiments (Moody and Paik 1993, Paik and Moody 1994), and torsion pendulum experiments (Moore and Boynton 1992, Moore *et al* 1993, Kolosnitsyn 1993b). The proposals of Moore and colleagues would seek to evaluate the inverse square law over the range 100 cm to 100 m.

Table 5. Some examples (including proposals) of measurements of the gravitational interaction at small intermass spacings, listed in order of increasing scale of distance.

Reference/date	Method/apparatus	Range
Zhang (1989)	Torsion balance suspending rod-shaped test masses inside of cylindrical cavities (at shortest distance)	50 μm $< r <$ 1 mm
Price (1988)	High-Q parallel-plate oscillator operating at cryogenic temperatures	100 μm
Zhang (1988a)	Torsion balance suspending rod-shaped test masses between pairs of similar rod-shaped source masses	500 μm $< r <$ 1 mm
Zhang (1988b)	Torsion balance suspending one test mass between rotatable, unequally-sized source masses	2.5 mm $< r <$ 10 mm
Oelfke (1984a, b)	Vacuum torsion balance using disk-shaped masses and capacitive feedback	3 mm $< r <$ 50 mm
Mio *et al* (1984, 1987)	Modified torsional-resonance gravity wave antenna driven by rotating cylindrical masses	7.1 mm
Feng and Zhang (1988)	Torsion balance suspended within a hollow cylinder, with pairs of small spherical attracting masses at either end	10 mm
Mackenzie (1895)	Torsion balance with a quartz suspension fibre and position-adjustable attracting masses	37 mm $< r <$ 74 mm

Measurements of G and investigations of the inverse square law at very small intermass spacings have also been largely lacking until quite recently. The last decade, however, has seen a number of interesting proposals put forth for experimental studies of the gravitational force at the millimetre and even submillimetre scales of distance, with apparatus subsequently designed, built and used in a few of these cases. Table 5 lists several of the relevant efforts and denotes the interaction range in each case. The rather modest densities of bulk terrestrial materials and the presence of strong intersurface (e.g., electrostatic and van der Waals) forces between bodies in close proximity place practical limits on the size, shape and relative nearness of the interacting masses used in experiments of this type.

5.2. Temporal constancy of G

Perhaps the most intriguing question about G is that of whether or not it is truly a constant at all, or if instead its value might be changing slowly with time. The question is a fundamental one and it has been the focus of much thought over the last several decades. The well known

Table 6. Laboratory- and satellite-based measurements of $\dot{G}/G$: proposals and experiments.

Reference/date	Method/apparatus	Results/comments
Hoffmann (1962)	High-Q quartz pendulum gravimeter operating at ≈ 27.9 Hz	$\dot{G}/G \leqslant 4 \times 10^{-8}$ per year
Currott (1965)	High-Q quartz pendulum gravimeter operating at ≈ 4.9 Hz	$\dot{G}/G \leqslant 6.2 \times 10^{-7}$ per year (estimated from pendulum decrement)
Weiss and Block (1965)	LaCoste-style gravimeter in high vacuum with electrostatic servo system	$\dot{G}/G \leqslant 3.6 \times 10^{-6}$ per year (estimated from the gravimetry data)
Weiss (1965)	Inverted Kelvin absolute electrometer used as a precision gravimeter	Test mass position sensed interferometrically
Wilk (1971)	Several proposed space-based instrumentation systems for measuring $\dot{G}/G$	Target accuracy of $\dot{G}/G \approx 1 \times 10^{-10}$ per year
Groten and Thyssen-Bornemisza (1972)	Modified-Beams version of a Cavendish balance in a spacecraft having a highly eccentric orbit	Target accuracy of $\dot{G}/G \approx 1 \times 10^{-11}$ per year
Beams (1973)	Rotating Cavendish balance kept at $1\,^\circ$K and superconductively shielded	Target accuracy of $\dot{G}/G \approx 1 \times 10^{-10}$ per year
Braginsky and Ginzburg (1974)	Low-temperature pendulum oscillator (method 1) or high-stability superconducting gravimetry (method 2)	Requires that $\dot{G}/G = \dot{g}/g$ (as in other gravimeter experiments)
Ritter *et al* (1976) Ritter and Beams (1978)	Symmetrized Cavendish balance rotated at constant speed with feedback between the large and small mass systems	Target accuracy of $\dot{G}/G \approx 1 \times 10^{-11}$ per year
Braginsky *et al* (1977)	Cryogenic torsion-balance oscillator made of single-crystal sapphire	Target accuracy of $\dot{G}/G \approx 1 \times 10^{-12}$ per month
Halpern and Long (1978)	Piezoelectric force gauge compared to Josephson effect voltage, and magnetic spring using persistent currents	Target accuracy of $\dot{G}/G \approx 1 \times 10^{-10}$ per year and 1×10^{-11} per year, respectively
Dannehold (1982)	Frequency shift of an iodine-stabilized laser caused by change in the length of the cavity due to variation of gravity	Target accuracy of $\dot{G}/G \leqslant 4 \times 10^{-12}$ per year
Hellings (1988) Anderson *et al* (1989)	Proposed Doppler and range tracking of Phobos landers (5 year observation time)	Target accuracy of $\dot{G}/G \approx 1.6 \times 10^{-12}$ per year
Sanders and Deeds (1992)	Project SEE (satellite energy exchange) using test masses moving in Darwin orbits to measure change in $M_\oplus G$	Target accuracy of $\dot{G}/G < 10^{-12}$ per year
Damour and Esposito-Farèse (1994)	Proposed orbital test of relativistic gravity using artificial satellites in Earth orbit	Target accuracy of $\dot{G}/G \sim 10^{-13}$ per year
Gong (1996)	Proposed modification to Hughes–Drever experiment, using NMR carried out on the ^{3}He nucleus	Target accuracy of $\lvert\dot{G}/G\rvert \leqslant 1.1 \times 10^{-15}$ per year

