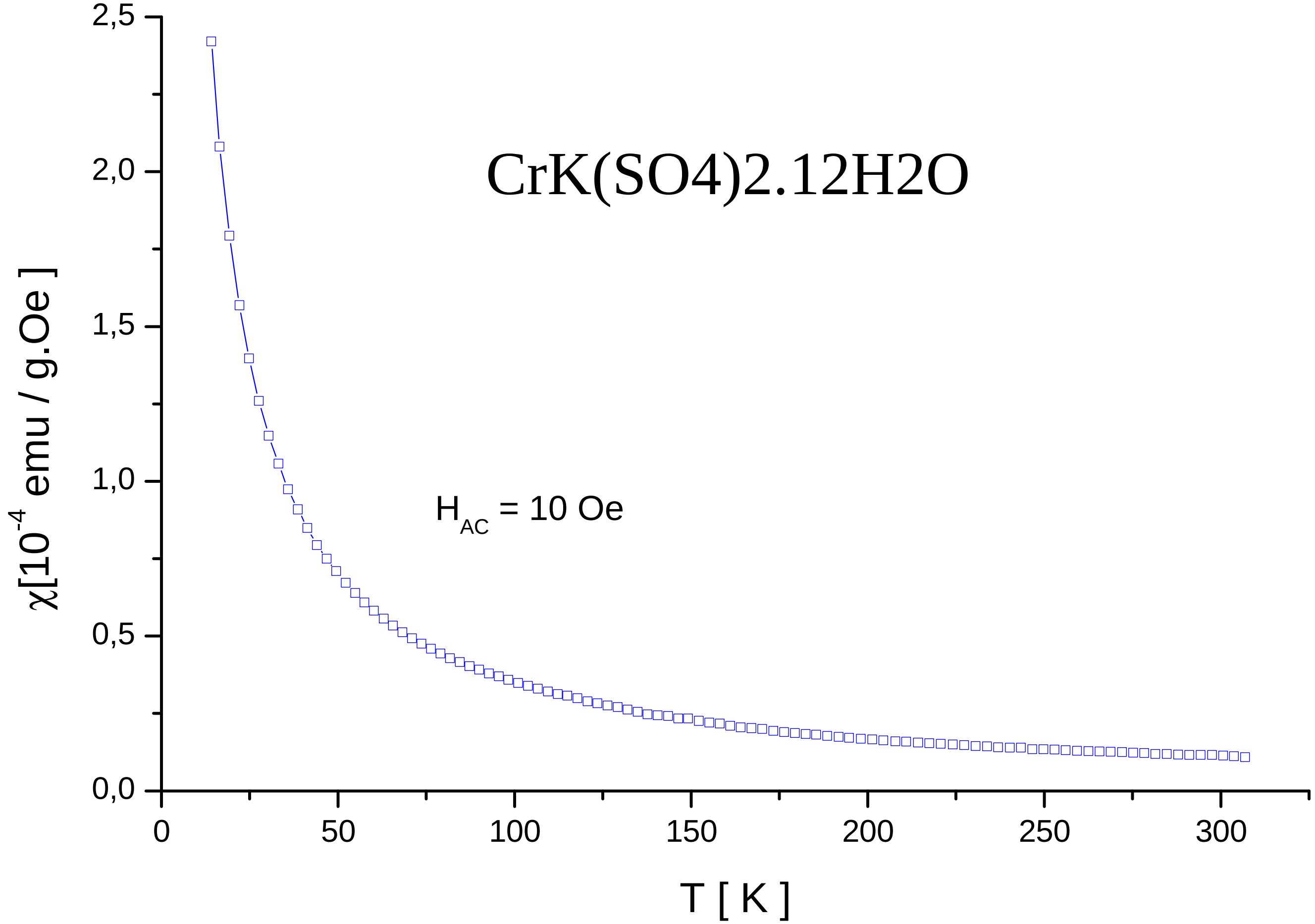
Ejercicio 6


CrK(SO4)2.12H2O
H_{AC} = 10 Oe
χ[10^{-4} emu / g.Oe]
T [K]
0,0
0,5
1,0
1,5
2,0
2,5
0
50
100
150
200
250
300

CrK(SO4)2.12H2O

$$\chi_p = M / H \approx \frac{N_{Cr}\mu_0\mu_B^2 g^2 s(s+1)}{3kT} = \frac{N_{Cr}\mu_0\mu_B^2 g^2}{3k} s(s+1)\frac{1}{T} = \frac{C}{T}$$

$$C = \frac{N_{Cr}\mu_0\mu_B^2 g^2}{3k} s(s+1) = 0{,}0202$$

$$\frac{N_{Cr}\mu_0\mu_B^2 g^2}{3k} = \frac{N_{Cr} \cdot 4\pi\times10^{-7} \cdot (9.27\times10^{-24})^2 \cdot 4}{3\cdot1.38\times10^{-23}}$$

$$1m_{Cr} = 51.9961\text{g/mol}$$

$$1\mathrm{m}_{\mathrm{K}} = 39.0983\text{g/mol}$$

$$2\mathrm{m}_{\mathrm{S}} = 2\times32.065\text{g/mol}$$

$$20\mathrm{m}_{\mathrm{O}} = 20\times15.9994\text{g/mol}$$

$$24\mathrm{m}_{\mathrm{H}} = 24\times1.00794\text{g/mol}$$

$$m_{\mathrm{CrK(SO4)2.12H2O}} = 499{,}40296\text{g/mol}$$

$$v_{\mathrm{CrK(SO4)2.12H2O}} = 272.898cm^3\text{/mol}$$

$$v_{\mathrm{CrK(SO4)2.12H2O}} = 2{,}729\times10^{-4}m^3\text{/mol}$$

$$N_{\mathrm{Cr}} = 6.02\times10^{23}\text{át/}2{,}729\times10^{-4}m^3 = 2.206\times10^{27}\,át/m^3$$

$$\frac{N_{Cr}\mu_0\mu_B^2 g^2}{3k} = \frac{2.206\times10^{27}\cdot 4\pi\times10^{-7}\cdot(9.27\times10^{-24})^2\cdot 4}{3\cdot 1.38\times10^{-23}} = 0.023$$

$$s(s+1) = \frac{0,0202}{0.023} = 0.876$$

$$s^2 + s - 0.876 = 0$$

$$s = \frac{-1\pm\sqrt{1+3.506}}{2} = \begin{cases} 0.561 \\ -1.561 \end{cases}$$

$$\left[Cr^0\right] = \ldots 3d^4 4s^2$$

$$\left[Cr^{+2}\right] = \ldots 3d^4$$

$$\left[Cr^{+3}\right] = \ldots 3d^3 \Rightarrow s = 3/2$$

$$\left[Cr^{+4}\right] = \ldots 3d^2$$

$$\left[Cr^{+5}\right] = \ldots 3d^1 \Rightarrow s = 1/2$$

$$\boxed{B_{1/2}(x) = \tanh(x)}$$

$$B_s(x) = \frac{2s+1}{2s}\coth\left(\frac{2s+1}{2s}x\right) - \frac{1}{2s}\coth\left(\frac{x}{2s}\right)$$

$$B_{1/2}(x) = 2\coth(2x) - \coth(x)$$

$$B_{1/2}(x) = 2\frac{e^{2x}+e^{-2x}}{e^{2x}-e^{-2x}} - \frac{e^{x}+e^{-x}}{e^{x}-e^{-x}}$$

$$B_{1/2}(x) = \frac{1}{e^{x}-e^{-x}}\left[2\frac{e^{2x}+e^{-2x}}{e^{x}+e^{-x}} - \left(e^{x}+e^{-x}\right)\right]$$

$$B_{1/2}(x) = \frac{1}{e^{x}-e^{-x}}\left[\frac{2e^{2x}+2e^{-2x}-\left(e^{x}+e^{-x}\right)^2}{e^{x}+e^{-x}}\right]$$