'large numbers hypothesis' (LNH) of Dirac opened the door to some insightful speculation in this area by offering a tie-point between the secularly increasing age of the universe and a possible concomitant secular decrease in the value of G. His introduction of two metrics, one for dealing with atomic phenomenon and the other for mechanical (i.e. gravitational) processes, created a framework in which variations of G and the possibility of spontaneous matter creation would seem not unreasonably related to the rest of physics, although both G and the sizes of rest masses are unchanging within general relativity. A host of subsequent studies has examined the theoretical ramifications of the LNH, and much of the experimental and observational interest in searching for a possible $\dot{G}/G$ (thought to be on the order of 10^{-11} per year) in the 1970s and early 1980s was likewise motivated by this interesting conjecture.

Scalar-tensor theories of gravity also predicted a change in the value of G; in this case one that would have a secular component as well as a modulation of the nominal value of G as determined on the surface of the Earth during the course of the annual orbit around the Sun. Extensive laboratory testing of possible experimental approaches to this problem were carried out at Princeton University in the 1960s (Hoffmann 1962, Currott 1965, Weiss and Block 1965), and the designs of several potential space-based investigations were discussed and analysed at MIT and elsewhere (Wilk 1971, Groten and Thyssen-Bornemisza 1972). More recently, the search for a 'theory of everything' (TOE) in which all of the fields are unified has led to predictions of $\dot{G}/G$ that derive from the compactification of extra dimensions in the universe. The measurement of $\dot{G}/G$ is thus a significant test of unification theories and the stringency with which the size of any such effect can be delimited by either laboratory experiment or astronomical/astrophysical observation serves to identify progress in the development of the theoretical predictions.

The status of the theoretical framework for understanding $\dot{G}/G$, the story of its evolution and the results of the then-contemporary measurements of it were reviewed by Wesson in several publications dating to *circa* 1980 (Wesson 1978, 1980a, b, 1981), and also by Ritter (1982), Narlikar (1983) and Coley (1985). The limits on the size of the effect had been set by lunar and solar-system ranging studies, and by a variety of astrophysical and geophysical phenomenologies. In a somewhat later paper, Hellings (1988) surveyed the more modern theoretical motivations for $\dot{G}/G$, discussed the results of the most recent determination of it (which was $\dot{G}/G = (0.2 \pm 0.4) \times 10^{-11}$ per year at the time), and described plans for some possible future studies. A comprehensive bibliography to the literature on determinations of $\dot{G}/G$ was also made available at that point by Gillies (1987).

The fundamental nature of the question, the challenge of measuring such a small effect and the potential need to separate the 'G-dot' from any 'm-dot' effects prompted several workers to consider the possibility of carrying out laboratory-based searches. Table 6 lists several such studies that were either proposed or undertaken, with some of the latter brought to very high stages of technology development. While no laboratory test to date has succeeded in measuring $\dot{G}/G$ at a cosmologically interesting level (e.g., relative to the predictions of the LNH), there may still be a possibility to do so with a space-based experiment (Sanders and Deeds 1992, Damour and Esposito-Farèse 1994).

Similar motivations have animated a number of other workers, of course, and table 7 lists many of the astronomical, astrophysical and phenomenological determinations of $\dot{G}/G$ that have been made during the last ten or so years. (Also included are the results of various other investigations that were not attempts at measuring $\dot{G}/G$ *per se*, but were nevertheless efforts at somehow characterizing the behaviour of this quantity.) As seen in this table, several of these studies in fact achieved the goal of testing $\dot{G}/G$ at cosmologically interesting levels. Where available, the uncertainties in the estimates have been included in the results. The preponderance of the evidence points to an effect which, if it exists at all, is indeed very weak. The overall situation with respect to $\dot{G}/G$ has been discussed in detail recently by Melnikov (1994). Comments on the role of $\dot{G}/G$ in theories of gravity with torsion and on the coupling between gravitational self-energy and $\dot{G}/G$ have been published by Rauch (1984) and Nordvedt (1990), respectively. The interesting possibility

Table 7. Several of the astronomical, astrophysical and phenomonological determinations or analyses of $\dot{G}/G$ made during approximately the past ten years.

Reference/date	Technique	Results/comments
Hellings *et al* (1983) Hellings *et al* (1989)	Ranging to Viking landers on Mars via the Deep Space Network (measurements made between 1976 and 1982)	$\dot{G}/G = (0.2 \pm 0.4) \times 10^{-11}$ per year
Burša (1984)	Analysis of the time variation of the angular momentum of the Earth–Moon system in its heliocentric motion	$\dot{G}/G < 5 \times 10^{-17}$ per year
Wu and Wang (1986, 1988)	Theoretical estimation of $\dot{G}/G$ from an assessment of the curvature of potentials in superstring theories	$\dot{G}/G \approx -1 \times 10^{-11\pm1}$ per year
Yabushita (1986)	Analysis of the secular accelerations of the Sun and the Moon	$\dot{G}/G$ ranges from $-(1.4$ to $3.3)$ $\times 10^{-11}$ per year
Aleshkina *et al* (1987)	Radar ranging to inner planets (A) and analysis of observations of lunar eclipses (B)	$\dot{G}/G = (3.7 \pm 0.8) \times 10^{-11}$ (A) $\dot{G}/G = (-0.5 \pm 0.5) \times 10^{-11}$ (B) per year, respectively
Barr and Mohapatra (1988)[a]	Theoretical investigation of cosmological evolution of the coupling constants due to a change in a dilaton field	Change in the constants implies long range forces that violate equivalence principle
Damour *et al* (1988)	Timing of the binary pulsar PSR 1913 + 16	$\dot{G}/G = (1.0 \pm 2.3) \times 10^{-11}$ per year
Rosen (1988)	Introduction of a background metric of constant curvature into the Weyl–Dirac theory of gravity	Effective gravitational constant, G_e, predicted to vary at the rate $\dot{G}_e/G_e = -5.5 \times 10^{-12}$ per year
Soldano (1988)	Theoretical estimate of $\dot{G}/G$ that arises within an expression for the terrestrial range dependency of G	$\dot{G}/G = 0.218 \times 10^{-11}$ per year
Taylor and Weisberg (1989)	Timing of the binar pulsar PSR 1913 + 16	$\dot{G}/G = (1.2 \pm 1.3) \times 10^{-11}$ per year
Abdel-Rahman (1990)	Evaluation of a critical density cosmology in which G increases in time	Model lets $G = \alpha t^2$, which leads to agreement with estimated age of universe
Accetta *et al* (1990)	Revision of primordial nucleosynthesis model by including new neutron half-life data and reaction rate uncertainties	$\dot{G}/G < \pm 9 \times 10^{-13}$ per year (at present times)
Damour (1990) Damour and Taylor (1991)	Timing of the binary pulsar PSR 1913 + 16	$\dot{G}/G = (1.10 \pm 1.07) \times 10^{-11}$ per year
Damour *et al* (1990)	Evaluation of a JBD model where the scalar couples differently to visible and dark matter	Observed $\dot{G}/G$ is compatible with a massless scalar field coupling to dark matter
Goldman (1990)	Timing of the binary pulsar PSR 0655 + 64	$\lvert\dot{G}/G\rvert \leqslant (2.2$ to $5.5) \times 10^{-11}$ per year
Liu and Wang (1990)	Numerical solution of ten-dimensional Einstein equations in superstring theories	Range of $\dot{G}/G$ estimated to be -1×10^{-11} to -6×10^{-12} per year
Sisterna and Vucetich (1990)	Analysis of several categories of observational evidence within the framework of the standard model	Bounds on $\dot{G}/G$ are established relative to the variation of other constants

Table 7. (Continued)