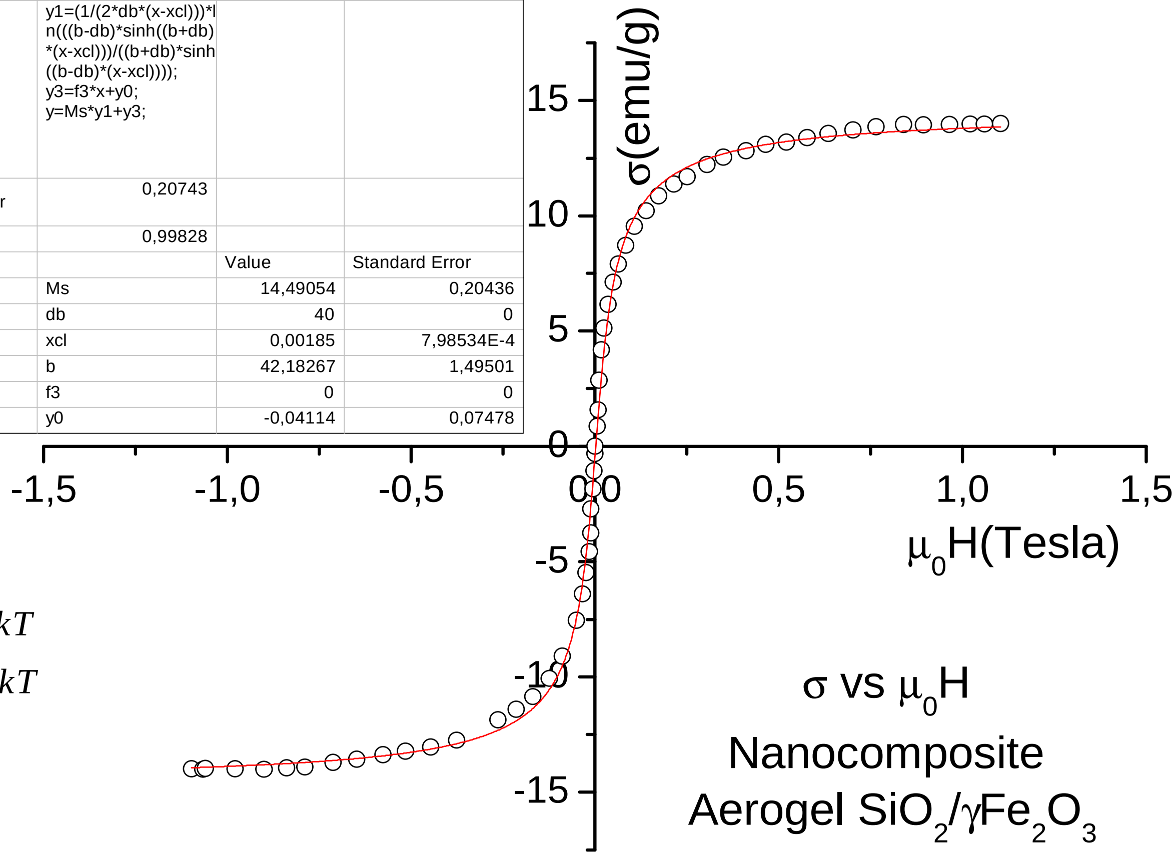
$$B_{1/2}(x) = \frac{1}{e^{x}-e^{-x}}\left[\frac{e^{2x}-2+e^{-2x}}{e^{x}+e^{-x}}\right]$$

$$B_{1/2}(x) = \frac{1}{e^{x}-e^{-x}}\frac{\left(e^{x}-e^{-x}\right)^2}{e^{x}+e^{-x}}$$

$$B_{1/2}(x) = \frac{e^{x}-e^{-x^{2}}}{e^{x}+e^{-x}} = \tanh(x)$$

Ejercicio 9

Model	LangevinDistr (User)		
Equation	y1=(1/(2*db*(x-xcl)))*ln(((b-db)*sinh((b+db)*(x-xcl)))/((b+db)*sinh((b-db)*(x-xcl)))); y3=f3*x+y0; y=Ms*y1+y3;		
Reduced Chi-Sqr	0,20743		
Adj. R-Square	0,99828		
		Value	Standard Error
Data3_Memug	Ms	14,49054	0,20436
Data3_Memug	db	40	0
Data3_Memug	xcl	0,00185	7,98534E-4
Data3_Memug	b	42,18267	1,49501
Data3_Memug	f3	0	0
Data3_Memug	y0	-0,04114	0,07478



$$x = B, H$$

$$b = \mu_g / kT$$

$$db = \sigma / kT$$

Análisis de M vs H con Langevin distribuida

Para una muestra monodispersa

$$M = N\mu L(\alpha\mu) \quad N = \frac{part.}{masa}; \quad L \equiv Langevin; \quad \alpha = \frac{B}{kT}$$

Para una muestra polidispersa

$$M = \frac{N\int_0^\infty \mu L(\alpha\mu)P(\mu)d\mu}{\int_0^\infty P(\mu)d\mu}; \quad P(\mu) \equiv distribución\ de\ momentos$$

Momento medio

$$\langle\mu\rangle = \frac{\int_0^\infty \mu P(\mu)d\mu}{\int_0^\infty P(\mu)d\mu} \Rightarrow \int_0^\infty P(\mu)d\mu = \frac{1}{\langle\mu\rangle}\int_0^\infty \mu P(\mu)d\mu$$

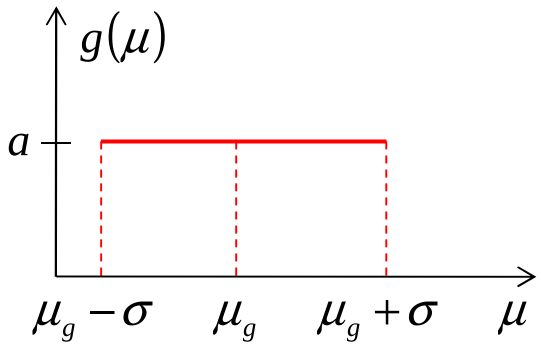
$$\Downarrow$$

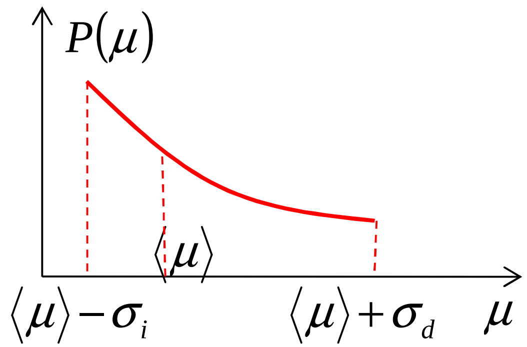
$$M = \frac{N\langle\mu\rangle\int_0^\infty \mu L(\alpha\mu)P(\mu)d\mu}{\int_0^\infty \mu P(\mu)d\mu}$$

$$Llamando\ g(\mu) = \mu P(\mu)$$

$$M = \frac{N\langle\mu\rangle \int_0^\infty L(\alpha\mu) g(\mu) d\mu}{\int_0^\infty g(\mu) d\mu} \quad (1)$$

Para poder resolver analíticamente:





$$\int_0^\infty P(\mu) d\mu = a \int_{\mu_1}^{\mu_2} \frac{d\mu}{\mu} = a \ln\left(\frac{\mu_2}{\mu_1}\right)$$

$$\int_0^\infty \mu P(\mu) d\mu = \int_0^\infty g(\mu) d\mu = a \int_{\mu_1}^{\mu_2} d\mu = 2a\sigma$$

$$\langle\mu\rangle = \frac{\int_0^\infty \mu P(\mu) d\mu}{\int_0^\infty P(\mu) d\mu} = \frac{2\sigma}{\ln\left(\frac{\mu_g + \sigma}{\mu_g - \sigma}\right)}$$

La resolución de (1) es

$$M = \frac{N\langle\mu\rangle}{2\sigma\alpha}\ln\left[\frac{(\mu_g-\sigma)\sinh\{(\mu_g+\sigma)\alpha\}}{(\mu_g+\sigma)\sinh\{(\mu_g-\sigma)\alpha\}}\right] \qquad N\langle\mu\rangle = M_s$$

A la expresión anterior le agregamos una componente ferro para tener en cuenta la coercitividad. En un próximo intento trataremos con dos Langevin. Se incluyó y_0 a fin de tener en cuenta un eventual desplazamiento vertical. La coercititividad xcl se fijó en cero.

```
y = Ms*(((1-f2)/(2*db*(x-xcl)))*ln(((b-db)*sinh((b+db)*(x-xcl)))/((b+db)*
    sinh((b-db)*(x-xcl)))))+f2*(2/Pi)*atan((x/xcs-1)*tan(Pi*S/2)))+f3*x+y0
```

$$x = B, H \qquad xcs = H_C^{FM}$$

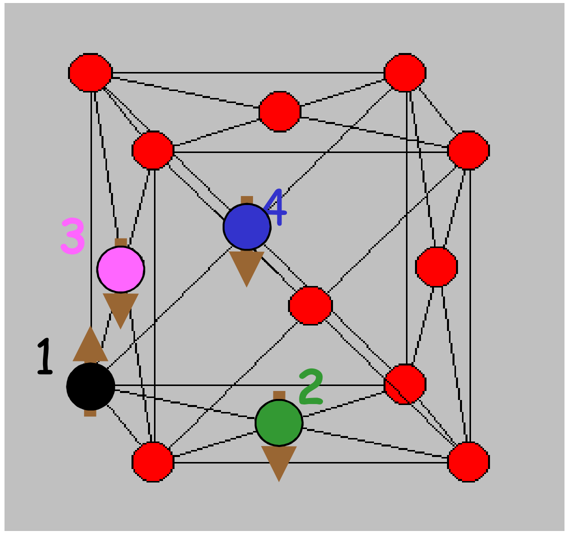
$$b = \mu_g / kT \qquad S = M_R / M_S^{FM} \qquad M_S = N_m^{SP}\langle\mu\rangle + M_S^{FM}$$

$$db = \sigma / kT \qquad f_3 = masa \cdot \chi_{PM}$$

Intercambio (c)