Reference/date	Technique	Results/comments
Soleng (1991)	Generalization of Einstein–Cartan theory allowing dilaton currents that couple algebraically to the torsion trace	Raidal variation of G within massive objects predicted; $\Delta G/G \approx 10^{-10}$ for Earth
Wang (1991)	Analysis of the Iben standard stellar evolution model yielding a 1% radius change and a 3% luminosity change	$\lvert\dot{G}/G\rvert < 10^{-12}$ per year
Anderson *et al* (1992)	Ranging to Mercury and Venus via Goldstone Facility of the Deep Space Network and spacecraft	Null result with a standard error of $\pm 2.0 \times 10^{-12}$ per year
Gasanalizade (1992b,1994, 1995)	Evaluation of Earth's perihelion advance and solar gravitational red shift at aphelion (a) and perihelion (p) of Earth	$(\Delta G_{a-p}/G_0) = 1.561\,16 \times 10^{-4}$
Chandler *et al* (1993)	Combination of spacecraft tracking, planetary ranging, lunar laser ranging and VLBI observations	$\dot{G}/G = (0 \pm 1) \times 10^{-11}$ per year
Damour and Nortvedt (1993)	Examination of the natural attractor mechanism in tensor-scalar theories of gravity	$\lvert\dot{G}/G\rvert \sim (4\beta - \gamma - 3)H_0$ where $\beta - 1$ and $\gamma - 1$ are PPN parameters
Fortini *et al* (1993)	Analysis of heavy element production in stars	$\lvert\dot{G}/G\rvert \leqslant 2 \times 10^{-12}$ per year
Goldman (1993)	Timing of the binar pulsar PSR 0655 + 64	$\lvert\dot{G}/G\rvert \leqslant (1.1$ to $4.4) \times 10^{-11}$ per year
Gundlach and Damour (1993)	Evaluation of variation of the coupling constants by assessment of standard big-bang nucleosynthesis	$\dot{G}/G = (-1.1$ to $4.0) \times 10^{-12}$ per year for a Hubble constant of 100 km s^{-1} Mpc^{-1}
Müller *et al* (1993)	Least-squares fit to lunar laser ranging data taken between 1969 and 1990	$\dot{G}/G = (-1 \pm 4) \times 10^{-13}$ per year
Pitjeva (1993)	Reanalysis of radar ranging data to inner planets, taking into account the topography of Mercury	$\dot{G}/G = (0.47 \pm 0.47) \times 10^{-11}$ per year
Demarque *et al* (1994)	Observations of solar p-mode spectrum compared against solar models in which G varies in time	For $G(t) \propto t^{-\beta}$, only $-0.4 \leqslant \beta \leqslant 0.4$ is allowed over the last 4.5×10^9 years
Dickey *et al* (1994)	Lunar laser ranging to corner-cube retroreflectors on the Moon	$\lvert\dot{G}/G\rvert < 1 \times 10^{-11}$ per year
Kaspi *et al* (1994)	Timing of pulsar PSR 1855 + 09	$\dot{G}/G = (-9 \pm 18) \times 10^{-12}$ per year
Massa (1994)	Evaluation of Einstein–Hilbert action term that includes variable-gravitational coupling	Resolves LNH rate ($G \propto t^{-1}$) with null observations by invoking matter creation
García-Berro *et al* (1995)	Luminosty of white dwarf stars against time-varying G; stratified models (A), constant distribution of carbon and oxygen (B)	$\dot{G}/G \leqslant -(1 \pm 1) \times 10^{-11}$ year^{-1} $\dot{G}/G \leqslant -(3^{+1}_{-3}) \times 10^{-11}$ year^{-1} for (A) and (B), respectively
Guenther *et al* (1995)	Observations of solar g-mode spectrum compared against solar models in which G varies in times	$\lvert\dot{G}/GH\rvert \lesssim 0.05$, where H is the Hubble constant
Ma (1995)	Modification of the large numbers hypothesis wherein $G \propto H^{2/(4+3\omega)}$, with H being the Hubble constant	$\dot{G}/G = -31H/320$

Table 7. (Continued)

Reference/date	Technique	Results/comments
Massa (1995)	Examines 'maximal power hypothesis' within Einstein–Cartan theory of gravity	Finds $\dot{G} > 0$ leads to an age of the universe consistent with observation
Williams *et al* (1996)	Lunar laser ranging to corner-cube retroreflectors on the Moon	$\lvert\dot{G}/G\rvert < 8 \times 10^{-12}$ per year
Thorsett (1996)	Evaluation of the Chandrasekhar mass limit for neutron stars in binary pulsars at (a) 68% and (b) 95% confidence limits	$\dot{G}/G = (-0.6 \pm 2.0) \times 10^{-12}$ and $\dot{G}/G = (-0.6 \pm 4.2) \times 10^{-12}$ per year for (a) and (b), respectively

[a] This is one example of several articles that discuss the possible time variation of the fundamental constants from the point of view of theory, and superstring theories in particular. Another example is the paper by Maeda (1988).

of temporal oscillations in the size of G (as produced, perhaps, by an inflationary expansion of the universe) has been discussed by Hill *et al* (1990), Accetta and Steinhardt (1991), Morikawa (1991) and Steinhardt and Will (1995).

5.3. Temperature-dependent gravity, gravitational absorption and other anomalous effects

Many of the laboratory investigations carried out on the force of gravity have been devoted to the search for a dependence of G on either a physical or chemical property of the masses under study, or to the test of possible couplings between gravity and one of the other fundamental forces of nature. The early part of the 20th century saw the publication of results from several experiments aimed at detecting a sensitivity of G to the electrification or magnetization of matter, searches for a preferred directive action of crystalline materials (tests of the local isotropy of gravity), studies of the gravitational strength of radioactive substances and inquiries into the possible temperature dependence of the gravitational force (Gillies and Ritter 1984, Gillies 1987). In spite of some initial controversies, all of these experiments were ultimately interpreted as producing null results, thus confirming the universality and fundamental character of G and the force of gravity.

Interest in a temperature dependence of G has persisted to recent times, however, stimulated mostly by conjectures that gauge-theory models predict such a phenomena. Linde (1980) produced one such model which also called for the sign of the effective gravitational constant to be reversed in the early universe, a scenario which would have resulted in a repulsive gravitational force ('antigravity') in that epoch. The general form of the temperature dependence of G in this class of models is

$$G = G_0(1 - \alpha G_0 T^2)^{-1}$$

where G_0 is the value of G at zero temperature (essentially that of the present laboratory value), T is temperature and α is a number derived from the values of the coupling constants in the gauge theory under consideration. In one indirect test of this model, Davies (1981) examined the temperature dependence of G within the context of black hole thermodynamics and found that this allowed him to use the second law of thermodynamics to put constraints on non-zero-temperature gauge theories. Massa (1989a) examined the physics of the critical temperature, T_C, that can be derived from this class of predictions for $G(T)$

$$T_{\mathrm{C}} = (\alpha G_0)^{-1/2}$$

and found that T_{C} is a universal upper bound on temperature and that it cannot be reached by physical processes. Massa also points out that a possible value for α is approximately

10^{-20} s^2 kg K^{-2} cm^{-3}, the ratio of the electromagnetic to gravitational coupling constants. A related paper (Massa 1989b), examines a generalized form of the Oppenheimer–Volkoff equation for hydrostatic equilibrium that incorporates the model for $G(T)$ given above. The result in this case, too, was a prediction of an upper bound on the temperature of physical processes. In a much different regime of physics, Assis and Clemente (1993) use the Dulong–Petit rule for specific heats and the Einstein mass–energy relationship to modify the Newtonian inverse square law so that they can estimate the fractional change in gravitational force per degree of change in temperature of the test masses. Their model leads to a prediction of $\delta G(T)/G = 10^{-14}$ K^{-1}, which is only a few orders of magnitude away from the level of sensitivity achieved by the highest-performance laboratory test to date: the beam-balance experiment of Poynting and Phillips (1905), which was able to place a limit of $\delta G(T)/G = 10^{-10}$ K^{-1} on the size of any such effect. In spite of these modern motivations for the possible existence of temperature-dependent gravity, there has been very little laboratory activity aimed at studying such phenomenon since 1925. Virtually all of the experiments relevant to this topic were carried out before then (Gillies 1987, 1988).

Table 8. Representative examples of the searches for a gravitational absorption effect that have been carried out over recent decades (λ is the Majorana absorption parameter, as defined in the text).

Reference/date	Technique	Results/comments
Harrison (1963)	Analysis of gravity-tide observations near the equator	$\lambda \leqslant 1 \times 10^{-13}$ cm^2 g^{-1}
Slichter *et al* (1965)	Gravity observations during the solar eclipse of February 15, 1961	$\lambda \leqslant 8.3 \times 10^{-16}$ cm^2 g^{-1} at the 50% confidence level
Braginsky and Martynov (1968)[a]	Torsion balance with externally driven chopper body revolving between the suspended and attracted masses	$\lambda \leqslant 2.8 \times 10^{-13}$ cm^2 g^{-1}
Crowley *et al* (1974)	Analysis of geothermal data to reveal any thermalization of attenuated gravitational fields	$\lambda \leqslant 1.9 \times 10^{-30}$ cm^2 g^{-1}
Steenbeck and Treder (1982)	Observations of the motions of nested spheres on board an artificial satellite (proposed experiment)	Target sensitivity expected: $\lambda \ll 1.0 \times 10^{-15}$ cm^2 g^{-1}
Liakhovets (1986a, b)	Gravimetric measurements carried out inside a large building and deep underground	$\lambda = 3.8 \times 10^{-12}$ and $\lambda = 2.8 \times 10^{-12}$ cm^2 g^{-1}, respectively
Šimon and Kostelecký (1988)	Spectral analysis of superconducting gravimeter data taken at the Brussels gravimetry station	$\lambda \leqslant 1.0 \times 10^{-15}$ cm^2 g^{-1}
Eckhardt (1990)	Re-interpretation of lunar laser ranging data	$\lambda \leqslant 1.0 \times 10^{-21}$ cm^2 g^{-1}

[a] The authors note that the probability of *not* detecting a value of λ at this level is approximately 2×10^{-37}.