Ferro y Antiferromagnetismo. Teoría del campo molecular

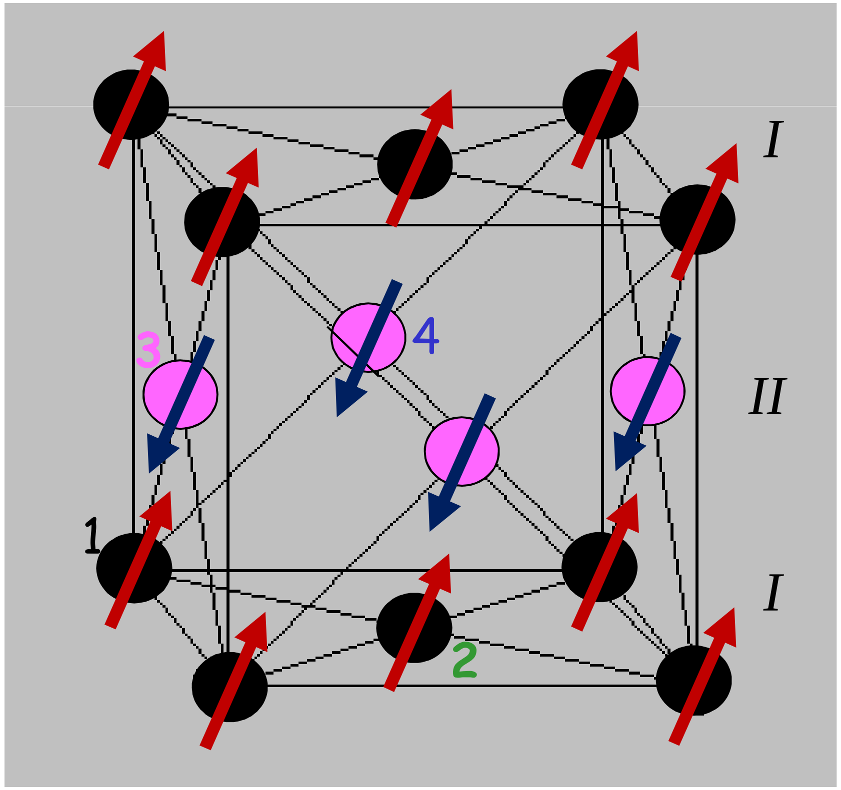
Red fcc



AF: Primeros vecinos con momentos antiparalelos a "1"

Frustración: "2" y "4" no son antiparalelos

Cómo establecer el orden antiferro ?



Se subdivide la red en 4 subredes, 1, 2, 3, 4

1, puntos (k, l, m)
2, puntos (k+1/2, l+1/2, m)
} *I*

3, puntos (k+1/2, l, m +1/2)
4, puntos (k, l +1/2, m +1/2)
} *II*

2 subredes *I*, *II*

Red fcc

$$C = \frac{4Ns(s+1)\mu_0 g^2 \mu_B^2}{3k}$$

$$\chi_{ini} = \frac{C}{(T+\Theta)}$$

$$\Theta_{afm} = -\frac{8s(s+1)J}{k}$$

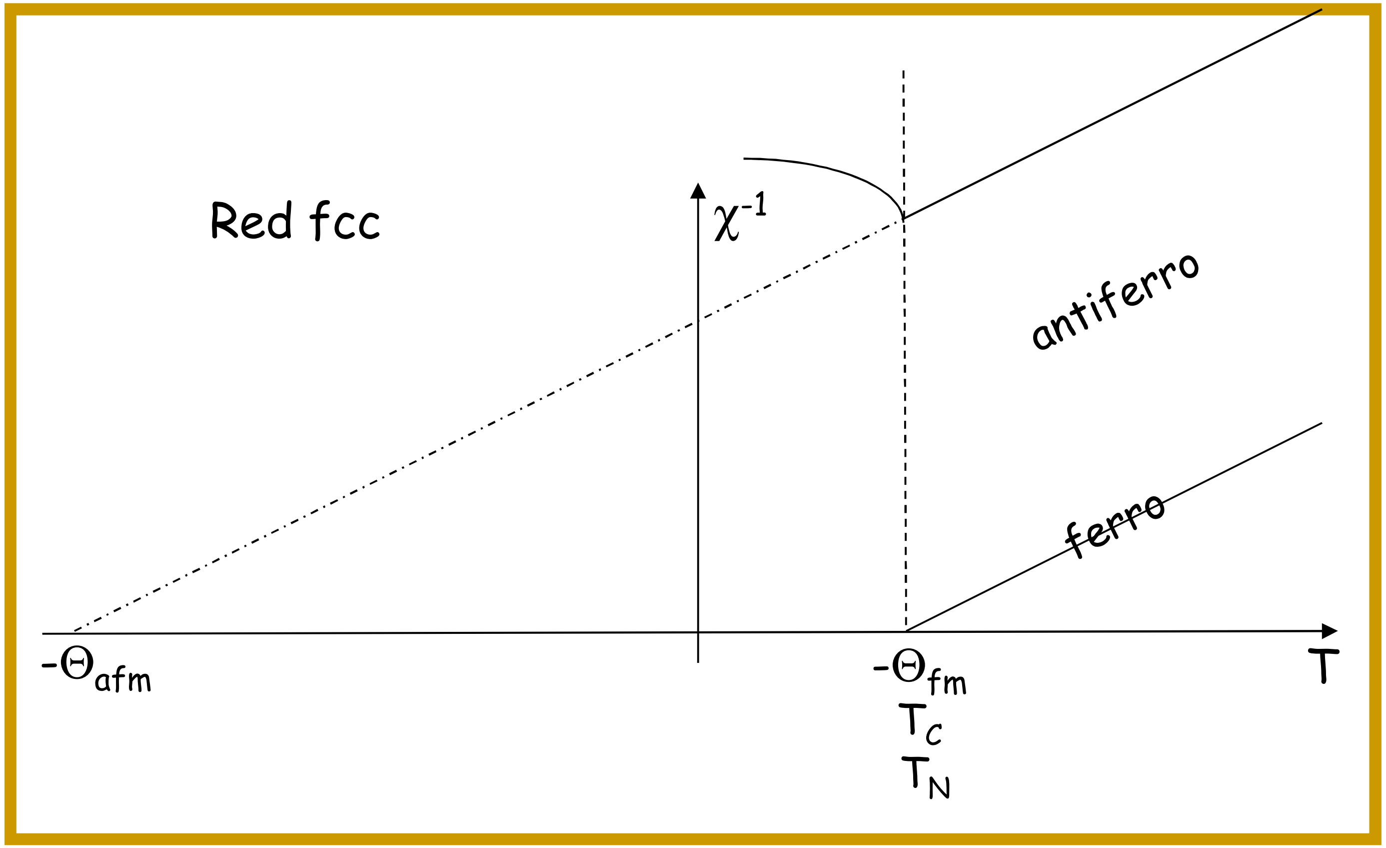
$$\Theta_{fm} = -\frac{8s(s+1)J}{3k}$$

J > 0 ⇨ $T_C = \frac{8s(s+1)J}{3k} > 0$

J < 0 ⇨ $T_N = -\frac{8s(s+1)J}{3k} > 0$

Red fcc

Resumen. Temperaturas características



$$\chi_{ini} = \frac{C}{(T+\Theta)}$$

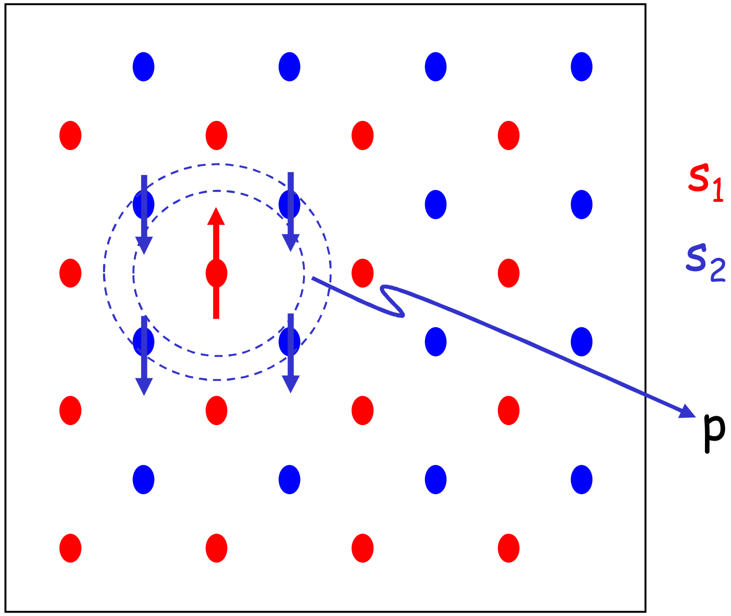
$$\Theta_{afm} = -\frac{8s(s+1)J}{k}$$

$$\Theta_{fm} = -\frac{8s(s+1)J}{3k}$$

$$T_C = \frac{8s(s+1)J}{3k}$$

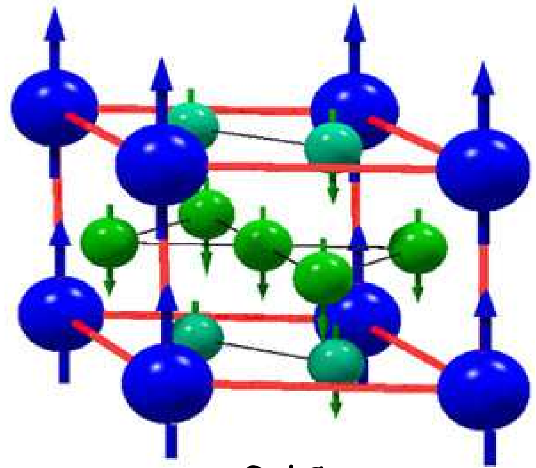
$$T_N = -\frac{8s(s+1)J}{3k}$$

Ferrimagnetismo. Teoría del campo molecular



$s_1 \neq s_2$

Ferrimagneto: momento diferente en cada subred



$GdCo_5$

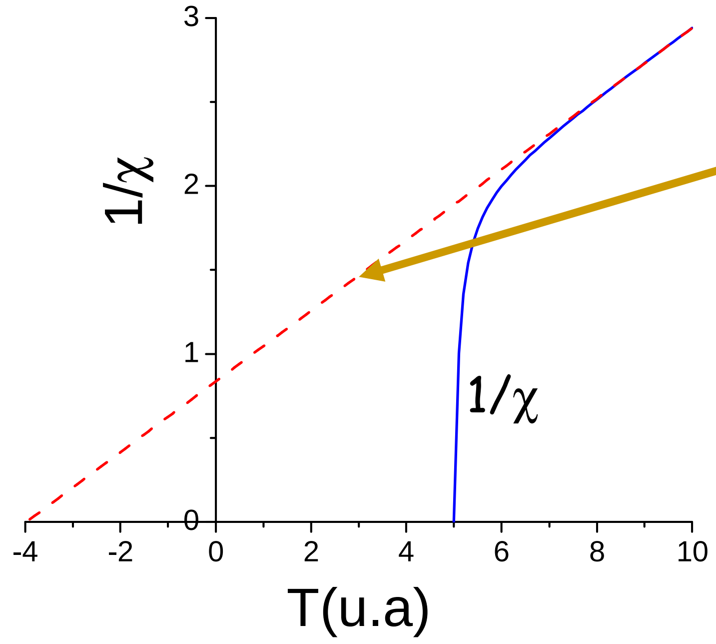
subredes diferentes

Ferrimagnetismo. Teoría del campo molecular

Procedimientos similares a los usados en los casos ferro y antiferro:

$$\chi_{ini} = \frac{(C_1 + C_2)T - 2\sqrt{C_1 C_2 \Theta_1 \Theta_2}}{T^2 - \Theta_1 \Theta_2} = \frac{(C_1 + C_2)T - 2\sqrt{C_1 C_2} T_N}{T^2 - T_N^2}$$

$$C_i = \frac{N s_i (s_i + 1) \mu_0 g_i^2 \mu_B^2}{3k} \qquad \Theta_i = \frac{2 p s_i (s_i + 1) J}{3k} \qquad T_N = \sqrt{\Theta_1 \Theta_2}$$



Para T >> T_N

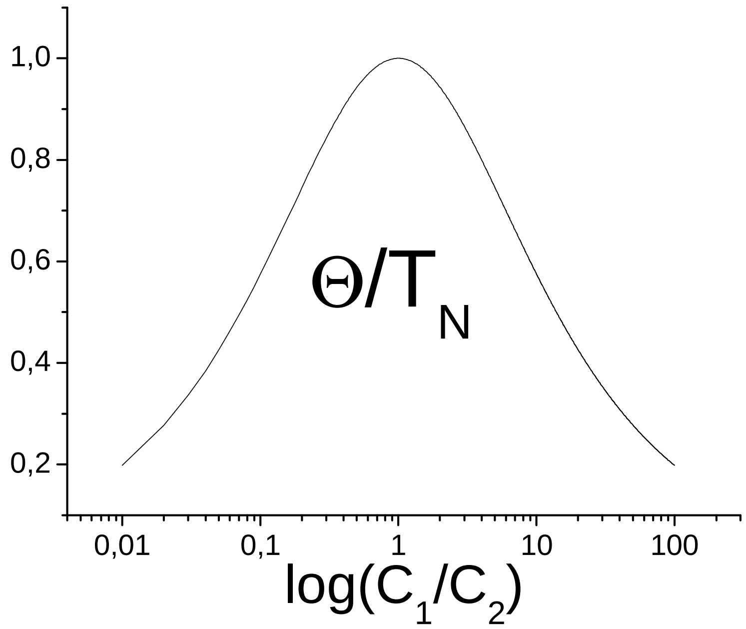
$$\chi_{ini}^{-1} \approx \frac{T + \Theta}{C_1 + C_2} \qquad \Theta = \frac{2\sqrt{C_1 C_2}}{C_1 + C_2} T_N$$

La relación entre Θ y T_N depende de la relación entre C_1 y C_2:

$$\frac{\Theta}{T_N} = \frac{2\sqrt{C_1 C_2}}{C_1 + C_2}$$

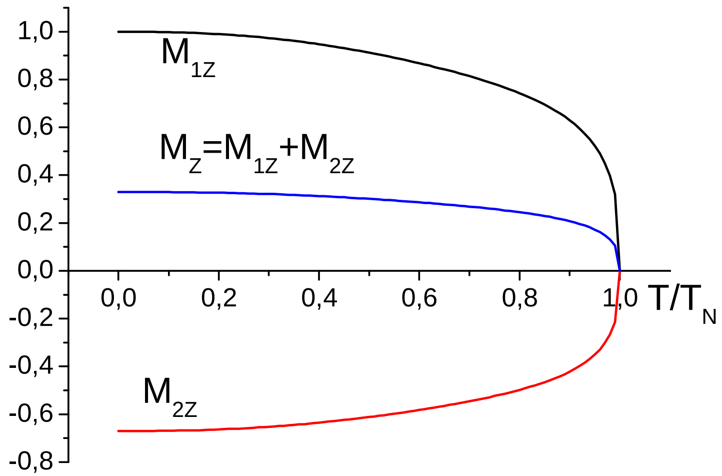
$\Theta \leq T_N$

$\Theta = T_N$ sólo si $C_1 = C_2 \Rightarrow$ antiferro

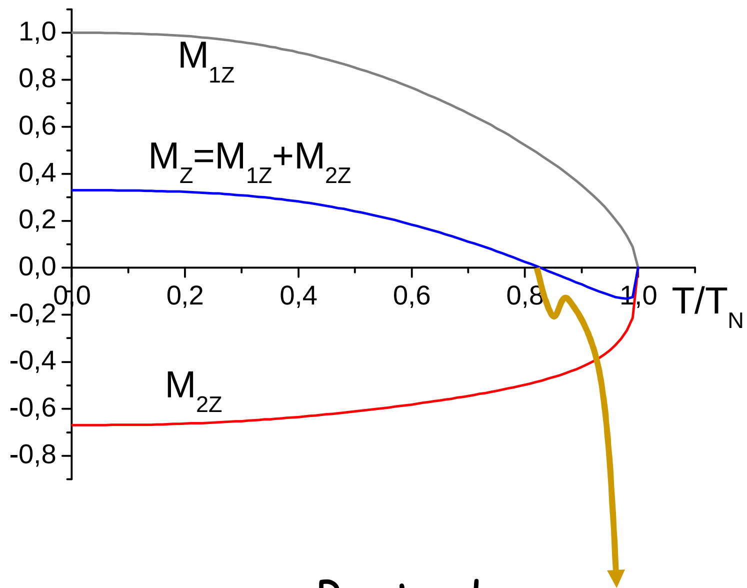


Magnetización de un ferrimagneto

Sub-redes con comportamiento térmico similar

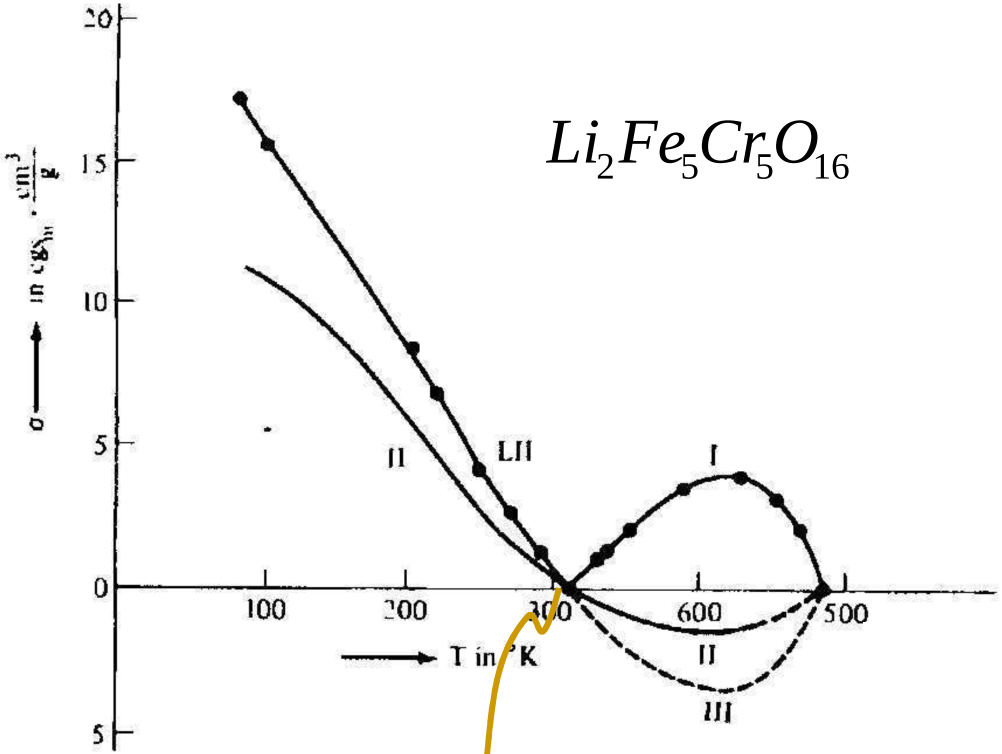


Sub-redes con diferente comportamiento térmico



Punto de compensación

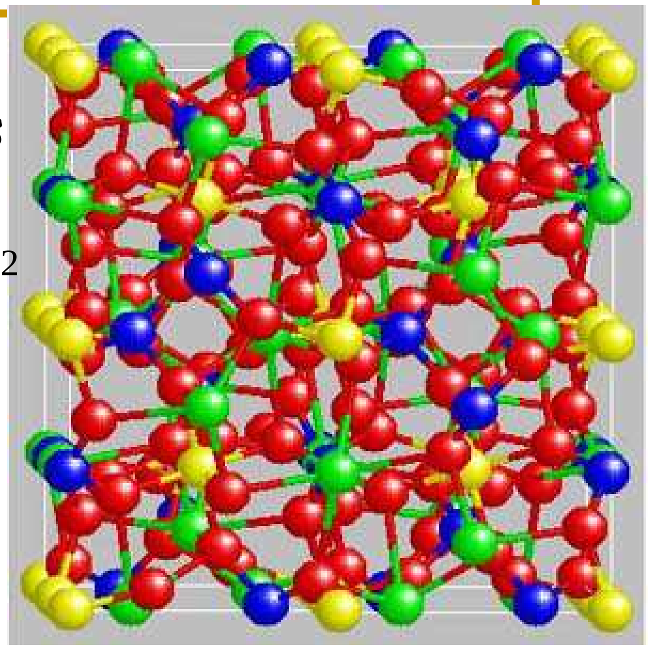
Ferrimagnetos reales



$Li_2Fe_5Cr_5O_{16}$

garnets

$Y_3Fe_5O_{12}$



garnets de tierras raras

$$5Fe_2O_3 - 3M_2O_3 = 2M_3Fe_5O_{12}$$

Gd, Tb, Dy, Ho

Er, Tm, Yb, Lu, Y

Punto de compensación

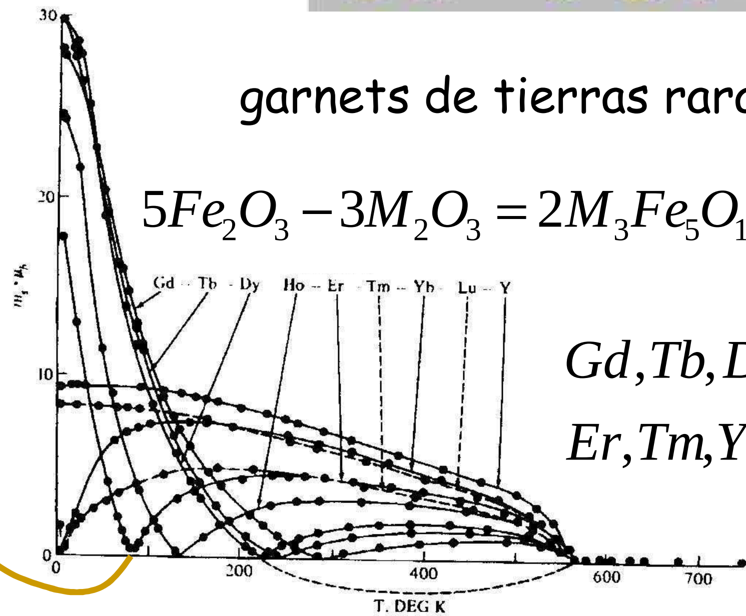
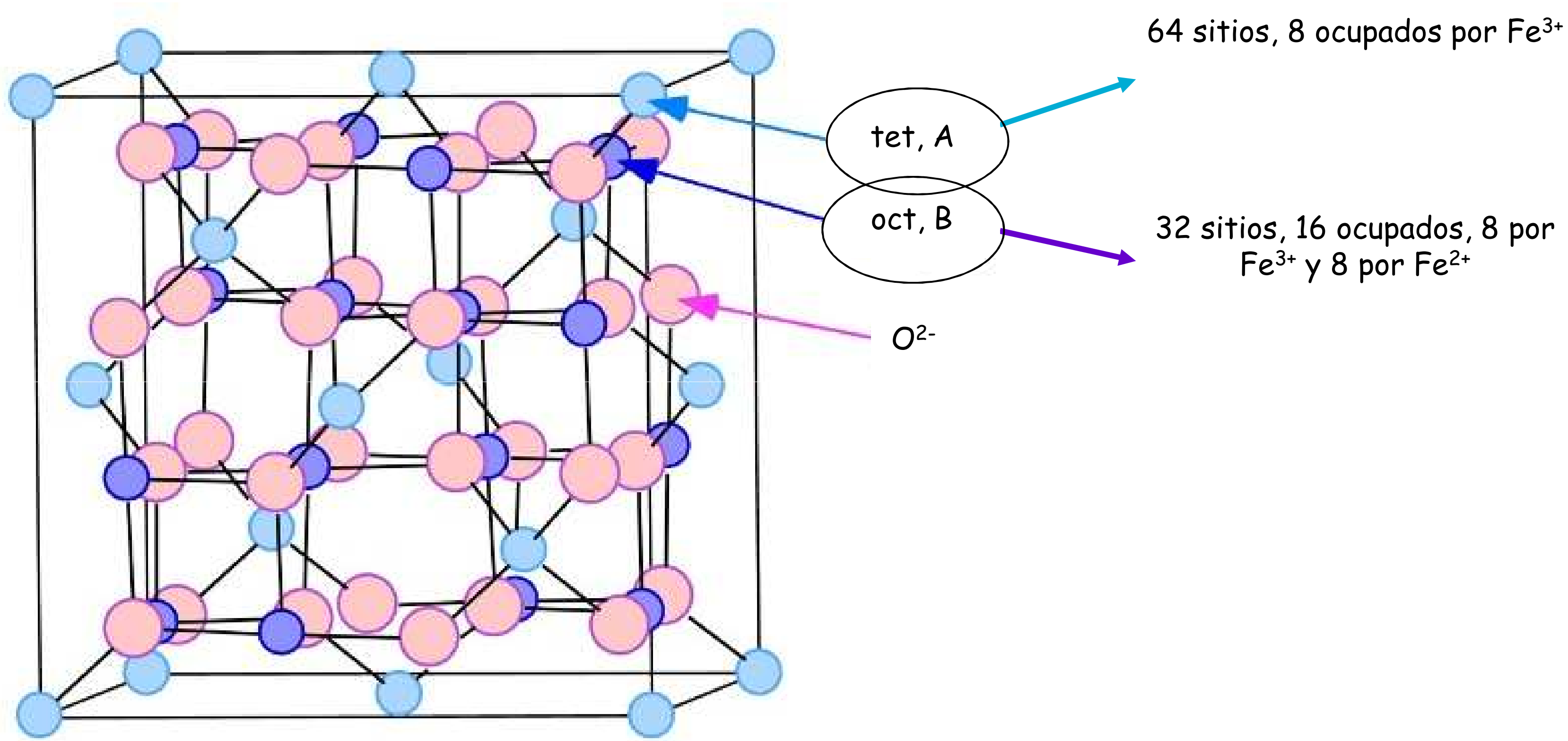