It has been a rather different story with respect to searches for gravitational absorption, however. In the 19th and early parts of the 20th centuries, several workers built torsion balances in which the circumferential gap between the suspended and attracting masses was made large enough to accommodate the presence of cylindrical (or other) shields constructed from materials of different densities. G would be measured in the usual way, first with a shield of one material in place and then with others, to see if the presence of an intervening medium somehow changed the strength of the gravitational interaction between the suspended and attracting bodies. A different approach to this problem was developed in the 1920s by Quirino Majorana, who began a long series of investigations with a high-sensitivity beam balance in which one of the suspended masses could be alternately shielded and then unshielded by dense materials such as mercury and lead. His goal was to test the hypothesis that there was a progressive absorption of the gravitational force acting between two bodies when a material medium screened one of them from the other. In essence, this was a search for the gravitational equivalent of magnetic permeability, and he defined a screening constant, λ, sometimes called the extinction coefficient, to be the physical measure of the medium's screening ability. The specific model he developed allowed the gravitational force F between two bodies having masses m and M and separated by a distance R to vary as a function of the density ρ and thickness L of the screen between them such that

$$F = GmMR^{-2}\exp\left[-\int \lambda\rho \, \mathrm{d}L\right].$$

Majorana concluded that the measurements made with the beam balance apparatus demonstrated that $\lambda = 6.7 \times 10^{-12}$ cm^2 g^{-1} for mercury and $\lambda = 2.8 \times 10^{-12}$ cm^2 g^{-1} for lead (Majorana 1920, 1930). His claim of finding a non-null result created a great deal of interest in the scientific community at the time and, in spite of many subsequent measurements finding no such effect, there has been active interest in this question down to present times. The more recent studies have included both experimental and observational efforts, and table 8 provides a representative listing of several of the investigations that have been undertaken since the 1960s. Interest in gravitational absorption has remained active for a number of reasons. First, others besides Majorana have claimed to observe the effect. Liakhovets (1986a, b), for instance, carried out gravimetry experiments that yielded results he found to be consistent with those of Majorana. Also, claims of gravitational anomalies observed during eclipses (see below) have been made from time to time, and gravitational absorption has sometimes been appealed to as a possible cause of or contributing factor to such effects (the papers by Adămuţi (1976, 1982) discuss this particular topic in detail). Moreover, there are methods of predicting that a gravitational equivalent of magnetic permeability should exist, albeit at a virtually unobservable level. Forward (1961), for instance, noted that a general relativistic analogue of magnetic permeability could be identified, and that it had the extremely small value $(16\pi G/c^2) \approx 3.7 \times 10^{-26}$ m kg^{-1}. Slabkii (1966) also studied the electromagnetic analogy and developed a version of it that included induction and gravimagnetic effects. Weber (1966) pointed out that quasistatic shielding could be predicted from a general relativistic analysis of tidal phenomenon, but he too noted that it was a very weak effect. Steenbeck and Treder (1982) conjectured that the size of the Majorana extinction coefficient might be $[(hG/c^3)(h/m_N^2c)] \approx 10^{-22}$ cm^2 g^{-1}, where h is Planck's constant, m_N is the mass of a nucleon and c is the speed of light, and De Sabbata and Sivaram (1991) discussed the theoretical motivations for gravitational shielding within the context of torsion in gravity. Some philosophical issues underlying the role that gravitational absorption might play in creating an understanding of the dimension of time have even been considered (Kozyrev 1971). Finally, as a practical issue, the existence of a non-null absorption effect would call for the modification of Kepler's third law, with consequences (at some level) for our understanding of orbital motions due to a central force. Thus, it is perhaps not surprising that measurements of λ have remained ongoing throughout the 20th century. Thorough historical surveys of the search for gravitational absorption in the early 1900s have been prepared by de Andrade Martins (1995a, b). Steenbeck and Treder (1984) have also documented the early history of this field and have provided commentary on some of the more recent studies, as well. (It is interesting to note that modern tests of the superposition of gravitational fields, for example Goodkind *et al* (1993), search for essentially the same type of phenomenon as would be sought in a gravitational absorption experiment, but within a different interpretive context.)

The weakness of the gravitational force makes it difficult to characterize all aspects of the interaction with great certainty. Therefore, the door must always be left open to the possibility that new gravitational effects may be discovered by future experiments or revealed from re-analysis of old ones. Partly because of this, the last few decades have seen a variety of claims advanced about the possible anomalous behaviour of gravity as observed during the course of a number of different kinds of laboratory investigations. As mentioned above, some of these have involved experiments carried out during solar eclipses. Recent examples of apparatus built especially for this purpose include the high-Q torsion pendulum of Chen *et al* (1982, 1983) and the sensitive paraconical pendulum of Savrov (1989), both of which were designed to search for anomalies of the type reported by Allais (1959a, b) and Saxl and Allen (1971) during the course of solar eclipses. Kuusela (1991), too, built a torsion pendulum for this purpose and made measurements of the pendulum's period during the solar eclipse of July 22, 1990. The data showed, however, that there were no significant variations in the pendulum's period during that eclipse. In fact the change in the period was less than 4.3×10^{-6} of its baseline value. (See the paper by Caputo (1977) for details of some of the older studies done during eclipses.)