Figure 11.13-Temperature dependencies of the magnetic moments of the rare earth garnets. Note the occurrence of a compensation point in several of the curves. From Bertaut F., and Pauthenet,R., Proc. IEE, 104, Suppl.#5, 261,(1957)

Ferrimagnetos reales

Magnetita

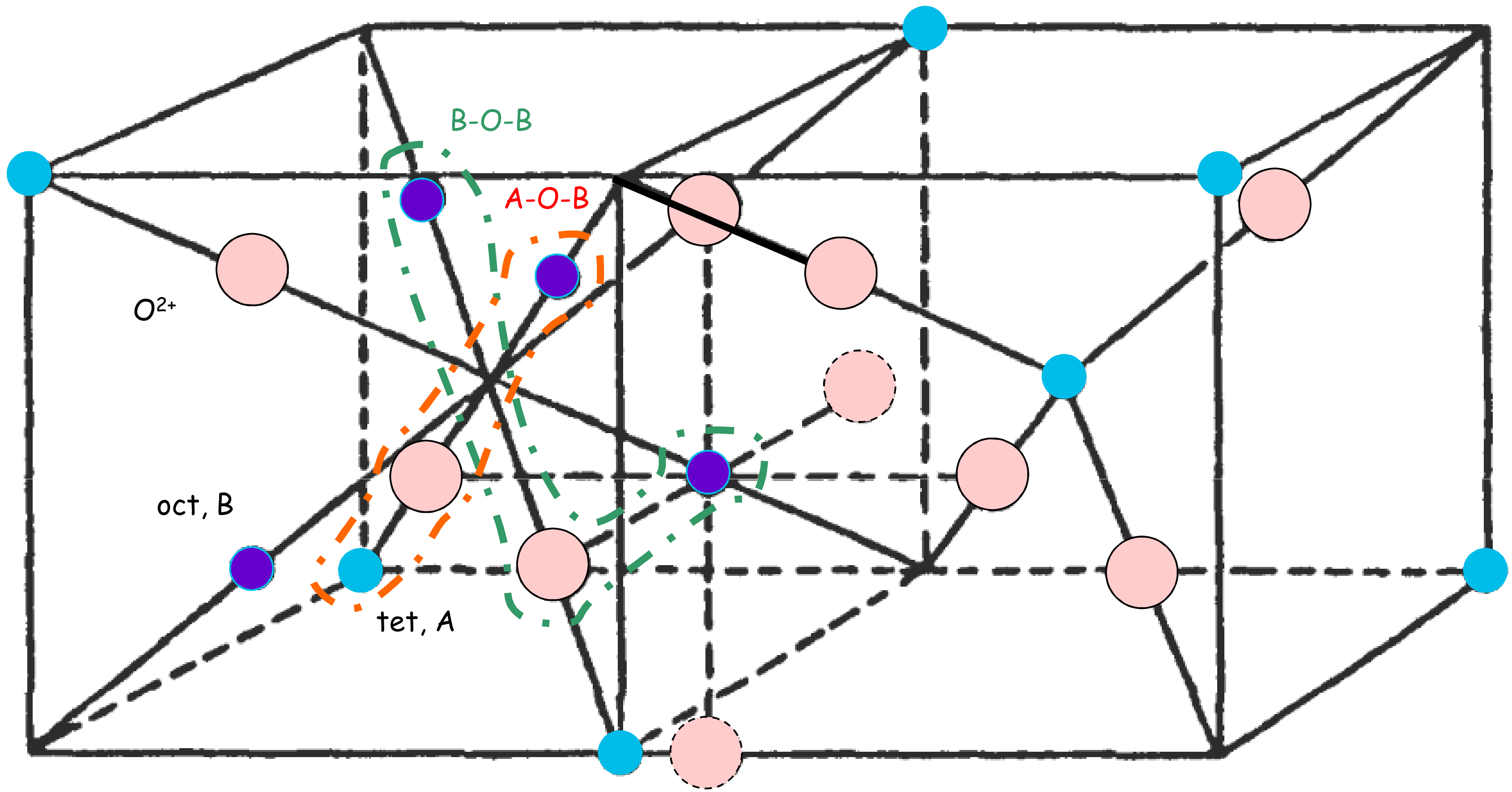


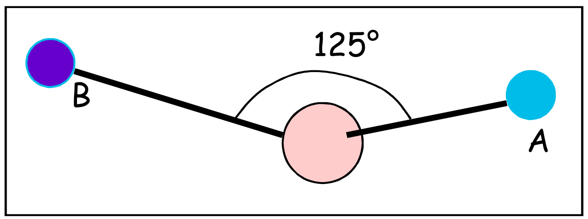
$$Fe_3O_4 = Fe^{2+}O.Fe_2^{3+}O_3$$

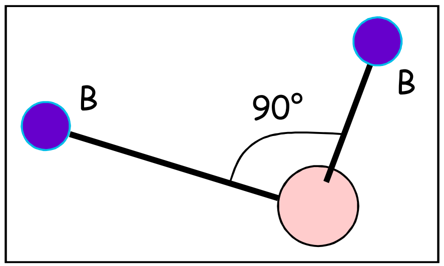
$$\langle \vec{m} \rangle_{Fe_3O_4} \approx m_{Fe^{3+}}(\Uparrow) + m_{Fe^{3+}}(\Downarrow) + m_{Fe^{2+}}(\uparrow) \approx m_{Fe^{2+}}(\uparrow) = 4\mu_B \vec{u}_z$$

Intercambio y superintercambio

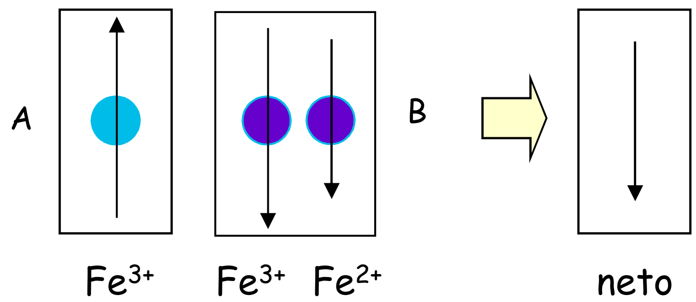
Magnetita







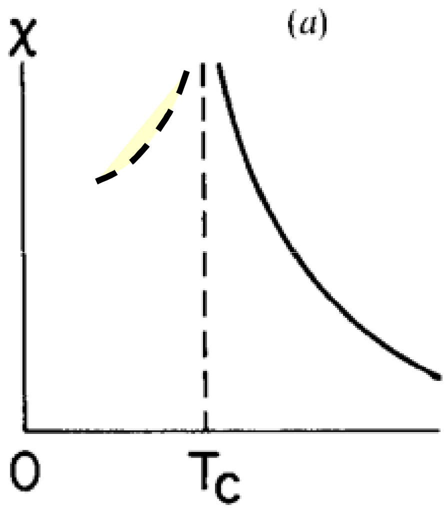
$$|J|_{AB} >> |J|_{BB}$$



Máximos de χ en T_C, T_N

$$\chi_{inic} = \frac{C}{T + \Theta}$$

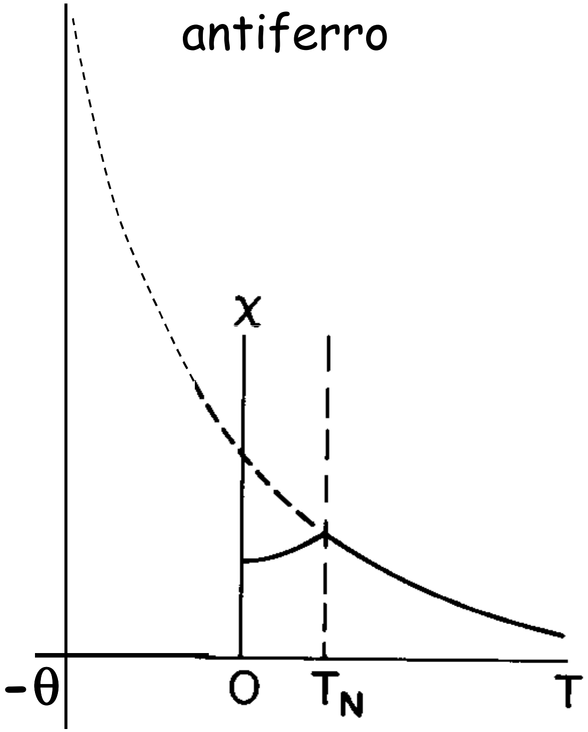
ferro



$$\chi = \frac{C}{T - T_C}$$

Curie-Weiss Law
$(T > T_C)$

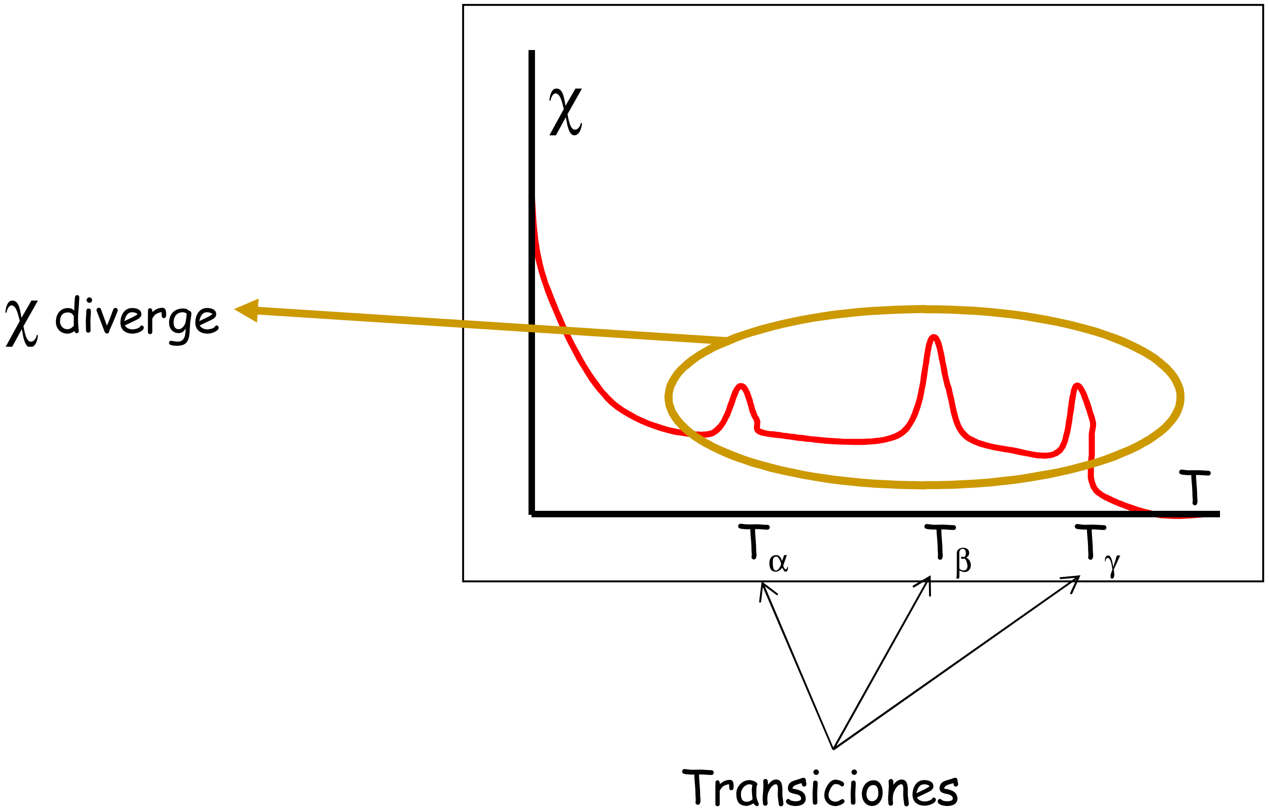
antiferro



$$\chi = \frac{C}{T + \theta}$$

$(T > T_N)$

Comportamiento de χ en puntos de transición de fase magnética



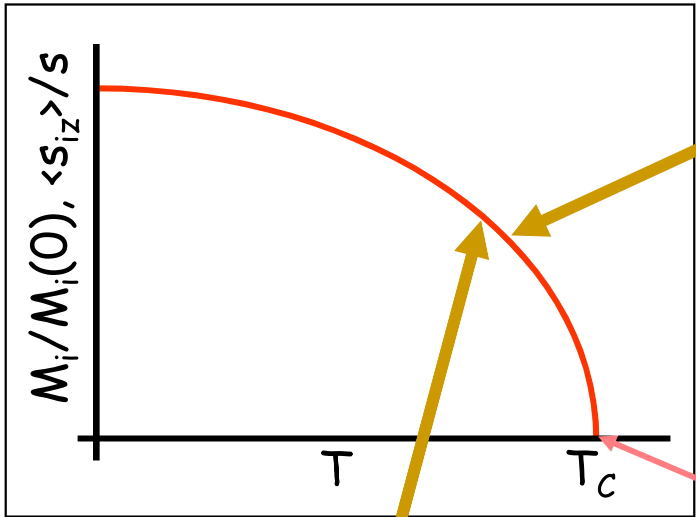
Valores medidos de m=gs(μ_B) en diferentes materiales

Tabla 3.2. Cristales ferro-ferrimagnéticos

Sustancia	Imanación de saturación $M_s/10^7$ A/m Temp. amb.	Imanación de saturación $M_s/10^7$ A/m 0 K	n_B(0 K) por molécula	Temperatura de Curie en K
Fe	0.1707	0,174	2,22	1043
Co	0,14	0,1446	1,72	1368
Ni	0,0485	0,051	0,606	627
Gd	-	0,206	7,63	292
Dy	-	0,292	10,2	88
MnAs	0,067	0,087	3,4	318
MnBi	0,062	0,068	3,52	630
MnSb	0,071	-	3,5	587
CrO_2	0,0515	-	2,03	386
$MnOFe_2O_3$	0,041	-	5,0	573
$FeOFe_2O_3$	0,048	-	4,1	858
$NiOFe_2O_3$	0,027	-	2,4	858
$CuOFe_2O_3$	0,0135	-	1,3	728
$MgOFe_2O_3$	0,011	-	1,1	713
EuO	-	0,192	6,8	69
$Y_3Fe_5O_{12}$	0,013	0,02	5,0	560

<gs>

Expresiones empíricas para $\langle s_z \rangle$; Comportamiento crítico



Solución aproximada de la ecuación trascendente - Ferromagnetos

$$\frac{\langle s_z \rangle}{s} = B_s\left(\frac{s}{kT}\left[\mu_0 g \mu_B H + 2pJ\langle s_z \rangle\right]\right)$$

$$t = \frac{T}{T_C} \xrightarrow{T<T_C} 1$$

$$\frac{\langle s_z \rangle}{s} = (1-t)^{\beta}$$

Comportamiento crítico

$$\beta = 3/2 \quad (3d)$$

$$\frac{\langle s_z \rangle}{s} = \frac{(1-t)^{\beta'}}{1-\beta' t + At^3 - Ct^{7/2}}$$

Expresión empírica de Aharoni

Introduction to the Theory of Ferromagnetism, Amikami Aharoni, Oxford Science Publications, 1998.

Órdenes de magnitud: Intercambio - Dipolar

Interación Dipolar

$$J_{dip}, T_{Cr\ dip}\ ?$$

$$U_{dip} = \frac{\mu_0}{4\pi r^3}\left[\vec{m}_1 \cdot \vec{m}_2 - 3(\vec{m}_1 \cdot \hat{r})(\vec{m}_2 \cdot \hat{r})\right]$$

$$U_{dip} \approx \frac{\mu_0}{4\pi r^3} m_1 m_2$$

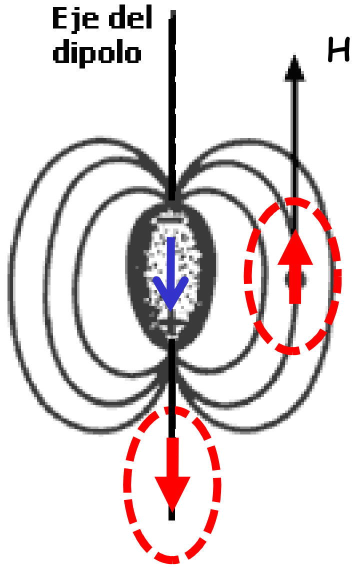
$$m_1 \approx m_2 \approx \mu_B \approx 10^{-23}\, Am^2$$

$$r \approx 0.2nm = 2\times 10^{-10}\, m$$

$$U_{dip} \approx \frac{4\pi\times 10^{-7}}{4\pi\times 8\times 10^{-30}} \times 10^{-46}\, Joules = 1.25\times 10^{-24}\, Joules = 0.78\times 10^{-5}\, eV$$

$$U_{exch} \approx 2J_{exch} = 10^{-21}\, Joules = 0.63\times 10^{-2}\, eV$$

$$\frac{(T_{Cr})_{dip}}{(T_{Cr})_{exch}} \approx \frac{U_{dip}}{U_{exch}} \approx 10^{-3} \xrightarrow{T_{exch}<10^3 K} (T_{Cr})_{dip} < 1K$$