Another recently purported gravitational anomaly was that of Hayasaka and Takeuchi (1989), who described an experiment in which a gyroscope was weighed while spinning and exhibited a decrease in weight during rotation with the spin vector oriented downwards. The effect was proportional to spin speed. The authors found no systematic effects in their experiment that could explain the anomaly, which was clearly outside of the predictions of standard spin-related gravitational effects such as the Lense–Thirring precession, the Einstein–Cartan spin–spin coupling, etc. Their finding was the subject of much discussion in the literature (Maddox 1990, Salter 1990, Baker 1990, Watson 1990, Tallarida 1990, Adelberger 1990b, Harvey 1990), but subsequent experimental tests of the effect were not able to replicate it (Faller *et al* 1990, Quinn and Picard 1990, Nitschke and Wilmarth 1990). The experiment of Quinn and Picard, for instance, put a limit of only approximately 60 μg on the mass change of the rotor used in their experiment, an amount equivalent to 2×10^{-7} of its nominal mass of 0.33 kg.

A couple of other anomalous effects have also been reported recently. The first, which remains largely undiscussed in the literature, is the unexplained observation by Lavrent'ev *et al* (1991) of changes in density and mass of test substances (e.g., a sample of distilled water) placed in the presence of an external, irreversible thermodynamic reaction (e.g., the evaporation of liquid nitrogen). Under certain experimental conditions, changes in mass in the range of approximately 10 ppm were noted for a variety of organic and inorganic materials weighed in sealed flasks suspended from an analytical balance, with measurement errors of ± 0.15 mg. An after-effect was also noted in some cases, wherein the change continued even though the external process presumably driving it had ceased. These workers cast their studies in terms of an exploration of how the inertial properties of matter might be coupled to the thermodynamic characteristics of the environment or milleau in which it is observed. Thus, their experiments might be viewed as a sort of test for a coupling between the second law of thermodynamics and the principle of equivalence. Of course, the size of the claimed effect would seem to counteract the validity of any interpretation that appeals to a statistical-mechanics description of neutral, bulk matter.

Another unusual result was that of Podkletnov and Nieminen (1992), who used a high-sensitivity electronic balance to weigh a 5.5 g sample of silicon dioxide while it was suspended 15 mm above a disk of $YBa_2Cu_3O_{7-x}$, a high-temperature superconductor maintained at temperatures $\lesssim 77\,^\circ$K. The sample was found to loose about 0.05% of its weight when suspended above the static disk and, when the disk was rotated, this loss eventually increased to 0.3% of the sample's weight. The authors suggested that the unusual grain structure induced in the superconducting material by rotation of the disk somehow led to modifications of the internal energy states, with a resulting shift in the balance of the internal electromagnetic, nuclear and gravitational forces thus producing the equivalent of a weak gravitational shielding effect as sensed by weighings of the mass suspended above the disk. The effect reached its maximum strength at temperatures below $40\,^\circ$K, it increased with increasing rotational speed of the superconducting disk and it showed a resonant behaviour when the frequency of the magnetic fields of the suspension and/or rotational drive systems was greater than 10^5 Hz. The authors claimed to have considered and then eliminated several possible alternative sources for their observations, but de Podesta and Bull (1995) have pointed out that the result can be explained simply by applying a buoyancy correction to the measured force. Moreover, Unnikrsihnan (1996) has noted that the data presented by Podkletnov and Nieminen are not internally consistent, and that a static-case experiment he carried out showed no evidence of shielding. As of this writing, however, the original experiment and the unpublished results of subsequent work by the Finnish group remain the subject of much discussion and speculation. This is partly because the analysis of Modanese (1996) found no explanation for the claimed result within the context of the standard theories of gravity, and also because of the recent predictions of novel gravitomagnetic effects that might be associated with superconductors. Definitive laboratory experiments would clearly settle the issue, and might serve as a useful means of exploring some of the predicted gravitomagnetic phenomenon, as well.

Most searches for gravitational anomalies have not typically involved the measurement of G, *per se*. Even so, the results of such experiments, particularly the investigations of gravitational shielding or absorption, are often interpreted in terms of either an aniotropy in or variation of G. As we have seen above, however, the general relativistic predictions of the gravitational analogue of magnetic permeability (which incorporates G) call for it to be an exceedingly weak effect, and this is typical of most such phenomenon that might otherwise give rise to gravitational anomalies. Therefore, while there is little doubt that future generations of ever-more-sensitive experiments will almost certainly reveal interesting things about the gravitational force, any claims to the discovery of relatively strong anomalous effects must be examined with very great care.

6. Discussion and concluding remarks

The measurement of the Newtonian gravitational constant, G, the characterization of its properties, and the testing of the inverse square law of gravity are problems that have been under study for many years. The spread in the values of G obtained by the recent high-precision determinations of it attests to the difficulty of the experiments. Interestingly, the differences in the published results replicates a similar situation that arose almost 140 years ago (Jacobs 1857), and which seems to have repeated itself every few decades since then.