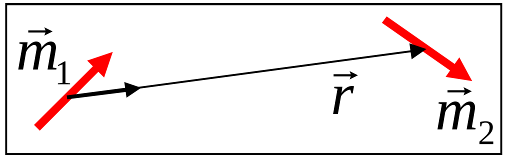


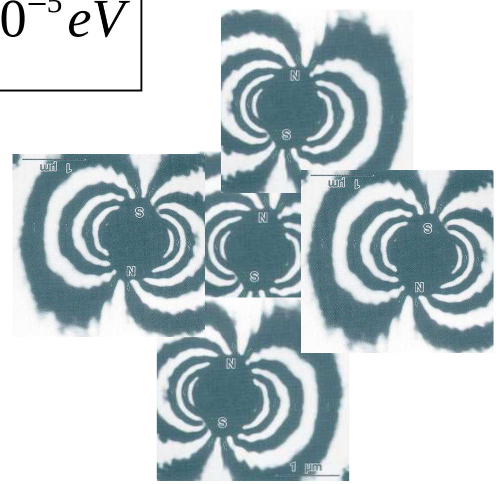
Intercambio

$$J_{exch} \approx 10^{-21}\, Joule$$

$$T_{Cr\ exch} < 10^3\, K$$

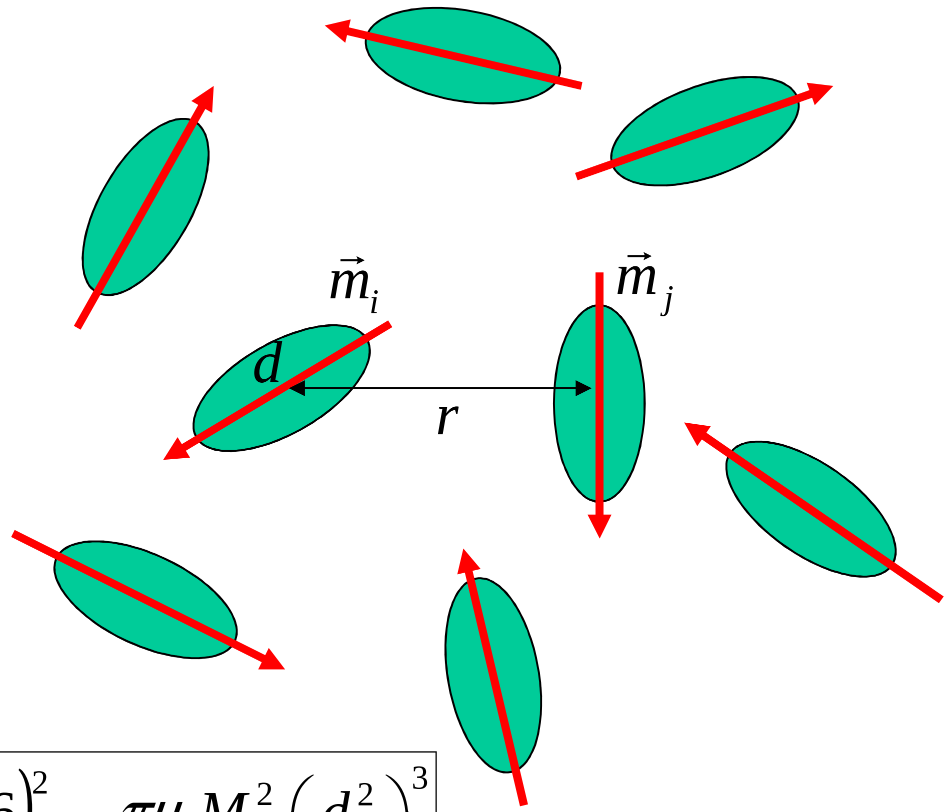
m_1, m_2





Órdenes de magnitud: Dipolar

Cómo depende $U_{dipolar}$ de $\langle r \rangle$ y $\langle d \rangle$?



$$\left|U_{dip}\right| \approx \frac{\mu_0}{4\pi r^3} m_i m_j$$

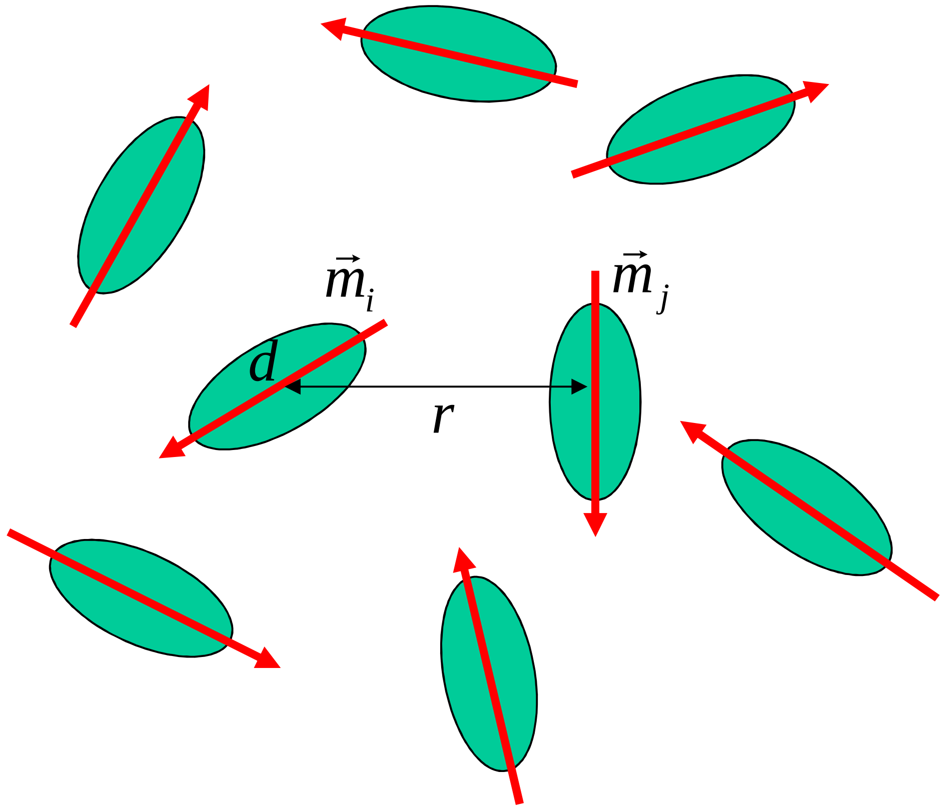
$$\left|U_{dip}\right| \approx \frac{\mu_0 m_i m_j}{4\pi r^3} = \frac{\mu_0 m^2}{4\pi r^3} = \frac{\mu_0 \left(M_S \pi d^3/6\right)^2}{4\pi r^3} \approx \frac{\pi \mu_0 M_S^2}{144} \left(\frac{d^2}{r}\right)^3$$

Si d = r (partículas en contacto) $\Rightarrow$

$$\left|U_{dip}\right| \approx \frac{\pi \mu_0 M_S^2}{144} d^3$$

Órdenes de magnitud: Dipolar - Zeeman

Para qué relación $\langle r\rangle/\langle d\rangle$ será $U_{Dipolar} \ll U_{Zeeman}$?



$$\left|U_{dip}\right| \approx \frac{\mu_0}{4\pi r^3} m_i m_j$$

$$U_Z = -\mu_0 \vec{m}_i \cdot \vec{H} \Rightarrow \left|U_Z\right| \approx \mu_0 m_i H$$

$$\frac{\left|U_{dip}\right|}{\left|U_Z\right|} \approx \frac{\mu_0 m_i m_j}{4\pi r^3 \mu_0 m_i H} = \frac{m_j}{4\pi r^3 H} = \frac{M_S V_j}{4\pi r^3 H} \approx \frac{M_S 4\pi d^3}{3\times 4\pi r^3 H} = \frac{M_S}{3H}\left(\frac{d}{r}\right)^3$$

$$\frac{\left|U_{dip}\right|}{\left|U_Z\right|} << 1 \Rightarrow \frac{r}{d} >> \left(\frac{M_S}{3H}\right)^{1/3} \xrightarrow{M_S = 10^6\,Am^{-1},\, H = 10^4\,Am^{-1}} \frac{r}{d} >> 3.2$$