As discussed at length in section 4, determinations of G are fraught with difficulty because of the universality of the gravitational force, its weakness compared to the other fundamental interactions and the sensitive nature of the apparatus used to make the measurements. Nothing can be done about the first two of these problems, but there are many efforts now aimed at achieving improved design of the instrumentation systems employed in the measurement of G. These include many attempts at refinement of the torsion balance, painstaking searches for sources of systematic error in that class of instruments and the development of completely new techniques of measurement, possibly including experiments in space, all in the hopes of arriving at a value of G with uncertainty approaching 1 ppm. The quest for improved instrumentation has recently shed much light on the torsion balance, and several new insights into its operation have been gained, for example, the observations that anelasticity of the torsion fibre can affect the value of G (Kuroda 1995). While such effects can explain part of the spread between some of the measurements, much additional work must still be done, not only to resolve the large differences that exist between the most recent results, but also to help interpret any internal inconsistencies within the data from a particular experiment. A recent example of an effort toward the latter end is the work of Vladimirsky (1995), who noted that the apparent temporal changes in the values of G obtained by Izmaylov *et al* (1993) could be correlated with periods of minimum solar activity. He also noted that solar flare activity could be correlated with the dispersion of the measurements. Both of these points led him to suggest that there may be a 'magneto-plasticity' in the suspension fibres which could be affected by background fields at extra-low frequencies. There is also the suggestion by Aspden (1982) that the absorption of thermal radiation by a conductive housing used to electrically shield a sensitive apparatus (e.g., a torsion balance) may lead to charge induction effects that could have a non-negligible impact on determinations of G.

Perhaps it may be necessary to re-investigate some long-held fundamental principle that affects the way in which the data from a measurement of G are interpreted. Horák (1984), for instance, developed a generalized form of Archimedes' principle which by his claim gave a more accurate prediction of the size of the gravitational force acting on a particle due to a mass immersed in a symmetric volume of fluid. Since the large masses in virtually all measurements of G are resting in air during the course of the experiment, he suggested that the correction he derived should lead to a 'vacuum' value of G somewhere between 10^{-5} to 10^{-4} larger than the published laboratory values. Vybíral (1987) built a dynamic-mode torsion balance to study this effect, and made measurements of the balance's oscillation period with the large masses in air and then in water. His data suggested that the type of buoyancy correction discussed by Horák is valid and he concluded that the correction should be applied whenever G is measured using an apparatus in which all of the interacting masses are not simultaneously in the same medium (e.g., all of them in vacuum or in a chamber at atmospheric pressure). While there is not yet any consensus regarding the validity of any of these conjectures, their analyses, like that of Kuroda, are representative of the struggle involved in resolving the instrumentational issues associated with the measurement of G.

Of course, all of this begs the question of establishing what the absolute value of G actually is. In the absence of a *bona fide* theoretical prediction, and with the experimental results exhibiting the scatter that they do, the question becomes largely one of deciding on an algorithm for weighting (if appropriate) and averaging a set of existing measurements that satisfy suitable selection criteria. This task is handled by those who carry out the least-squares adjustment of the values of the physical constants, a process that is presently underway, with publication of the recommended values (under the auspices of CODATA and allied institutions and organizations) scheduled for 1997. It is likely that the evaluation of G will include the results from all of the recent high-precision experiments that are well documented and that have produced data of demonstrable self-consistency, with the question being one of deciding how far back in time to extend the window of inclusion. A related issue is the assignment of an uncertainty to the absolute value of G. While there are well known statistical techniques for combining uncertainties to arrive at a composite value, it may necessarily be the case that a completely different approach be followed here. This was what happened in the previous evaluation (Cohen and Taylor 1987), where the absolute value of G was taken to be essentially that of Luther and Towler (1982), with the size of the uncertainty being simply double the amount reported by Luther and Towler, a straightforward method of accounting for the lack of agreement with the two other measurements considered to be competitive at the time. Without a clear, confirmed relationship between G and any other fundamental constant of nature or measured physical quantity, there is very little else that can be done.

The situation with respect to searches for variations of G is far less ambiguous: within the weak field limit (and to some extent for matter at high densities), there are no confirmed laboratory experiments or astronomical observations that demonstrate unequivocally that G varies in either space or time, that its value depends on the temperature or any other physical property of the test masses, that G is modified by the presence of matter placed between gravitationally interacting test masses, or that any other anomalous phenomenon indeed exists, at least at presently achievable levels of measurement sensitivity and stability. This is not to say that exciting discoveries relating to gravity will never be found. Lunar laser ranging may eventually be able to detect spatial anisotropies of G predicted by metric theories of gravity to occur at a level of $\Delta G/G \approx 5\times10^{-12}$ (Nordtvedt 1996). Furthermore, the question of the behaviour of antimatter in gravitational fields is presently being addressed (Nieto and Goldman 1991) and the search continues for a spin-component in the gravitational force (see Ritter *et al* (1993), for one example). Finally, although the many different searches for a 'fifth force' in nature did not reveal the presence any new, long-range interactions as conceived originally, that episode serves to remind that care must be taken to not overlook the possible presence of any experimental signals that might reveal new physics.

G is indeed a mysterious constant, seemingly eternal and unchanging, but veiled from view by our inability to seize it. A century from now, will we be any closer?

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